

Linear Equations

Then

In Chapter 1 you learned to simplify algebraic expressions.

Now/

In Chapter 2, you will:

- Solve equations by using the four basic operations.
- Solve equations by using multiple steps.
- Solve proportions.Use formulas to solve
- real-world problems.



HS-AT-S-EI3 Students will solve one-variable equations ... using manipulatives, symbols, procedures and graphing.

HS-NPO-S-PNO4 Students will justify the solution steps in simplifying expressions or solving an equation.

Why?

SHOPPING In recent years, the percent of change in sales per year at shopping malls in the U.S. averaged 5%. This change over a period of time is called the rate of change. On a graph, it is called the slope.

Math in Motion, Animation glencoe.com



KY Math Online

Online Option

Take a self-check Chapter Readiness Quiz at **glencoe.com**.

Get Started on Chapter 2

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 2. To get ready, identify important terms and organize your resources. You may wish to refer to **Chapter 0** to review prerequisite skills.



Ten recabala	- /	
English		Español
formula	• p. 76 •	fórmula
solve an equation	• p. 83 •	resolver una ecuación
equivalent equations	• p. 83 •	ecuaciones equivalentes
multi-step equation	• p. 91 •	ecuación de varios pasos
identity	• p. 98 •	identidad
ratio	• p. 111 •	razón
proportion	• p. 111 •	proporción
rate	• p. 113 •	tasa
unit rate	• p. 113 •	tasa unitaria
scale model	• p. 114 •	modelo de escala
percent of change	• p. 119 •	porcentaje de cambio
literal equation	• p. 127 •	ecuación literal
dimensional analysis	• p. 128 •	análisis dimensional
weighted average	• p. 132 •	promedio ponderado

Review Vocabulary

algebraic expression • p. 5 • expression algebraica an expression consisting of one or more numbers and variables along with one or more arithmetic operations

coordinate system • p. 38 • sistema de coordenedas the grid formed by the intersection of two number lines, the horizontal axis and

function • p. 45 • funcion a relation in which each element of the domain is



Then

You evaluated and simplified algebraic expressions. (Lesson 1-2)

Now/

- Translate sentences into Equations.
- Translate Equations into sentences.

KY Program of Studies

HS-AT-S-VEO1 Students will write expressions, equations, inequalities and relations in equivalent forms. HS-AT-S-EI1 Students will write equivalent forms of equations, inequalities and systems of equations and inequalities and solve them with fluency - mentally or with paper and pencil in simple cases and using technology in all cases. Also addresses HS-AT-S-VEO2.

New Vocabulary

KY Math Online

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- Personal Tutor
- Self-Check Quiz
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Writing Equations

Why?

The Daytona 500 is widely considered to be the most important event of the NASCAR circuit. The distance around the track is 2.5 miles, and the race is a total of 500 miles. We can write an equation to determine how many laps around the track it takes to finish the race.



Write Verbal Expressions To write an equation, identify the unknown for which you are looking and assign a variable to it. Then, write the sentence as an equation. Look for key words such as *is, is as much as, is the same as,* or *is identical to* that indicate where you should place the equals sign.

Consider the Daytona a 500 example above.

Words	The length of each lap times the number of laps is the length of the race.				
Variable	Let ℓ represent the number of laps in the race.				
Equation	2.5	×	l	=	500

EXAMPLE 1 Translate Sentences into Equations

Translate each sentence into an equation.

a. Seven times a number squared is five times the difference of *k* and *m*.

Seven	times	n squared	is	five	times	the difference of <i>k</i> and <i>m</i>
7	•	n^2	=	5	•	(k-m)
The eq	nuation	is $7n^2 = 50$	k - n	n)		

The equation is 7n = 5(k - m).

b. Fifteen times a number subtracted from 80 is 25.

You can rewrite the verbal sentence so it is easier to translate. Let *n* represent the number. *Fifteen times a number subtracted from 80* is the same as *80 minus 15 times a number is 25*.

80	minus	15	times	a number	is	25.
80	_	15	•	п	=	25

The equation is 80 - 15n = 25.

Check Your Progress

- **1A.** Two plus the quotient of a number and 8 is the same as 16.
- **1B.** Twenty-seven times *k* is *h* squared decreased by 9.

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Real-World Link

In 1919, Britain and France offered a flight that carried two passengers at a time. Now there are approximately 45,000 flights each day in the U.S., carrying hundreds of passengers on each flight.

Source: Flightaware

Translating sentences to algebraic expressions and equations is a valuable skill in solving real-world problems.

Real-World EXAMPLE 2 Use the Four-Step Problem-Solving Plan

AIR TRAVEL Refer to the information at the left. In how many days will 180,000 flights have occurred in the United States?

- **Understand** The information given in the problem is that there are approximately 45,000 flights per day in the United States. We are asked to find how many days it will take for 180,000 flights to have occurred.
 - **Plan** Write an equation. Let *d* represent the number of days needed.

= 180,000
by asking, "What number times 45,000 is 180,000

Check Check your answer by substituting 4 for *d* in the equation.

$45,000(4) \stackrel{?}{=} 180,000$	Substitute 4 for	d.

 $180,000 = 180,000 \checkmark$ Multiply.

The answer makes sense and works for the original problem.

Check Your Progress

2. GOVERNMENT There are 50 members in the North Carolina Senate. This is 70 fewer than the number in the North Carolina House of Representatives. How many members are in the North Carolina House of Representatives?

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A rule for the relationship between certain quantities is called a **formula**. These equations use variables to represent numbers and form general rules.

EXAMPLE 3 Write a Formula

GEOMETRY Translate the sentence into a formula.

The area of a triangle equals the product of $\frac{1}{2}$ the length of the base and the height.

Words	The area of a triangle	equals	product of $\frac{1}{2}$ the length of the base and the height.
Variables	Let $A = $ area, b	= base, a	and $h =$ height.
Equation	A	=	$\frac{1}{2}bh$

The formula for the area of a triangle is $A = \frac{1}{2}bh$.

Check Your Progress

3. GEOMETRY Translate the sentence into a formula. In a right triangle, the square of the measure of the hypotenuse *c* is equal to the sum of the squares of the measures of the legs, *a* and *b*.



Math History Link

Ahmes

(about 1680–1620 в.с.) Ahmes was the Egyptian mathematician and scribe who copied the Rhind Mathematical Papyrus. The papyrus contains 87 algebra problems of the same type. The first set of problems asks how to divide *n* loaves of bread between 10 people. **Write Sentences from Equations** If you are given an equation, you can write a sentence or create your own word problem.



Translate each equation into a sentence.



When given a set of information, you can create a problem that relates a story.

EXAMPLE 5 Write a Problem

Write a problem based on the given information.

t = the time that Maxine drove; t + 4 = the time that Tia drove; 2t + (t + 4) = 28

Sample problem:

Maxine and Tia went on a trip, and they took turns driving. During her turn, Tia drove 4 hours more than Maxine. Maxine took 2 turns, and Tia took 1 turn. Together they drove for 28 hours. How far did Maxine drive during each of her turns?

Check Your Progress

5. p = Beth's salary; 0.1p = bonus; p + 0.1p = 525

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🗹 Check Your Understanding

Example 1	Translate each sentence into an equation.
p. 75	1. Three times r less than 15 equals 6.
	2. The sum of <i>q</i> and four times <i>t</i> is equal to 29.
	3 A number n squared plus 12 is the same as the quotient of p and 4.
	4. Half of <i>j</i> minus 5 is the sum of <i>k</i> and 13.
	5. The sum of 8 and three times <i>k</i> equals the difference of 5 times <i>k</i> and
	6. Three fourths of w plus 5 is one half of w in addition to nine.

- **7.** The quotient of 25 and *t* plus 6 is the same as twice *t* plus 1.
- **8.** Thirty-two divided by *y* is equal to the product of three and *y* minus four.

3.

Example 2 p. 76	9. SAVINGS Samuel has \$1900 total of \$2500 by depositing solve an equation to find he	in the bank. He wishes to increase his account to a \$30 dollars per week from his paycheck. Write and ow many weeks he needs to reach his goal.			
	10. PAINTING Miguel is earning fee plus \$12 per can of pain equation to find how many	g extra money by painting homes. He charges a \$200 t needed to complete the job. Write and use an cans of paint he needs for a \$260 job.			
Example 3	Translate each sentence into a	formula.			
р. 76	11. The perimeter of a regular	pentagon is 5 times the length of each side.			
	12. The area of a circle is the pr	12. The area of a circle is the product of π and the radius <i>r</i> squared.			
	13. Four times π times the radi	us squared is the surface area of a sphere.			
	14. One third the product of th volume of a pyramid with a	e length of the side squared and the height is the a square base.			
Example 4	Translate each equation into a	sentence.			
p. 77	15. $7m - q = 23$	16. $6 + 9k + 5j = 54$			
	17. $3(g+8) = 4h - 10$	18. $6d^2 - 7f = 8d + f^2$			
Example 5	Write a problem based on the	given information.			
p. 77	19. $g = \text{gymnasts on a team; } 3g = 45$				
	20. $c = \cos t$ of a notebook; 0.25	c = markup; c + 0.25c = 3.75			

Practice and Problem Solving

Example 1 Translate each sentence into an equation. p. 75 **21.** The difference of *f* and five times *g* is the same as 25 minus *f*. **22.** Three times *b* less than 100 is equal to the product of 6 and *b*. **23.** Four times the sum of 14 and *c* is *a* squared. Example 2 **24.** MUSIC The eight-note interval of white keys between two notes with the same p. 76 name is called an octave. A piano has 52 white keys. Write and use an equation to find the number of octaves on a piano keyboard. **25.** GARDENING A flat of plants contains 12 plants. Yoshi wants to plant a garden that has three rows of plants with 10 plants in each row. Write and solve an equation to decide the number of flats Yoshi should to buy. **Example 3** Translate each sentence into a formula. p. 76 **26.** The perimeter of a rectangle is equal to 2 times the length plus twice the width. Celsius temperature *C* is five ninths times the difference of the Fahrenheit 27 temperature *F* and 32. **28.** The density of an object is the quotient of its mass and its volume. **29.** Simple interest is computed by finding the product of the principal amount *p*, the interest rate *r*, and the time *t*. **Example 4** Translate each equation into a sentence. p. 77 **30.** j + 16 = 35**31.** 4*m* = 52 **32.** 7(p + 23) = 102**34.** $\frac{2}{5}v + \frac{3}{4} = \frac{2}{3}x^2$ **35.** $\frac{1}{3} - \frac{4}{5}z = \frac{4}{3}y^3$ **33.** $r^2 - 15 = t + 19$

= Step-by-Step Solutions begin on page R12.

Extra Practice begins on page 815.

Example 5 Write a problem based on the given information.

- **36.** q =quarts of strawberries; 2.50q = 10
- **37.** p = the principal amount; 0.12p = the interest charged; p + 0.12p = 224
- **38.** m = number of movies rented; 10 + 1.50m = 14.50
- **39.** p = the number of players in the game; 5p + 7 = number of cards in the game

For Exercises 41–43, match each sentence with an equation.

A. $g^2 = 2(g - 10)$	C. $g^3 = 24g + 4$
B. $\frac{1}{2}g + 32 = 15 + 6g$	D. $3g^2 = 30 + 9g$

- **40.** One half of *g* plus thirty-two is as much as the sum of fifteen and six times *g*.
- **41.** A number *g* to the third power is the same as the product of 24 and *g* plus 4.
- **42.** The square of *g* is the same as two times the difference of *g* and 10.
- **43.** The product of 3 and the square of *g* equals the sum of thirty and the product of nine and *g*.
- **44. MONEY** Tim's bank contains quarters, dimes, and nickels. He has three more dimes than quarters and 6 fewer nickels than quarters. If he has 63 coins in all, write and solve an equation to determine how many quarters Tim has.
- SHOPPING Pilar is stocking up on supplies for her camping trip. She bought 17 items, including tent stakes, packets of drink mix, and bottles of water. She bought 3 times as many packets of drink mix as she did tent stakes. She also bought 2 more bottles of water than tent stakes. Write and solve an equation to discover how many tent stakes she bought.
- **46. MULTIPLE REPRESENTATIONS** In this problem, you will explore how to translate relations with powers.

X	2	3	4	5	6
У	5	10	17	26	37

- **a. VERBAL** Write a sentence to describe the relationship between *x* and *y* in the table.
- **b. ALGEBRAIC** Write an equation that represents the data in the table.
- **c. GRAPHICAL** Graph each ordered pair and draw the function. Describe the graph as *discrete* or *continuous*.

H.O.T. Problems

Use Higher-Order Thinking Skills

- **47. OPEN ENDED** Write a problem about your favorite television show that uses the equation x + 8 = 30.
- **48. REASONING** The surface area of a three-dimensional object is the sum of the areas of the faces. If ℓ represents the length of the side of a cube, write a formula for the surface area of the cube.
- **49. CHALLENGE** Given the perimeter *p* and width *w* of a rectangle, write a formula to find the length ℓ .
- **50.** WRITING IN MATH Explain how to translate a verbal sentence into an algebraic equation. Include any tips that you may have for your fellow students.



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Real-World Link

There are more than 16,000 commercial and public campgrounds nationwide. Camping is the number one outdoor vacation activity in America.

Source: Travel Industry Association of America

KCCT PRACTICE MA-HS-5.3.1

51. Which equation *best* represents the relationship between the number of hours an electrician works *h* and the total charges *c*?

Cost of Electrician		
Emergency House Call	\$30 one time fee	
Rate	\$55/hour	

- A c = 30 + 55
- **B** c = 30h + 55
- **C** c = 30 + 55h
- **D** c = 30h + 55h
- **52.** A car traveled at 55 miles per hour for 2.5 hours and then at 65 miles per hour for 3 hours. How far did the car travel in all?
 - F 300.5 mi H 330 mi G 305 mi I 332.5 mi

Spiral Review

Write a counterexample for each conditional statement. (Lesson 1-8)

- **55.** If you were born in Florida, then you live in Florida.
- **56.** If the product of two numbers is an even number, then both factors must be even numbers.
- **57.** If a number is divisible by 2, then it is divisible by 4.
- **58. SHOPPING** For every two pairs of earrings that you buy at the regular price of \$29 each, you get a third pair free. (Lesson 1-7)
 - **a.** Make a table that shows the cost of buying 1 to 5 pairs of earrings.
 - **b.** Write the data as a set of ordered pairs.
 - **c.** Graph the data.
- 59. GEOMETRY Refer to the table below. (Lesson 1-6)

Polygon	triangle	quadrilateral	pentagon	hexagon	heptagon
Number of Sides	3	4	5	6	7
Interior Angle Sum	180	360	540	720	900

a. Identify the independent and dependent variables.

61. 10⁶

- **b.** Identify the domain and range for this situation.
- c. State whether the function is *discrete* or *continuous*. Explain.

Skills Review

Evaluate each expression. (Lesson 1-1)

60. 9²

53. GEOMETRY Suppose each dimension of rectangle *ABCD* is doubled. What is the perimeter of the new ABCD?



- **54. STATISTICS** Stacy's first five science test scores were 95, 86, 83, 95, and 99. Which of the following is a true statement?
 - **A** The mode is the same as the median.
 - **B** The median is the same as the mean.
 - **C** The range is the same as the mode.
 - **D** The mode is the same as the mean.

80 Chapter 2 Linear Equations

Algebra Lab Solving Equations



EXPLORE

HS-AT-S-EI3 Students will solve one-variable equations and inequalities using manipulatives, symbols, procedures and graphing, including graphing the solution set on a number line. Also addresses HS-NPO-S-NO13, HS-AT-S-EI1, and HS-AT-S-EI4.

You can use **algebra tiles** to model solving equations. To **solve an equation** means to find the value of the variable that makes the equation true. An $\begin{bmatrix} x \\ 1 \end{bmatrix}$ tile represents the variable *x*. The **1** tile represents a positive 1. The **1** tile represents a negative 1. And, the **1** tile represents the variable negative *x*. The goal is to get the *x*-tile by itself on one side of the mat by using the rules stated below.



ACTIVITY 1 Addition Equation

Use an equation model to solve x + 3 = -4.

- **Step 1** Model the equation. Place 1 *x*-tile and 3 positive 1-tiles on one side of the mat. Place 4 negative 1-tiles on the other side of the mat.
- **Step 2** Isolate the *x*-term. Add 3 negative 1-tiles to each side. The resulting equation is x = -7.



ACTIVITY 2 Subtraction Equation

Use an equation model to solve x - 2 = 1.

Step 1



Place 1 *x*-tile and 2 negative 1-tiles on one side of the mat. Place 1 positive 1-tile on the other side of the mat. Then add 2 positive 1-tiles to each side. Step 2



Group the tiles to form zero pairs. Then remove all the zero pairs. The resulting equation is x = 3.

Model and Analyze

Use algebra tiles to solve each equation.

5. WRITING IN MATH If a = b, what can you say about a + c and b + c? about a - c and b - c?

When solving multiplication equations, the goal is still to get the *x*-tile by itself on one side of the mat by using the rules for dividing stated below.



ACTIVITY 3 Multiplication Equation

Use an equation model to solve 3x = 12.

- **Step 1** Model the equation. Place 3 *x*-tiles on side of the mat. Place 12 positive 1-tiles on the other side of the mat.
- **Step 2** Isolate the *x*-term. Separate the tiles into 3 equal groups to match the 3 *x*-tiles. Each *x*-tile is paired with 4 positive 1-tiles. The resulting equation is x = 4.



Model and Analyze

Use algebra tiles to solve each equation.

6. 5x = -15

7. -3x = -9

8. 4x = 8

```
9. -6x = 18
```

10. MAKE A CONJECTURE How would you use algebra tiles to solve $\frac{x}{4} = 5$? Discuss the steps you would take to solve this equation algebraically.

Then

You translated sentences into equations. (Lesson 2-1)

Now

- Solve equations by using addition and subtraction.
- Solve equations by using multiplication and division.

KY Program

of Studies **HS-NPO-S-PNO4** Students will justify the solution steps in simplifying expressions or solving an equation. HS-AT-S-EI3 Students will

solve one-variable equations and inequalities using manipulatives, symbols, procedures and graphing, including graphing the solution set on a number line. Also addresses HS-AT-S-VEO1, HS-AT-S-EI1, and HS-AT-S-EI4.

New Vocabulary solve an equation equivalent equations

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Solving One-Step Equations

Why?

A record for the most snow angels made at one time was set in Michigan when 3784 people participated. North Dakota had 8910 people register to break the record. To determine how many more people North Dakota had than Michigan, solve the equation 3784 + x = 8910.



Solve Equations Using Addition or Subtraction In an equation, the variable represents the number that satisfies the equation. To solve an equation means to find the value of the variable that makes the equation true.

The process of solving an equation involves isolating the variable (with a coefficient of 1) on one side of the equation. Each step in this process results in equivalent equations. Equivalent equations have the same solution.

Key C	Concept Addition Property of Equality For Your
Words	If an equation is true and the same number is added to each side of the equation, the resulting equivalent equation is also true.
Symbols	For any real numbers a , b , and c , if $a = b$, then $a + c = b + c$.
Examples	$ \begin{array}{rcl} 14 = 14 & -3 = -3 \\ 14 + 3 = 14 + 3 & +9 = +9 \\ 17 = 17 & 6 = 6 \end{array} $

EXAMPLE 1 Solve by Adding

Solve c - 22 = 54.

Horizontal Method

c - 22 = 54
c - 22 + 22 = 54 + 22
c = 76

	Vertical Metho	bd
Original equation	c - 22 = 5	4
Add 22 to each side.	+22 = +2	2
Simplify.	<i>c</i> = 76	

To check that 76 is the solution, substitute 76 for *c* in the original equation.

CHECK c - 22 = 54**Original equation** $76 - 22 \stackrel{?}{=} 54$ Substitute 76 for c. 54 = 54Subtract

Check Your Progress Solve each equation.

1A. 113 = g - 25

1B. i - 87 = -3

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Similar to the Addition Property of Equality, the **Subtraction Property of Equality** can also be used to solve equations.

StudyTip

Subtraction Subtracting a value is equivalent to adding the opposite of the value.

For Your Key Concept **Subtraction Property of Equality** FOLDARIE If an equation is true and the same number is subtracted Words from each side of the equation, the resulting equivalent equation is also true. For any real numbers a, b, and c, if a = b, then a - c = b - c. **Symbols** 13 = 1387 = 87 Examples 87 - 17 = 87 - 17 $\frac{-28 = -28}{-15 = -15}$ 70 = 70

StudyTip

Solving an Equation When solving equations you can use either the horizontal method or the vertical method. Both methods will produce the same result.

EXAMPLE 2 Solve by Subtracting

Solve 63 + m = 79.

Horizontal Method		Vertic	al Met	hod
63 + m = 79	Original equation	63 +	m =	79
63 - 63 + m = 79 - 63	Subtract 63 from each side.	-63	= -	-63
m = 16	Simplify.		m = 1	16

To check that 16 is the solution, replace *m* with 16 in the original equation.

CHECK 63 + m = 79 **Original equation** $63 + 16 \stackrel{?}{=} 79$ **Substitution**, m = 16. $79 = 79 \checkmark$ **Simplify**.

Check Your Progress Solve each equation.

2B. −12 = *p* + 16

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Solve Equations Using Multiplication or Division In an equation $\frac{x}{3} = 9$, the variable *x* is divided by 3. To solve for *x*, undo the division by multiplying each side by 3. This is an example of the **Multiplication Property of Equality**.

Key	Concept Multiplication Property of Equality For Your
Words	If an equation is true and each side is multiplied by the same nonzero number, the resulting equation is equivalent.
Symbols	For any real numbers a , b , and c , if $a = b$, then $ac = bc$.
Example	If $x = 5$, then $3x = 15$.
	Division Property of Equality
Words	If an equation is true and each side is divided by the same nonzero number, the resulting equation is equivalent.
Symbols	For any real numbers a, b, and c, $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.
Example	If $x = -20$, then $\frac{x}{5} = \frac{-20}{5}$ or -4 .

The reciprocal of a number can be used to solve equations.

Review Vocabulary reciprocal the multiplicative inverse of a number (Lesson 1-4)

EXAMPLE 3	Solve by Multiplying and Dividing	
Solve each equ	ation.	
a. $\frac{2}{3}q = \frac{1}{2}$		
$\frac{2}{3}q = \frac{1}{2}$	Original equation	
$\frac{3}{2}\left(\frac{2}{3}\right)q = \frac{3}{2}\left(\frac{1}{2}\right)$	Multiply each side by $\frac{3}{2}$, the reciprocal of $\frac{2}{3}$.	
$q = \frac{3}{4}$	Check the result.	
b. $39 = -3r$		
39 = -3r	Original equation	
$\frac{39}{-3} = \frac{-3r}{-3}$	Divide each side by —3.	
-13 = r	Check the result.	
Check Your Pr	ogress	
3A. $\frac{3}{5}k = 6$	3B. $-\frac{1}{4} = \frac{2}{3}b$	
		Personal Tutor glencoe.com



Real-World Link

Almost half of 10- to 18-year-olds in the U.S. use a cell phone. Of those, 53% play games on their phones, more than 33% download games, 52% use the calendar/organizer, and nearly all teens with camera phones snap pictures.

Source: Lexdon Business Library

We can also use reciprocals and properties of equality to solve real-world problems.

Real-World EXAMPLE 4 Solve by Multiplying

SURVEYS A recent survey of 13- to 15-year-old girls was conducted. Of those surveyed, 225, or about $\frac{9}{20}$ said they talk on the telephone while they watch television. About how many girls were surveyed?



About 500 girls were surveyed.

Check Your Progress

4. STAINED GLASS Allison is making a stained glass panel for a window. She knows her pattern requires that one fifth of the glass should be blue. She has 288 square inches of blue glass. If she intends to use all of her blue glass, how much glass will she need for the entire project?

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Check Your Understanding

Examples 1 and 3 Solve each equation. Check your solution.

pp. 83–85

1. $g + 5 = 33$	2. $104 = y - 67$	3. $\frac{2}{3} + w = 1\frac{1}{2}$
4. $-4 + t = -7$	5. $a + 26 = 35$	6. $-6 + c = 32$
7. $1.5 = y - (-5.6)$	8. $3 + g = \frac{1}{4}$	9. $x + 4 = \frac{3}{4}$
10. $\frac{t}{7} = -5$	11. $\frac{a}{36} = \frac{4}{9}$	12. $\frac{2}{3}n = 10$
13. $\frac{8}{9} = \frac{4}{5}k$	14. $12 = \frac{x}{-3}$	15. $-\frac{r}{4} = \frac{1}{7}$

Example 4

p. 85

- 16. FUNDRAISING The television show "Idol Gives Back" raised money for relief organizations. During this show, viewers could call in and vote for their favorite performer. The parent company contributed money for each of the 50 million votes cast. If the total donation was \$5 million, what did they pay for each vote?
- **17. SHOPPING** Hana decides to buy her cat a bed from an online fund that cares for stray animals. She finds that $\frac{7}{8}$ of her purchase goes to the shelters that care for the animals. How much of the money that Hana spent actually went to the animal shelter?



= Step-by-Step Solutions begin on page R12.

Extra Practice begins on page 815.

Practice and Problem Solving

Examples 1 and 3	Solve each equation. Che	eck your solution.	
рр. 83–85	18. $v - 9 = 14$	19. $44 = t - 72$	20. $-61 = d + (-18)$
	21. $18 + z = 40$	22. $-4a = 48$	23. 12 <i>t</i> = −132
	24. $18 - (-f) = 91$	25 $-16 - (-t) = -45$	26. $\frac{1}{3}v = -5$
	27. $\frac{u}{8} = -4$	28. $\frac{a}{6} = -9$	29. $-\frac{k}{5} = \frac{7}{5}$
	30. $\frac{3}{4} = w + \frac{2}{5}$	31. $-\frac{1}{2} + a = \frac{5}{8}$	32. $-\frac{t}{7} = \frac{1}{15}$
	33. $-\frac{5}{7} = y - 2$	34. $v + 914 = -23$	35. $447 + x = -261$
	36. $-\frac{1}{7}c = 21$	37. $-\frac{2}{3}h = -22$	38. $\frac{3}{5}q = -15$
	39. $\frac{n}{8} = -\frac{1}{4}$	40. $\frac{c}{4} = -\frac{9}{8}$	41. $\frac{2}{3} + r = -\frac{4}{9}$

Example 4

p. 85

- **42.** CATS A domestic cat can run at speeds of 27.5 miles per hour when chasing prey. A cheetah can run 42.5 miles per hour faster when chasing prey. How fast can the cheetah go?
- **43.** CARS The average time t it takes to manufacture a car in the United States is 24.9 hours. This is 8.1 hours longer than the average time it takes to manufacture a car in Japan. Write and solve an equation to find the average time to manufacture a car in Japan.

Solve each equation. Check your solution.

44.
$$\frac{x}{9} = 10$$
45. $\frac{b}{7} = -11$
46. $\frac{3}{4} = \frac{c}{24}$
47. $\frac{2}{3} = \frac{1}{8}y$
48. $\frac{2}{3}n = 14$
49. $\frac{3}{5}g = -6$
50. $4\frac{1}{5} = 3p$
51. $-5 = 3\frac{1}{2}x$
52. $6 = -\frac{1}{2}n$
53. $-\frac{2}{5} = -\frac{z}{45}$
54. $-\frac{g}{24} = \frac{5}{12}$
55. $-\frac{v}{5} = -45$

Write an equation for each sentence. Then solve the equation.

- **56.** Six times a number is 132.
- **57.** Two thirds equals negative eight times a number.
- 58. Five elevenths times a number is 55.
- **59.** Four fifths is equal to ten sixteenths of a number.
- **60.** Three and two thirds times a number equals two ninths.
 - Four and four fifths times a number is one and one fifth.
- **62. SHOPPING** Adelina is comparing prices for two brands of health and energy bars at the local grocery store. She wants to get the best price for each bar.
 - **a.** Write an equation to find the price for each bar of the Feel Great brand.
 - **b.** Write an equation to find the price of each bar for the Super Power brand.
 - c. Which bar should Adelina buy? Explain.
- **63. MEDIA** The world's largest passenger plane, the Airbus A380, was first used by Singapore Airlines in 2005. The following description appeared on a news Web site after the plane was introduced.

"That airline will see the A380 transporting some 555 passengers, 139 more than a similarly set-up 747." How many passengers will a similarly set-up 747 transport?

- **64. FUEL** In 2004, approximately 5 million cars and trucks were classified as flex-fuel, which means they could run on gasoline or ethanol. In 2006, that number increased to 7.5 million. How many more cars and trucks were flex-fuel in 2006?
- **65. CHEERLEADING** At a certain cheerleading competition the maximum time per team, including the set up, is 3 minutes. The Ridgeview High School squad's performance time is 2 minutes and thirty four seconds. How much time does the squad have left for their set up?
- **66. COMIC BOOKS** An X-Men #1 comic book in mint condition recently sold for \$45,000. An Action Comics #63 (Mile High), also in mint condition, sold for \$15,000. How much more did the X-Men comic book sell for than the Action Comics book?
- **67. MOVIES** A certain movie made \$1.6 million in ticket sales. Its sequel made \$0.8 million in ticket sales. How much more did the first movie make than the sequel?
- **68.** CAMERAS An electronics store sells a certain digital camera for \$126. This is $\frac{2}{3}$ of the price that a photography store charges. What is the cost of the camera at the photography store?



Real-World Link

Ethanol is produced from corn and is considered energy efficient because it yields 25% more energy than the process to create it.

Source: U.S. Department of Energy



BLOGS In 2006, 57 million American adults read online Weblogs, or blogs. However, 45 million fewer American adults say that they maintain their own blog. How many American adults maintain a blog?



Real-World Link

Schools have begun using an online voting system that allows students to log in and vote for homecoming king and queen.

Source: NewBay Media

70. SCIENCE CAREERS According to the Bureau of Labor and Statistics, approximately 65,000,000 women were employed in the United States in 2004.

- **a.** The number of women in the computer science fields times 26 is the number of working women. Write an equation to represent the number of women employed in the computer sciences in 2004. Then solve the equation.
- **b.** The number of women in natural science fields is 2,266,000 less than the number of women in computer science fields. How many women are in natural science fields?
- **71. DANCES** Student Council has a budget of \$1000 for the homecoming dance. So far, they have spent \$350 dollars for music.
 - **a.** Write an equation to represent the amount of money that they have remaining to spend. Then solve the equation.
 - **b.** They then spent an additional \$225 on decorations. Write an equation to represent the amount of money that they have remaining.
 - **c.** Assuming that the Student Council spent their entire budget, write an equation to represent how many \$6 tickets they must sell to make a profit.

H.O.T. Problems

Use Higher-Order Thinking Skills

72. WHICH ONE DOESN'T BELONG? Identify the equation that does not belong with the other three. Explain your reasoning.

$$n + n = 25$$

```
n - 16 = 29
```

n - 4 = 9

- **73. OPEN ENDED** Write an equation involving addition and demonstrate two ways to solve it.
- **74. REASONING** Which triangle does not show the relationship between the height and base as $4\frac{1}{2}b$?
- **75. CHALLENGE** Determine whether each sentence is *sometimes, always,* or *never true.* Explain your reasoning.

a. x + x = x **b.** x + 0 = x

- **76. REASONING** Determine the value for each statement below.
 - **a.** If x 7 = 14, what is the value of x 2?
 - **b.** If t + 8 = -12, what is the value of t + 1?
- **77. CHALLENGE** Discuss why the equations $\frac{2}{3}b = 16$ and 48 = 2c have the same solution.
- **78.** WRITING IN MATH Compare and contrast the Multiplication Property of Equality and the Division Property of Equality. Explain why they can be considered the same property. Which one do you think is easier to use?

inangie	Dase (cm)	mengine (cini)
$\triangle ABC$	3.8	17.1
$\triangle MQP$	5.4	24.3
$\triangle RST$	6.3	28.5
$\triangle TRW$	1.6	7.2

KCCT PRACTICE MA-HS-5.3.1

- **79.** Which of the following best represents the equation w 15 = 33?
 - A Jake added *w* ounces of water to his water bottle, which originally contained 33 ounces of water. How much water did he add?
 - **B** Jake added 15 ounces of water to his water bottle, for a total of 33 ounces of water. How much water *w* was originally in the bottle?
 - **C** Jake drank 15 ounces of water from his water bottle and 33 ounces were left. How much water *w* was originally in the bottle?
 - **D** Jake drank 15 ounces of water from his water bottle, which originally contained 33 ounces. How much water *w* was left?
- **80. SHORT RESPONSE** Charlie's company pays him for every mile that he drives on his trip. When he drives 50 miles, he is paid \$30. How many miles did he drive if he was paid \$275?

81. The table shows the results of a survey given to 500 international travelers. Based on the data, which statement about international travelers is true?

Vacation Plans					
Destination Percent					
The Tropics	37				
Europe	19				
Asia	17				
Other	17				
No Vacation	10				

- **F** Fifty have no vacation plans.
- **G** Fifteen are going to Asia.
- H One third are going to the tropics.
- J One hundred are going to Europe.
- **82. GEOMETRY** The amount of water needed to fill a pool represents the pool's _____.
 - A volume C circum
 - **B** surface area
- **C** circumference
- D perimeter

Spiral Review

Translate each sentence into an equation. (Lesson 2-1)

- **83.** The sum of twice *r* and three times *k* is identical to thirteen.
- **84.** The quotient of *t* and forty is the same as twelve minus half of *u*.
- **85.** The square of *m* minus the cube of *p* is sixteen.
- **86.** Two times *z* is equal to two times the sum of *v* and *x*.

Write each statement in if-then form. (Lesson 1-8)

- 87. The trash is picked up on Monday.
- **88.** Vito will call after school.
- **89.** For x = 8, $x^2 3x = 40$.
- **90.** 4q + 6 > 42 when q > 9.

Skills Review

91. COMMUNICATION Sato is keeping track of how he communicates with his friends for a math project. In a week, he averages 5 hours using e-mail, 18 hours on the phone, and 12 hours meeting with them in person. Write and evaluate an expression to predict how many hours he will spend communicating with his friends over the next 12 weeks. (Lesson 1-4)

Algebra Lab Solving Multi-Step Equations





EXPLORE

HS-AT-S-EI3 Students will solve one-variable equations and inequalities using manipulatives, symbols, procedures and graphing, including graphing the solution set on a number line. *Also addresses HS-NPO-S-NO13, HS-AT-S-EI1, and HS-AT-S-EI4*.

You can use algebra tiles to model solving multi-step equations.



Place 4 *x*-tiles and 3 positive 1-tiles on one side of the mat. Place 5 negative 1-tiles on the other side.



Group the tiles to form zero pairs and remove the zero pairs.



Since there are 3 positive 1-tiles with the *x*-tiles, add 3 negative 1-tiles to each side to form zero pairs.



Separate the remaining tiles into 4 equal groups to match the 4 *x*-tiles. Each *x*-tile is paired with 2 negative 1-tiles. The resulting equation is x = -2.

Model Use algebra tiles to solve each equation.

1.	3x - 7 = -10	2. $2x + 5 = 9$	3. $5x - 7 = 8$	4. $-7 = 3x + 8$
5.	5 + 4x = -11	6. $3x + 1 = 7$	7. $11 = 2x - 5$	8. $7 + 6x = -11$

- **9.** What would be your first step in solving 8x 29 = 67?
- **10.** What steps would you use to solve 9x + 14 = -49?
- **90** Chapter 2 Linear Equations

2-3

Then

You solved single-step equations. (Lesson 2-2)

Now/

- Solve equations involving more than one operation.
- Solve equations involving consecutive integers.

KY Program of Studies

HS-NPO-S-PNO4 Students will justify the solution steps in simplifying expressions or solving an equation. HS-AT-S-EI3 Students will solve one-variable equations and inequalities using manipulatives, symbols, procedures and graphing, including graphing the solution set on a number line. Also addresses HS-AT-S-VEO1, HS-AT-S-EI1, and HS-AT-S-EI4.

New Vocabulary

multi-step equation consecutive integers number theory

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- Homework Help

Solving Multi-Step Equations

Why?

The Tour de France is the premier cycling event in the world. The course for the 2007 Tour de France is shown. If the length of the shortest portion of the race can be represented by k, the expression 4k + 20 is the length of the longest stage. The longest stage is 236 kilometers.

Solve Multi-Step Equations The situation above can be described by the equation 4k + 20 = 236. Because this equation requires more than one step to solve, it is called a **multi-step equation**. To solve this equation, we must undo each operation by working backward.



EXAMPLE 1 Solve Multi-Step Equations Solve each equation.

a.	11x - 4 = 29	
	11x - 4 = 29	Original equation
	11x - 4 + 4 = 29 + 4	Add 4 to each side.
	11x = 33	Simplify.
	$\frac{11x}{11} = \frac{33}{11}$	Divide each side by 11.
	x = 3	Simplify.
b.	$\frac{a+7}{8} = 5$	
	$\frac{a+7}{8} = 5$	Original equation
	$8\left(\frac{a+7}{8}\right) = 8(5)$	Multiply each side by 8.
	a + 7 = 40	Simplify.

 $\frac{-7 = -7}{a = 33}$ Subtract 7 from each side. Simplify.

You can check your solutions by substituting the results back into the original equations.

Check Your Progress

Solve each equation. Check your solution.

1A. 2a - 6 = 4

1B. $\frac{n+1}{-2} = 15$

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Real-World EXAMPLE 2 Write and Solve a Multi-Step Equation

SHOPPING Hiroshi is buying a pair of water skis that are on sale for $\frac{2}{3}$ of the original price. After he uses a \$25 gift certificate, the total cost before taxes is \$115. What was the original price of the skis? Write an equation for the problem. Then solve the equation.



The original price of the skis was \$210.

Check Your Progress

V

- **2A. RETAIL** A music store has sold $\frac{3}{5}$ of their hip-hop CDs, but 10 were returned. Now the store has 62 hip-hop CDs. How many hip-hop CDs did the music store originally have?
- **2B. READING** Len read $\frac{3}{4}$ of a graphic novel over the weekend. Monday, he read 22 more pages. If he has read 220 pages, how many pages does the book have?

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Solve Consecutive Integer Problems Consecutive integers are integers in counting order, such as 4, 5, and 6 or n, n + 1, and n + 2. Counting by two will result in *consecutive even integers* if the starting integer n is even and *consecutive odd integers* if the starting integer n is odd.

Concept Summary Consecutive Integers For Your FOLDABLE					
Туре	Words	Symbols	Example		
Consecutive Integers	Integers that come in counting order.	n, n + 1, n + 2,	, -2, -1, 0, 1, 2,		
Consecutive Even Integers	Even integer followed by the next even integer.	n, n + 2, n + 4,	, -2, 0, 2, 4,		
Consective Odd Integers	Odd integer followed by the next even integer.	<i>n</i> , <i>n</i> + 2, <i>n</i> + 4,	, –1, 1, –3, 5,		

Number theory is the study of numbers and the relationships between them.

StudyTip

Leading coefficients You can use the same steps to solve a multistep equation if the leading coefficient is a fraction or an integer.



Real-World Link

Shoppers in Shanghai, China, can pay for purchased items at a terminal that can match the buyers' fingerprints with their bank accounts. Source: Shanghai Daily

92 Chapter 2 Linear Equations

EXAMPLE 3 Solve a Consecutive Integer Problem

NUMBER THEORY Write an equation for the following problem. Then solve the equation and answer the problem.

Find three consecutive odd integers with a sum of -51.

Let n = the least odd integer.

Then n + 2 = the next greater odd integer, and n + 4 = the greatest of the three integers.

Words	The sum of three co	is —51.	
Equation	n + (n + 1)	= -51	
n + (n + 2) -	+(n+4) = -51	Original equation	
	3n + 6 = -51	Simplify.	
	-6 = -6	Subtract 6 from each s	ide.
	3n = -57	Simplify.	
	$\frac{3n}{3} = \frac{-57}{3}$	Divide each side by 3.	
	n = -19	Simplify.	
$n \perp 2 = -10$	$\pm 2 \text{ or } -17 n$	1 - 10 + 10r	15

n + 2 = -19 + 2 or -17 n + 4 = -19 + 4 or -15The consecutive odd integers are -19, -17, and -15.

CHECK -19, -17, and -15 are consecutive odd integers. -19 + (-17) + (-15) = $-51 \checkmark$

Check Your Progress

3. Write an equation for the following problem. Then solve the equation and answer the problem.

Find three consecutive integers with a sum of 21.

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🗹 Check Your Understanding

Example 1

le 1 Solve each equation. Check your solution.

1	5	
3m + 4 = -11	2. $12 = -7f - 9$	3. $-3 = 2 + \frac{a}{11}$
4. $\frac{3}{2}a - 8 = 11$	5. $8 = \frac{x-5}{7}$	6. $\frac{c+1}{-3} = -21$

Example 2 p. 92

- **7. NUMBER THEORY** Twelve decreased by twice a number equals -34. Write an equation for this situation and then find the number.
- **8. BASEBALL** Among the career home run leaders for Major League Baseball, Hank Aaron has 175 fewer than twice the number that Dave Winfield has. Hank Aaron hit 755 home runs. Write an equation for this situation. How many home runs did Dave Winfield hit in his career?

Example 3 p. 93

- **3** Write an equation and solve each problem.
 - **9.** Find three consecutive odd integers with a sum of 75.
 - **10.** Find three consecutive integers with a sum of -36.

StudyTip

Representing Consecutive Integers You can use the same expressions to represent either consecutive even integers or consecutive odd integers. It is the value of *n* (odd or even) that differs between the two expressions.

Practice and Problem Solving

p. 91

Example 1 Solve each equation. Check your solution.

11. $3t + 7 = -8$	12. $8 = 16 + 8n$	13. $-34 = 6m - 4$
14. $9x + 27 = -72$	15. $\frac{y}{5} - 6 = 8$	16. $\frac{f}{-7} - 8 = 2$
17. $1 + \frac{r}{9} = 4$	18. $\frac{k}{3} + 4 = -16$	19. $\frac{n-2}{7} = 2$
20. $14 = \frac{6+z}{-2}$	21. $-11 = \frac{a-5}{6}$	22. $\frac{22-w}{3} = -7$

Example 2 p. 92 **BUSINESS** The Cell+ Cellular Phone store offers the plans shown in the table. Raul chose the business plan and has budgeted \$100 per month. Write an equation for this situation, and determine how many minutes per month he can use the phone and stay within budget.

Plan	Flat Monthly Fee	Anytime Minutes	Cost per Minute After Anytime Minutes
personal	\$29.99	250	\$0.20
business	\$49.99	650	\$0.15
executive	\$59.99	1200	\$0.10

Example 3

p. 93

Write an equation and solve each problem.

- 24. Fourteen less than three fourths of a number is negative eight. Find the number.
- 25. Seventeen is thirteen subtracted from six times a number. What is the number?
- **26.** Find three consecutive even integers with the sum of -84.
- **27.** Find three consecutive odd integers with the sum of 141.
- 28. Find four consecutive integers with the sum of 54.
- **29.** Find four consecutive integers with the sum of -142.

Solve each equation. Check your solution.

~

0

0.4

30.	-6m - 8 = 24	51.	45 = 7 - 5n
32.	$\frac{2b}{3} + 6 = 24$	33.	$\frac{5x}{9} - 11 = -51$
34.	$65 = \frac{3}{4}c - 7$	35.	$9 + \frac{2}{3}x = 81$
36.	$-\frac{5}{2} = \frac{3}{4}z + \frac{1}{2}$	37.	$\frac{5}{6}k + \frac{2}{3} = \frac{4}{3}$
38.	$-\frac{1}{5} - \frac{4}{9}a = \frac{2}{15}$	39.	$-\frac{3}{7} = \frac{3}{4} - \frac{b}{2}$

Write an equation and solve each problem.

- **40. FAMILY** The ages of three brothers are consecutive integers with the sum of 96. How old are the brothers?
- **41. VOLCANOES** Lava movement can build up and form beaches at the coast of an island. The growth of an island in a seaward direction may be modeled as 8y + 2 centimeters, where *y* represents the number of years that the lava flows. An island has expanded 60 centimeters seaward. How long has the lava flowed?

Solve each equation. Check your solution.

42.
$$-5x - 4.8 = 6.7$$

4.
$$0.6a + 9 = 14.4$$

4

46.
$$9 = \frac{-6p - (-3)}{-8}$$

43
$$3.7q + 26.2 = 111.67$$

45. $\frac{c}{2} - 4.3 = 11.5$
47. $3.6 - 2.4m = 12$

Real-World Link

The top amusement parks in the world for attendance are:

- Magic Kingdom Orlando, FL
- 2. Disneyland Anaheim, CA
- 3. Tokyo Disneyland Urayasu, Chiba, Japan
- 4. Tokyo Disney Sea Urayasu, Chiba, Japan
- Disneyland Paris Cedex, Marne La Valle, France

- **48.** If 7m 3 = 53, what is the value of 11m + 2?
- **49.** If 13y + 25 = 64, what is the value of 4y 7?
- **50.** If -5c + 6 = -69, what is the value of 6c 15?
- **-51. AMUSEMENT PARKS** An amusement park offers a yearly membership of \$275 that allows for free parking and admission to the park. Members can also use the water park for an additional \$5 per day. Nonmembers pay \$6 for parking, \$15 for admission, and \$9 for the water park.
 - **a.** Write and solve an equation to find the number of visits it would take for the total cost to be the same for a member and a nonmember if they both use the water park at each visit.
 - **b.** Make a table for the costs of members and nonmembers after 3, 6, 9, 12, and 15 visits to the park.
 - **c.** Plot these points on a coordinate graph and describe things you notice from the graph.
- **52. SHOPPING** At The Family Farm, you can pick your own fruits and vegetables.
 - **a.** The cost of a bag of potatoes is \$1.50 less than $\frac{1}{2}$ of
 - the price of apples. Write and solve an equation to find the cost of potatoes.
 - **b.** The price of each zucchini is 3 times the price of winter squash minus \$7. Write and solve an equation to find the cost of zucchini.
 - **c.** Write an equation to represent the cost of a pumpkin using the cost of the blueberries.

H.O.T. Problems Use Higher-Order Thinking Skills

- **53. OPEN ENDED** Write a problem that can be modeled by the equation 2x + 40 = 60. Then solve the equation and explain the solution in the context of the problem.
- **54. REASONING** Describe the steps you can use to solve $\frac{w+3}{5} 4 = 6$.
- **55. CHALLENGE** To find the measure of an interior angle of a regular polygon, you can use the formula $m = \frac{180(n-2)}{n}$, where *m* represents the measure of each angle, and *n* represents the number of sides in the polygon. If m = 156, how many sides does the polygon have?
- **56. CHALLENGE** Determine whether the following statement is *sometimes, always,* or *never* true. Explain your reasoning.

The sum of three consecutive odd integers equals an even integer.

57. WRITING IN MATH Write a paragraph explaining the order of the steps that you would take to solve a linear equation.





KCCT PRACTICE MA-HS-5.3.1



59. GRIDDED RESPONSE The scale factor for two similar triangles is 2 : 3. The perimeter of the smaller triangle is 56 cm. What is the perimeter of the larger triangle in centimeters?

60. Mr. Morrison is draining his cylindrical pool. The pool has a radius of 10 feet and a standard height of 4.5 feet. If the pool water is pumped out at a constant rate of 5 gallons per minute, about how long will it take to drain the pool? (1 $ft^3 = 7.5$ gal)

F	37.8 min	Η	25.4 h
G	7 h	J	35.3 h

61. STATISTICS Look at the golf scores for the five players in the table.

Player	1	2	3	4	5
Score	80	91	103	79	78

Which of these is the range of the golf scores?

A	10	С	35
В	25	D	40

Spiral Review

- **62.** GAS MILEAGE A midsize car with a 4-cylinder engine travels 34 miles on a gallon of gas. This is 10 miles more than a luxury car with an 8-cylinder engine travels on a gallon of gas. How many miles does a luxury car travel on a gallon of gasoline? (Lesson 2-2)
- **63. DEER** In a recent year, 1286 female deer were born in Clark County. That is 93 fewer than the number of male deer born. How many male deer were born that year? (Lesson 2-2)

Translate each equation into a verbal sentence. (Lesson 2-1)

64. <i>f</i> − 15 = 6	65. $3h + 7 = 20$	66. $k^2 + 18 = 54 - m$
67. $3p = 8p - r$	68. $\frac{3}{5}t + \frac{1}{3} = t$	69. $\frac{1}{2}v = \frac{2}{3}v + 4$

70. GEOGRAPHY The Pacific Ocean covers about 46% of Earth. If *P* represents the area of the Pacific Ocean and *E* represents the area of Earth, write an equation for this situation. (Lesson 2-1)

Find the value of n in each equation. Then name the property that is used. (Lesson 1-3)

71.	1.5 + n = 1.5	72. 8 <i>n</i> = 1
73.	4 - n = 0	74. 1 = 2 <i>n</i>

Skills Review

Evaluate each expression. (Lesson 1-2) **76.** $\frac{38-12}{2\cdot 13}$

75.	5	+	$3(4^2)$	
-----	---	---	----------	--

77. $[5(1+1)]^3$

Solving Equations with the Variable on Each Side

Then

You solved multi-step equations. (Lesson 2-3)

Now/

- Solve equations with the variable on each side.
- Solve equations involving grouping symbols.

KY Program of Studies

HS-NPO-S-PNO4 Students will justify the solution steps in simplifying expressions or solving an equation. HS-AT-S-EI3 Students will solve one-variable equations and inequalities using manipulatives, symbols, procedures and graphing, including graphing the solution set on a number line. Also addresses HS-AT-S-EI1, HS-AT-S-EI4, and HS-AT-S-EI18.

New Vocabulary

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Why?

Eating habits in the United States have changed since 1985. The equation y = 1.3x + 19 represents the number of times Americans eat in their cars each year, where x is the number of years since 1985, and y is the number of times that they eat in their car. The equation y = -1.3x + 93 represents the number of times Americans eat in restaurants each year, where x is the number of years since 1985, and y is the number of times that they eat in their car. The equation y = -1.3x + 93 represents the number of times Americans eat in restaurants each year, where x is the number of years since 1985, and y is the number of times that they eat in a restaurant.

The equation 1.3x + 19 = -1.3x + 93 represents the year when the number of Americans eating in their cars will equal the number of Americans who eat in restaurants each year.



Variables on Each Side To solve an equation that has variables on each side, use the Addition or Subtraction Property of Equality to write an equivalent equation with the variable terms on one side.

EXAMPLE 1 Solve an Equation with Variables on Each Side

Solve 2 + 5k = 3k - 6. Check your solution.

2+5k = 3k-6	Original equation
-3k = -3k	Subtract 3k from each side.
2 + 2k = -6	Simplify.
-2 = -2	Subtract 2 from each side.
2k = -8	Simplify.
$\frac{2k}{2} = \frac{-8}{2}$	Divide each side by 2.
k = -4	Simplify.
CHECK $2 + 5k = 3k - 6$	Original equation
$2 + 5(-4) \stackrel{?}{=} 3(-4) - 6$	Substitution, $k = -4$
$2 + -20 \stackrel{?}{=} -12 - 6$	Multiply.
-18 = -18 🖌	Simplify.

Check Your Progress

Solve each equation. Check your solution.

1A.
$$3w + 2 = 7w$$

1C. $\frac{x}{2} + 1 = \frac{1}{4}x - 6$

1B. 5a + 2 = 6 - 7a

1D. 1.3c = 3.3c + 2.8

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Grouping Symbols If equations contain grouping symbols such as parentheses or brackets, use the Distributive Property first to remove the grouping symbols.

StudyTip

Solving an Equation You may want to eliminate the terms with a variable from one side before eliminating a constant.

EXAMPLE 2	Solve an Equ	lation with (Grouping S	vmbols
	JUIVE all Lyu		JI UUPIIIK J	YIIIDUIS

```
Solve 6(5m - 3) = \frac{1}{3}(24m + 12).
      6(5m-3) = \frac{1}{3}(24m+12)
                                           Original equation
       30m - 18 = 8m + 4
                                           Distributive Property
30m - 18 - 8m = 8m + 4 - 8m
                                           Subtract 8m from each side.
       22m - 18 = 4
                                           Simplify.
22m - 18 + 18 = 4 + 18
                                           Add 18 to each side.
             22m = 22
                                           Simplify.
             \frac{22m}{22} = \frac{22}{22}
                                           Divide each side by 22.
                m = 1
                                           Simplify.
```

Check Your Progress

Solve each equation. Check your solution.

2A. 8s - 10 = 3(6 - 2s)

2B. 7(n-1) = -2(3+n)Personal Tutor glencoe.com

Some equations may have no solution. That is, there is no value of the variable that will result in a true equation. Some equations are true for all values of the variables. These are called **identities**.

EXAMPLE 3 Find Special Solutions

 Solve each equation.

 a. 5x + 5 = 3(5x - 4) - 10x

 5x + 5 = 3(5x - 4) - 10x Original equation

 5x + 5 = 15x - 12 - 10x Distributive Property

 5x + 5 = 5x - 12 Simplify.

 -5x = -5x Subtract 5x from each side.

 $5 \neq -12$ This statement is false.

Since $5 \neq -12$, this equation has no solution.

b. 3(2b-1) - 7 = 6b - 10

3(2b - 1) - 7 = 6b - 10	Original equation
6b - 3 - 7 = 6b - 10	Distributive Property
6b - 10 = 6b - 10	Simplify.
0 = 0	Subtract $6b - 10$ from each side.

Since the expressions on each side of the equation are the same, this equation is an identity. It is true for all values of *b*.

Check Your Progress

3A. 7x + 5(x - 1) = -5 + 12x

3B. 6(y-5) = 2(10+3y)

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Personal Tutor glencoe.com
```

ReadingMath

No Solution The symbol that represents no solution is Ø.

The steps for solving an equation can be summarized as follows.

Concep	Concept Summary Steps for Solving Equations				
Step 1	Simplify the expressions on each side. Use the Distributive Property as needed.				
Step 2	Use the Addition and/or Subtraction Properties of Equality to get the variables on one side and the numbers without variables on the other side. Simplify.				
Step 3	Use the Multiplication or Division Property of Equality to solve.				
	Math in Motion, BrainPOP [®] glencoe.com				

There are many situations in which variables are on both sides of the equation.



Read the Test Item

The area of the first rectangle is 10x, and the area of the second is 6(3 + x). The equation 10x = 6(3 + x) represents this situation.

Solve the Test Item

$\mathbf{A} 10x = 6(3+x)$	$\mathbf{B} \qquad 10x = 6(3+x)$
$10(3) \stackrel{?}{=} 6(3+3)$	$10(4.5) \stackrel{?}{=} 6(3+4.5)$
$30 \stackrel{?}{=} 6(6)$	$45 \stackrel{?}{=} 6(7.5)$
30 ≠ 36 ×	45 = 45 ✓

Since the value 4.5 results in a true statement, you do not need to check 6.5 and 7. The answer is B.

Check Your Progress

4. Find the value of *x* so that the figures have the same perimeter.



Test-TakingTip

Choose a Method There is often more than one way to solve a problem. In this example, you can write an algebraic equation and solve for *x*. Or you can substitute each answer choice into the formulas to find the correct answer.

🗹 Check Your Understanding



36. 5[2v - 4(v + 5)] = 25

35. 3.2k - 4.3 = 12.6k + 14.5



Real-World Link

About 25% of American adults have Internet access and give time or money to charity. Of those, 56% have never visited the Web site of a charity, and only 7% say they have given online.

Source: Craver, Mathews, Smith & Co.

- **37. NUMBER THEORY** Three times the lesser of two consecutive even integers is 6 less than six times the greater number. Find the integers.
- **38. MONEY** Chris and Nora have been saving their quarters to give to a charity. So far, Chris has saved twice the number of quarters that Nora saved plus 6. The number of quarters Chris saved is also five times the difference of the number of quarters and 3 that Nora has saved. Write and solve an equation to find the number of quarters they each have saved.
- **DVD** A company that replicates DVDs spends \$1500 per day in building overhead plus \$0.80 per DVD in supplies and labor. If the DVDs sell for \$1.59 per disk, how many DVDs must the company sell each day before it makes a profit?
- **40. INTERNET ACCESS** The table shows the percent of households that have broadband Internet access and the average growth rates for two age groups. How long will it take for the percents to be the same?

Age Group	Percent with Broadband in Their Homes in 2006	Growth Rate Percentage per Year
18–49	52.5	42
50+	25.5	52

Source: Pe	w Internet	&	American	Life	Project
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- **41. Solution MULTIPLE REPRESENTATIONS** In this problem, you will explore the equation 2x + 4 = -x 2.
 - **a. GRAPHICAL** Make a table of values with five points for the equations y = 2x + 4 and y = -x 2. Graph the points from the tables.
 - **b. ALGEBRAIC** Solve the equation 2x + 4 = -x 2.
 - **c. VERBAL** Explain how the solution you found in part b is related to the intersection point of the graphs in part a.

H.O.T. Problems Use Higher-Order Thinking Skills

42. REASONING Solve the following equation. Describe each step in your solution.

t = 2 - 2[2t - 3(1 - t)]

- **43. CHALLENGE** Write an equation with the variable on each side of the equals sign, at least one fractional coefficient, and a solution of -6. Discuss the steps you used to write the equation.
- **44. OPEN ENDED** Create an equation with at least two grouping symbols for which there is no solution.
- **45. REASONING** Determine whether each solution is correct. If the solution is not correct, describe the error and give the correct solution.

a.
$$2(g+5) = 22$$

 $2g+5 = 22$
 $2g+5 = 22$
 $2g+5 - 5 = 22$
 $2g = 17$
 $2g = 8.5$
b. $5d = 2d - 18$
 $5d - 2d = 2d - 18 - 2d$
 $d = -6$
c. $-6z + 13 = 7z$
 $-6z + 13 - 6z = 7z - 6z$
 $13 = z$

46. CHALLENGE Find the value of *k* for which each equation is an identity.

```
a. k(3x - 2) = 4 - 6x
b. 15y - 10 + k = 2(ky - 1) - y
```

47. WRITING IN MATH Compare and contrast solving equations with variables on both sides of the equation to solving one-step or multi-step equations with a variable on one side of the equation.

KCCT PRACTICE MA-HS-5.3.1

48. A hang glider 25 meters above the ground starts to descend at a constant rate of 2 meters per second. Which equation could be used to determine *h*, the hang glider's height after *t* seconds of descent?

A h = 25t + 2t **B** h = -25t + 2 **C** h = 2t + 25**D** h = -2t + 25

49. GEOMETRY Two rectangular walls each with a length of 12 feet and a width of 23 feet need to be painted. It costs \$0.08 per square foot for paint. How much will it cost to paint the walls?

F	\$22.08	Н	\$34.50
G	\$23.04	J	\$44.16

50. SHORT RESPONSE Maddie works at Game Exchange. They are having a sale on used video games and DVDs.

ltem	Price	Special
video games	\$20	Buy 2 get 1 Free
DVDs	\$15	Buy 1 get 1 Free

She purchases four video games and uses her employee discount of 15%. If sales tax is 7.25%, how much does she spend on the games?

> **54.** $\frac{1}{2}z + 7 = 16 - \frac{3}{5}z$ **57.** $9 + \frac{y}{5} = 6$

51. Solve the equation $\frac{4}{5}x + 7 = \frac{3}{15}x - 3$.

A $-16\frac{2}{3}$	C $-6\frac{2}{3}$
B $-14\frac{4}{9}$	D -10

Spiral Review

Solve each equation. Check your solution. (Lesson 2-3)

52.	5n + 6 = -4	53. $-1 = 7 + 3c$
55.	$\frac{2}{5}x + 6 = \frac{2}{3}x + 10$	56. $\frac{a}{7} - 3 = -2$

- **58.** WORLD RECORDS In 1998, Winchell's House of Donuts in Pasadena, California, made the world's largest donut. It weighed 5000 pounds and had a circumference of 298.3 feet. What was the donut's diameter to the nearest tenth? (*Hint:* $C = \pi d$) (Lesson 2-2)
- **59. ZOO** At a zoo, the cost of admission is posted on the sign. Find the cost of admission for two adults and two children. (Lesson 1-3)

Find the value of *n* in each equation. Then name the property that is used in each step. (Lesson 1-3)

60.	25n = 25	61. $n \cdot 1 = 2$	62. $12 \cdot n = 12 \cdot 6$
63.	$n+0 = \frac{2}{3}$	64. $4 \cdot \frac{1}{4} = n$	65. $(10 - 8)(7) = 2(n)$

Skills Review

Translate each sentence into an equation. (Lesson 2-1)

66. Twice a number *t* decreased by eight equals seventy.

67. Five times the sum of *m* and *k* is the same as seven times *k*.

68. Half of *p* is the same as *p* minus 3.

Evaluate each expression. (Lesson 0-3)

69. -9 - (-14)	70. -10 + (20)	71. -15 - 9
72. 5(14)	73. -55 ÷ (-5)	74. -25(-5)

102 Chapter 2 Linear Equations



Solving Equations Involving Absolute Value

Then

You solved equations with the variable on each side. (Lesson 2-5)

Now/

- Evaluate absolute value expressions.
- Solve absolute value equations.

KY Program of Studies

HS-AT-S-EI4 Students will solve linear equations and inequalities in one variable including those involving the absolute value of a linear function. HS-AT-S-EI19 Students will use the skills learned to solve linear equations and inequalities to solve numerically, graphically or symbolically non-linear equations (e.g., absolute value, quadratic, exponential equations). Also addresses HS-AT-S-VEO1, HS-AT-S-EI1, and HS-AT-S-EI3.

KY Math Online

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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Why?

In 2007, a telephone poll was conducted to determine the reading habits of Americans. People in this survey were allowed to select more than one type of book.

The survey had a margin of error of 3 percentage points. This means that the results could be three points higher or lower. So, the percent of people who read religious material could be as high as 69% or as low as 63%.



Absolute Value Expressions Expressions with absolute values define an upper and lower range in which a value must lie. Expressions involving absolute value can be evaluated using the given value for the variable.

EXAMPLE 1	Expressions with	th Absolute Value	
Evaluate <i>m</i> +	6 -14 if $m = 4$.		
m + 6 − 14 =	= 4 + 6 - 14	Replace <i>m</i> with 4.	
=	= 10 - 14	4 + 6 = 10	
=	= 10 - 14	10 = 10	
=	= -4	Simplify.	
Check Your P	rogress	2	
1. Evaluate 23	- 3-4x if $x =$	2.	N
			-> Personal Tutor glencoe.com

Absolute Value Equations Looking at the example at the top of the page, we notice that the margin of error in the bar graph is an example of absolute value. The distance between 66 and 69 on a number line is the same as the distance between 63 and 66.



There are three types of open sentences involving absolute value, |x| = n, |x| < n, and |x| > n. In this lesson, we will consider only the first type. Look at the expression |x| = 4. This means that the distance between 0 and *x* is 4.

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- 1	!	1				1	1	1		1	_
	·	1				Т					
-5-	4—;	3—	2—1	()	1	2	3	4	5	

If |x| = 4, then x = -4 or x = 4. Thus, the solution set is $\{-4, 4\}$.

For each absolute value equation, we must consider both cases. To solve an absolute value equation, first isolate the absolute value on one side of the equals sign if it is not already by itself.

ReadingMath

Absolute Values The expression |f + 5| is read the *absolute value of the quantity f plus 5.*

Key	Concept Absolute Value Equations For Your FOLDABLE
Words	When solving equations that involve absolute values, there are two cases to consider.
	Case 1: The expression inside the absolute value symbol is positive.
	Case 2: The expression inside the absolute value symbol is negative.
Symbols	For any real numbers a and b, if $ a = b$, then $a = b$ or $a = -b$.
Example	d = 10, so $d = 10$ or $d = -10$.

EXAMPLE 2 Solve Absolute Value Equations

Solve each equation. Then graph the solution set.

a.
$$|f + 5| = 17$$

|f+5| = 17 Original equation

Case 1		Case 2
f + 5 = 17		f + 5 = -17
<i>f</i> + 5 - 5 = 17 - 5	Subtract 5 from each side.	f + 5 - 5 = -17 - 5
f = 12	Simplify.	f = -22
<mark>→ ●</mark> -25-2	<u>+ </u>	► 20 25

b. |b - 1| = -3

|b - 1| = -3 means the distance between *b* and 1 is -3. Since distance cannot be negative, the solution is the empty set \emptyset .

Check Your Progress

2A. |y + 2| = 4

2B. |3n - 4| = -1

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Absolute value equations occur in real-world situations that describe a range within which a value must lie.

Real-World EXAMPLE 3 Solve an Absolute Value Equation

SNAKES The temperature of an enclosure for a pet snake should be about 80°F, give or take 5°. Find the maximum and minimum temperatures.

You can use a number line to solve.



The distance from 80 to 75 is 5 units. The distance from 80 to 85 is 5 units.

The solution set is $\{75, 85\}$. The maximum and minimum temperatures are 85° and 75° .



Real-World Link

In 2001, the number of households in the U.S. that had either a turtle, snake, lizard, or other reptile as a pet was 1,678,000.

Source: American Veterinary Medical Association

Check Your Progress

3. ICE CREAM Ice cream should be stored at 5°F with an allowance for 5°. Write and solve an equation to find the maximum and minimum temperatures at which the ice cream should be stored.

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When given two points on a graph, you can write an absolute value equation to describe the distance between those two points.

StudyTip /

Find the Midpoint To find the point midway between two points, add the values together and divide by 2. For Example 4, 11 + $19 = 30, 30 \div 2 = 15$. So 15 is the point halfway between 11 and 19.

EXAMPLE 4 Write an Absolute Value Equation

Write an equation involving absolute value for the graph.

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Find the point that is the same distance from 11 and from 19. This is the midpoint between 11 and 19, which is 15.



The distance from 15 to 11 is 4 units. The distance from 15 to 19 is 4 units.

So an equation is |x - 15| = 4.

Check Your Progress

4. Write an equation involving absolute value for the graph.

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🗹 Check Your Understanding

Example 1	Evaluate each expression	on if $f = 3$, $g = -4$, and h	= 5.	
р. 103	1. $ 3 - h + 13$	2. $16 - g + 9 $	3. $ f + g - h$	
Example 2	Solve each equation. T	hen graph the solution se	t.	
р. 104	4. $ n+7 = 5$	5. $ 3z - 3 = 9$	6. $ 4n-1 = -6$	
	7. $ b+4 = 2$	8. $ 2t - 4 = 8$	9. $ 5h+2 = -8$	
Example 3 p. 104	10. PROFIT For a compa a 12% return on inve the least and the gre	ny to invest in a product, estment (ROI) plus or min eatest ROI they will receive	they must believe they will receiv us 3%. Write an equation to find e before they invest.	ve
Example 4	Write an equation invo	lving absolute value for o	each graph.	
р. 105	11 -4 - 3 - 2 - 1 0 1 2 3	↓ ↓ ↓ ↓ 12. ↓ 4 5 6 −1	· • · · · · · · · · · · · · · · · · · ·	•

Practice and Problem Solving

Step-by-Step Solutions begin on page R12.
 Extra Practice begins on page 815.

Example 1	Evaluate each expression	if $a = -2$, $b = -3$, $c = 2$, x	= 2.1, y = 3, and $z = -4.2$.		
р. 103	13 $ 2x + z + 2y$	14. $4a - 3b + 2c $	15. $- 5a + c + 3y + 2z $		
	16. $-a + 2x - a $	17. $ y - 2z - 3$	18. $3 3b - 8c - 3$		
	19. $ 2x - z + 6b$	20. $-3 z + 2(a + y)$	21. $-4 c-3 +2 z-a $		
Example 2	Solve each equation. The	n graph the solution set.			
р. 104	22. $ n-3 = 5$	23. $ f + 10 = 1$	24. $ v-2 = -5$		
	25. $ 4t - 8 = 20$	26. $ 8w + 5 = 21$	27. $ 6y-7 = -1$		
	28. $\left \frac{1}{2}x+5\right = -3$	29. $ -2y+6 = 6$	30. $\left \frac{3}{4}a - 3\right = 9$		
Example 3 p. 104	31. SURVEY The circle grap of a survey that asked, be rich some day?" If t is the range of the pero likely that they will be	where the right shows the rest, "How likely is it that you where margin of error is $\pm 4\%$, where the teens who say it is very rich?	vills Not Likely vill at All what ery Very Likely		
	32. CHEERLEADING For comis preparing a dance rorrules, the routine must of ±5 seconds.	2. CHEERLEADING For competition, the cheerleading team is preparing a dance routine. According to competition rules, the routine must last 4 minutes, with a variation of ± 5 seconds. a Fig. 1 the last tendence is the time of			
	a. Find the least and grather routine.	reatest possible times for			
	b. Find the least and g	reatest possible times in sec	onds.		
Example 4	Write an equation involv	ing absolute value for each	graph.		
p. 100	33.	↓ ↓ 34. 4 5 -10-8-	<mark>◆ ↓ ↓</mark> -6-4-2 0 2 4 6 8 10		
	35.	36. → 36. → − 7−6−!	5-4-3-2-1 0 1 2 3		
	Solve each equation. The	n graph the solution set.			
	37. $\left -\frac{1}{2}b - 2 \right = 10$	38. $ -4d+6 = 12$	39. $ 5f-3 = 12$		
	40. $2 h - 3 = 8$	41. $4 - 3 q = 10$	42. $\frac{4}{ p } + 12 = 14$		
	43. TRACK The 4×400 rela	y is a race where 4 runners f	take turns running 400 meters,		

- **43. TRACK** The 4×400 relay is a race where 4 runners take turns running 400 meters, or one lap around the track. During the race, one runner has to successfully pass a baton to the next runner.
 - **a.** If a runner runs the first leg in 52 seconds plus or minus 2 seconds, write an equation to find the fastest and slowest times.
 - **b.** If the runner of the second leg runs it in 53 seconds plus or minus 1 second, write an equation to find the fastest and slowest times.
 - **c.** Suppose the runner of the fourth leg is the fastest on the team. If he runs an average of 50.5 seconds plus or minus 1.5 seconds, what are the team's fastest and slowest times?



Real-World Link

An increasing number of fashion designers are using computer-aided design (CAD). CAD allows designers to view designs on virtual models.

Source: Bureau of Labor Statistics

- **44. FASHION** To allow for a model's height, a designer is willing to use models that require him to change hems either up or down 2 inches. The length of the skirts is 20 inches.
 - **a.** Write an absolute value equation that represents the length of the skirts.
 - **b.** What is the range of the lengths of the skirts?
 - **c.** If a 20-inch skirt was fitted for a model that is 5 feet 9 inches tall, will the designer use a 6-foot-tall model?
- **45. CARS** Speedometer accuracy can be affected by many details such as tire diameter and axle ratio. For example, there is variation of ± 3 miles per hour when calibrated at 50 miles per hour.
 - a. What is the range of actual speeds of the car if calibrated at 50 miles per hour?
 - **b.** A speedometer calibrated at 45 miles per hour has an accepted variation of ± 1 mile per hour. What can we conclude from this?

Write an equation involving absolute value for each graph.

46.	-5 -4 -3 -2 -1 0 1 2 3 4 5
47.	<u>-5</u> -4 -3 -2 -1 0 1 2 3 4 5
48.	<u>-5</u> -4 -3 -2 -1 0 1 2 3 4 5
49.	-2 -1 0 1 2
50.	-3 -2 -1 0 1 2 3
51.	-3 -2 -1 0 1 2 3

- **52. MUSIC** An audio playback recorder will record an hour and a half at a time with an allowance of plus or minus 3 minutes for time to change tracks.
 - **a.** Write an absolute value equation that represents the recording time.
 - **b.** What is the range of time in minutes that the playback recorder could run?
 - c. Graph the possible times on a number line.
- ACOUSTICS The Red Rocks Amphitheater located in the Red Rock Park near Denver, Colorado, is the only naturally occurring amphitheater. The acoustic qualities here are such that a maximum of 20,000 people, plus or minus 1000, can hear natural voices clearly.
 - **a.** Write an equation involving an absolute value that represents the number of people that can hear natural voices at Red Rocks Amphitheater.
 - **b.** Find the maximum and minimum number of people that can hear natural voices clearly in the amphitheater.
 - **c.** What is the range of people that can clearly hear natural voices here?


Real-World Link

Each spring since 1997, Gallaudet University holds it National Academic Bowl for Deaf and Hard of Hearing High School Students. Here, teams of high school students compete in local, regional, and national events throughout the U.S.

Source: Galluadet University

- **54. BOOK CLUB** A book club meets to discuss a portion of a book that they are reading. The members of the club agree to read within ten pages of the last page of the chapter. The chapter ends on page 203.
 - **a.** Write an absolute value equation that represents the pages where club members could stop reading.
 - **b.** Write the range of the pages where the club members could stop reading.
 - **SCHOOL** Washington High School and McKinley High School are competing in an academic challenge game. The team with a correct response is awarded 10 points. An incorrect response has a point value of -10. There are 5 mathematics questions.
 - **a.** Write an equation that represents the scoring for the challenge.
 - **b.** Make a table of values for the possible points that a school could receive during the mathematics portion of the challenge.
 - **c.** Write about how absolute values can be used in classes other than math.

H.O.T. Problems

Use Higher-Order Thinking Skills

56. OPEN ENDED Describe a real-world situation that could be represented by the absolute value equation |x - 4| = 10.

REASONING Determine whether the following statements are *sometimes*, *always*, or *never* true, if *c* is a whole number. Explain your reasoning.

- **57.** The value of |x + 1| is greater than zero.
- **58.** The solution of |x + c| = 0 is greater than 0.
- **59.** The inequality |x| + c < 0 has no solution.
- **60.** The value of |x + c| + c is greater than zero.
- 61. **REASONING** Explain why an absolute value can never be negative.
- **62. CHALLENGE** Use the sentence $x = 7 \pm 4.6$.
 - **a.** Describe the values of *x*.
 - **b.** Translate the sentence into an expression involving absolute value.
- **63.** FIND THE ERROR Alex and Wesley are solving |x + 5| = -3. Is either of them correct? Explain your reasoning.

 Alex
 Wesley

 |x + 5| = 3 or |x + 5| = -3 |x + 5| = -3

 x + 5 = 3 x + 5 = -3

 $\frac{-5}{x} = -5$ $x = \emptyset$

64. WRITING IN MATH Explain why there are either two, one, or no solutions for equations involving absolute values. Demonstrate an example of each possibility.

KCCT PRACTICE MA-HS-5.3.1

- **65.** Which equation represents the second step of the solution process? Step 1 4(2x + 7) - 6 = 3xStep 2 Step 3 5x + 28 - 6 = 0Step 4 5x = -22Step 5 x = -4.4A 4(2x-6) + 7 = 3x**B** 4(2x + 1) = 3x**C** 8x + 7 - 6 = 3x**D** 8x + 28 - 6 = 3x**66. GEOMETRY** The area of a circle is 25π square centimeters. What is the circumference? F 625π cm G 50π cm H 5π cm J 10π cm
- **67.** Tanya sells cosmetics door-to-door. She makes \$5 an hour and 15% commission of the total dollar value on whatever she sells. If Tanya's commission is increased to 17%, how much money will she make if she sells \$300 worth of product and works 30 hours?
 - **A** \$201
 - **B** \$226
 - **C** \$255
 - **D** \$283
- **68. EXTENDED RESPONSE** John just received his learner's permit. His mother has agreed to take him driving every day for two weeks. On the first day, John drives for 20 minutes. Each day after that, John's mother has agreed he can drive 5 minutes more than the day before.
 - **a.** Write a formula for the *n*th term of the sequence. Explain how you found the formula.
 - **b.** For how many minutes will John drive on the last day? Show how you found the number of minutes.
 - **c.** John's driver's education teacher requires that each student drive for 30 hours with an adult outside of class. Will John fulfill this requirement?

Spiral Review

Write and solve each equation. (Lesson 2-4)

- 69. One half of a number increased by 16 is four less than two thirds of the number.
- **70.** The sum of one half of a number and 6 equals one third of the number.
- **71. SHOE** If ℓ represents the length of a man's foot in inches, the expression $2\ell 12$ can be used to estimate his shoe size. What is the approximate length of a man's foot who wears a size 8? (Lesson 2-3)

Skills Review

Write an equation for each problem. Then solve the equation. (Lesson 2-2)

- **72.** Seven times a number equals -84. What is the number?
- **73.** Two fifths of a number equals -24. Find the number.
- **74.** Negative 117 is nine times a number. Find the number.
- **75.** Twelve is one fifth of a number. What is the number?

Translate each sentence into an equation. (Lesson 2-1)

- 1. The sum of three times *a* and four is the same as five times *a*.
- **2.** One fourth of *m* minus six is equal to two times the sum of *m* and 9.
- **3.** The product of five and *w* is the same as *w* to the third power.
- **4. MARBLES** Drew has 50 marbles in three colors: red, green, and blue. Drew has six more red marbles than blue marbles and four fewer green marbles than blue marbles. Write and solve an equation to determine how many blue marbles Drew has. (Lesson 2-1)

Solve each equation. Check your solution. (Lesson 2-2)

5. p + 8 = 13

CHAPTER

- **6.** -26 = b 3
- **7.** $\frac{t}{6} = 3$
- 8. MULTIPLE CHOICE Solve the equation $\frac{3}{5}a = \frac{1}{4}$. (Lesson 2-2) A $a = \frac{3}{20}$ B a = 2C $a = \frac{5}{12}$ D a = -3

Solve each equation. Check your solution. (Lesson 2-3)

- **9.** 2x + 5 = 13
- **10.** -21 = 7 4y
- **11.** $\frac{m}{6} 3 = 8$
- **12.** $-4 = \frac{d+3}{5}$
- **13. ANIMALS** The average length of a yellow-banded angelfish is 12 inches. This is 4.8 times as long as an average common goldfish. (Lesson 2-3)
 - **a.** Write an equation you could use to find the length of the common goldfish.
 - **b.** What is the length of an average common goldfish?

Write an equation and solve each problem. (Lesson 2-3)

- **14.** Three less than three fourths of a number is negative 9. Find the number.
- **15.** Thirty is twelve added to six times a number. What is the number?
- **16.** Find four consecutive integers with a sum of 106.

Solve each equation. Check your solution. (Lesson 2-4)

17.
$$8p + 3 = 5p + 9$$

18. $\frac{3}{4}w + 6 = 9 - \frac{1}{4}w$
19. $\frac{z+6}{3} = \frac{2z}{4}$

20. PERIMETER Find the value of *x* so that the triangles have the same perimeter. (Lesson 2-4)



21. PRODUCTION ABC Sporting Goods Company produces baseball gloves. Their fixed monthly production cost is \$8000 with a per glove cost of \$5. XYZ Sporting Goods Company also produces baseball gloves. Their fixed monthly production cost is \$10,000 with a per glove cost of \$3. Find the value of *x*, the number of gloves produced monthly, so that the total monthly production cost is the same for both companies. (Lesson 2-4)

Evaluate each expression if x = -4, y = 7, and z = -9. (Lesson 2-5)

- **22.** |3x 2| + 2y
- **23.** |-4y+2z|-7z
- **24.** MULTIPLE CHOICE Solve |6m 3| = 9. (Lesson 2-5)
 - F {2}
 H {-3, 6}

 G {-1, 2}
 I {-3, 3}
- **25. COFFEE** Some say to brew an excellent cup of coffee, you must have a brewing temperature of 200° F, plus or minus 5 degrees. Write and solve an equation describing the maximum and minimum brewing temperatures for an excellent cup of coffee.

Then

You evaluated percents by using a proportion. (Lesson 0-5)

Now/

- Compare ratios.
- Solve proportions.



HS-NPO-S-RP1 Students will calculate and apply ratios, proportions, rates and percentages to solve problems. Also addresses HS-NPO-S-NO9.

New Vocabulary

ratio proportion means extremes rate unit rate scale scale model

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Math in Motion

Ratios and Proportions

Why?

Ratios allow us to compare many items by using a common reference. The table below shows the number of a certain popular fast food restaurant, per 10,000 people, in the United States, as well as other countries. This allows us to compare the number of these restaurants using an equal reference.



Countries	United States	New Zealand	Canada	Australia	Japan	Singapore
Number of Restaurants per 10,000 People	0.433	0.369	0.352	0.349	0.282	0.273

Ratios and Proportions The comparison between the number of restaurants and the number of people is a ratio. A **ratio** is a comparison of two numbers by division. The ratio of *x* to *y* can be expressed in the following ways.

> x to yx:yų

Suppose you wanted to determine the number of restaurants per 100,000 people in Australia. Notice that this ratio is equal to the original ratio.



An equation stating that two ratios are equal is called a **proportion**. So, we can state that $\frac{0.349}{10,000} = \frac{3.49}{100,000}$ is a proportion.

EXAMPLE 1 **Determine Whether Ratios Are Equivalent**

Determine whether $\frac{2}{3}$ and $\frac{16}{24}$ are equivalent ratios. Write *yes* or *no*. Justify your answer.



When expressed in simplest form, the ratios are equivalent.

Check Your Progress

Determine whether each pair of ratios are equivalent ratios. Write yes or no. Justify your answer.

1A.
$$\frac{6}{10}, \frac{2}{5}$$

1B. $\frac{1}{6}, \frac{5}{30}$

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StudyTip

Means and Extremes To solve a proportion using cross products, write an equation that sets the product of the means equal to the product of the extremes.

There are special names for the terms in a proportion.



Key Concept		
Means-	Extremes Property of Proportion	
Words	In a proportion, the product of the extremes is equal to the product of the means.	
Symbols	If $\frac{a}{b} = \frac{c}{d}$ and b , $d \neq 0$, then $ad = bc$.	
Example	Since $\frac{2}{4} = \frac{1}{2}$, 2(2) = 4(1) or 4 = 4.	

Another way to determine whether two ratios form a proportion is to use cross products. If the cross products are equal, then the ratios form a proportion.

This is the same as multiplying the means, and multiplying the extremes.

EXAMPLE 2 Cross	s Products
Use cross products to	determine whether each pair of ratios forms a proportion.
a. $\frac{2}{3.5}, \frac{8}{14}$	
$\frac{2}{3.5} \stackrel{?}{=} \frac{8}{14}$	Original proportion
$2(14) \stackrel{?}{=} 8(3.5)$	Cross products
28 = 28 ✓	Simplify.
The cross products	are equal, so the ratios form a proportion.
b. $\frac{0.3}{1.5}, \frac{0.5}{2.0}$	
$\frac{0.3}{1.5} \stackrel{?}{=} \frac{0.5}{2.0}$	Original proportion
$0.3(2.0) \stackrel{?}{=} 0.5(1.5)$	Cross products
0.6 ≠ 0.75 ×	Simplify.
The cross products	are not equal, so the ratios do not form a proportion.
Check Your Progress	c)
$24 \frac{0.2}{2} 1$	DR <u>15</u> 35
2n. <u>1.8</u> ′ <u>0.9</u>	Personal Tutor glencoe.com

Solve Proportions To solve proportions, use cross products.



The ratio of two measurements having different units of measure is called a rate. For example, a price of \$9.99 per 10 songs is a rate. A rate that tells how many of one item is being compared to 1 of another item is called a unit rate.



Real-World Career

RETAIL BUYER

A retail buyer purchases goods for stores, primarily from wholesalers, for resale to the general public. Buyers use math to determine the amount of each product to order. A bachelor's degree with an emphasis on business studies is usually required.

Real-World EXAMPLE 4 Rate of Growth

RETAIL A clothing retailer has been rapidly expanding the number of retail stores they have in the United States. In the past two years they have opened 232 retail stores. If their rate of growth remains constant, how many retail stores will they have opened in 3 years?

Understand Let *r* represent the number of retail stores.

Plan	Write a proportion for the	he problem.
	<u>232 retail stores</u> _ <i>r</i> retail	stores
	2 years -3 ye	ears
Solve	$\frac{232}{2} = \frac{r}{3}$	Original proportion
	232(3) = (r)2	Find the cross products.
	696 = 2r	Simplify.
	$\frac{696}{2} = \frac{2r}{2}$	Divide each side by 2.
	348 = r	Simplify.
	They will have opened '	348 stores in 3 years

They will have opened 348 stores in 3 years.

Check If the clothing retailer continues to open 232 stores every 2 years, then in 3 years, they will have opened 348 stores.

Check Your Progress

4. EXERCISE It takes 7 minutes for Isabella to walk around the gym track twice. At this rate, how many times can Isabella walk around the track in a half hour?

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A rate called a **scale** is used when making a model of something that is too large or too small to be convenient at actual size. The scale compares the model to the actual size of the object using a proportion. A **scale model** is a three-dimensional reproduction of an item that has been reduced or increased in size proportionally.

Real-World EXAMPLE 5 Scale and Scale Models

MOUNTAIN TRAIL The scale on a map of the Great Smoky Mountains National Park is 3 inches = 10 miles. The length of the Ramsey Cascades Trail is about $1\frac{1}{9}$ inches on the map. What is the actual length of the trail?

Let ℓ represent the actual length.

scale \rightarrow $\frac{3}{10} = \frac{1\frac{1}{8}}{\ell}$ scale actual \rightarrow $\frac{3}{10} = \frac{1\frac{1}{8}}{\ell}$ scale $3(\ell) = 1\frac{1}{8} (10)$ Find the cross products. $3\ell = \frac{45}{4}$ Simplify. $3\ell \div 3 = \frac{45}{4} \div 3$ Divide each side by 3. $\ell = \frac{15}{4} \text{ or } 3\frac{3}{4}$ Simplify.

The actual length is about $3\frac{3}{4}$ miles.

Check Your Progress

5. AIRPLANES On a model airplane, the scale is 5 centimeters = 2 meters. If the wingspan of the scale model is 28.5 centimeters, what is the wingspan of the actual airplane?

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Check Your Understanding

Examples 1 and 2	Determine whether each pair of ratios are equivalent ratios. Write <i>yes</i> or <i>no</i> .			
рр. 111–112	1. $\frac{3}{7}, \frac{9}{14}$	2. $\frac{7}{8}, \frac{42}{48}$	3 $\frac{2.8}{4.4}, \frac{1.4}{2.1}$	
Examples 3	Solve each propor	rtion. If necessary, round to r	nearest hundredth.	
p. 113	4. $\frac{n}{9} = \frac{6}{27}$	5. $\frac{4}{u} = \frac{28}{35}$	6. $\frac{3}{8} = \frac{b}{10}$	
Example 4 p. 113	7. RACE Jennie ra maintain the s	In the first 6 miles of a marath ame pace, how long will it tak	ion in 58 minutes. If she is able to ke her to finish the 26.2 miles?	
Example 5 p. 114	8. MAPS On a roa 8 inches apart. are the two cit	ad map of North Carolina, Ra If the scale on the map is 1 in ies?	leigh and Asheville are about hch = 12 miles, how far apart	



Real-World Link

The Great Smoky Mountains National Park in Tennessee is home to several waterfalls. The Ramsey Cascades is 100 feet tall. It is the tallest in the park.

Source: National Park Service

Math in Motion, Interactive Lab glencoe.com

Practice and Problem Solving

= Step-by-Step Solutions begin on page R12. Extra Practice begins on page 815.

Examples 1 and 2	Determine whether e	each pair of ratios are equiva	lent ratios. Write yes or no.
pp. 111-112	9. $\frac{9}{11}, \frac{81}{99}$	10. $\frac{3}{7'} \frac{18}{42}$	11. $\frac{8.4}{9.2}, \frac{8.8}{9.6}$
	12. $\frac{4}{3}, \frac{6}{8}$	13. $\frac{29.2}{10.4}, \frac{7.3}{2.6}$	14. $\frac{39.68}{60.14}, \frac{6.4}{9.7}$
Example 3	Solve each proportion	n. If necessary, round to the	nearest hundredth.
р. 113	15. $\frac{3}{8} = \frac{15}{a}$	16. $\frac{t}{2} = \frac{6}{12}$	17. $\frac{4}{9} = \frac{13}{q}$
	18. $\frac{15}{35} = \frac{g}{7}$	19. $\frac{7}{10} = \frac{m}{14}$	20. $\frac{8}{13} = \frac{v}{21}$
	21. $\frac{w}{2} = \frac{4.5}{6.8}$	22. $\frac{1}{0.19} = \frac{12}{n}$	23. $\frac{2}{0.21} = \frac{8}{n}$
	24. $\frac{2.4}{3.6} = \frac{k}{1.8}$	25 $\frac{t}{0.3} = \frac{1.7}{0.9}$	26. $\frac{7}{1.066} = \frac{z}{9.65}$
	27. $\frac{x-3}{5} = \frac{6}{10}$	28. $\frac{7}{x+9} = \frac{21}{36}$	29. $\frac{10}{15} = \frac{4}{x-5}$
Example 4 p. 113	30. CAR WASH The B-128 cars in 3 hours cars can they wash	Clean Car Wash washed s. At that rate, how many h in 8 hours?	
Example 5 p. 114	31. MENU A restaurar from selling 110 h 53 hamburgers du their profit?	nt made \$545 in profit amburgers. If they sold uring lunch, what was	
	32. MODELS An artist blocks to build a s Space Center, Flor equals 1.67 feet of The model is 110.3 the actual space sl nearest tenth.	used interlocking building scale model of Kennedy tida. In the model, 1 inch an actual space shuttle. 3 inches tall. How tall is huttle? Round to the	

33. GEOGRAPHY On a map of Florida, the distance between Jacksonville and Tallahassee is 7.5 centimeters. If 2 centimeters = 40 miles, what is the distance between the two cities?



- 40. ATHLETES At Piedmont High School, 3 out of every 8 students are athletes. If there are 1280 students at the school, how many are not athletes?
- 41. BRACES Two out of five students in the ninth grade have braces. If there are 325 students in the ninth grade, how many have braces?
- **42. PAINT** Joel found that it took a half gallon of paint to cover 84 square feet of wall. He has 932 square feet of wall to paint. How many gallons of paint should he purchase?





Real-World Link

At a drive-in theater, the film is projected onto a large outdoor screen while the moviegoers sit in their cars. The sound for the film is broadcast on a radio station. In 1987, there were over 2000 drive-in theaters. By 1989, half of them had closed down.

Source: North American Theater Owners

MOVIE THEATERS Use the table at the right.

- **a.** Write a ratio of the number of indoor theaters to the total number of theaters for each year.
- **b.** Do any two of the ratios you wrote for part a form a proportion? If so, explain the real-world meaning of the proportion.
- **44. DIARIES** In a survey, 36% of the students said that they kept an electronic diary. There were 900 students who kept an electronic diary. How many students were in the survey?

Year	Indoor	Drive-In	Total
2000	35,567	683	36,280
2001	34,490	683	35,173
2002	35,170	666	35,836
2003	35,361	634	35,995
2004	36,012	640	36,652
2005	37,092	648	37,740
2006	37,776	649	38,415

Source: North American Theater Owners

- **45. Solution MULTIPLE REPRESENTATIONS** In this problem, you will explore how changing the lengths of the sides of a shape by a factor changes the perimeter of that shape.
 - **a. GEOMETRIC** Draw a square *ABCD*. Measure and label the sides. Draw a second square *MNPQ* with sides twice as long as *ABCD*. Draw a third square *FGHJ* with sides half as long as *ABCD*.
 - **b. TABULAR** Complete the table below using the appropriate measures.

AB	CD	M	IPQ	FG	HJ
Side length		Side length		Side length	
Perimeter		Perimeter		Perimeter	

c. VERBAL Make a conjecture about the change in the perimeter of a square if the side length is increased or decreased by a factor.

H.O.T. Problems Use Higher-Order Thinking Skills

- **46. OPEN ENDED** Write a real-life example of a ratio.
- 47. **REASONING** Compare and contrast ratios and rates.
- **48.** CHALLENGE If $\frac{a+1}{b-1} = \frac{5}{1}$ and $\frac{a-1}{b+1} = \frac{1}{1}$, find the value of $\frac{b}{a}$. (*Hint:* Choose different values of *a* and *b* for which the proportions are true and evaluate the expression $\frac{b}{a}$.)
- **49. FIND THE ERROR** Tim and Aisha are solving the following problem. Is either of them correct? Explain.

Two years ago, 78 women were enrolled in a dance class, while 162 men were enrolled. This year 193 men enrolled, while the ratio of women to men did not change. How many women enrolled this year?

Tim	Aisha
$\frac{78}{162} = \frac{193}{x}$ 78x = (162)(193)	$\frac{162}{78} = \frac{x}{193}$ $162(193) = 78x$
78×=31,266	31,266 = 78x
× ≈ 400.8	× = 400.8

50. WRITING IN MATH Describe how businesses can use ratios. Write about a real-world situation in which a business would use a ratio.

KCCT PRACTICE MA-HS-1.4.1

51. In the figure, *x* : *y* = 2 : 3 and *y* : *z* = 3 : 5. If *x* = 10, find the value of *z*.

- **A** 15
- **B** 20
- **C** 25
- **D** 30
- **52. GRIDDED RESPONSE** A race car driver records the finishing times for recent practice trials.

Trial	Time (seconds)
1	5.09
2	5.10
3	4.95
4	4.91
5	5.05

What is the mean time, in seconds, for the trials?

Spiral Review

Solve each equation. (Lesson 2-5)

55. <i>x</i> =	5 = -8	56.	b+9 = 2
57. 2 <i>p</i> –	-3 = 17	58.	5c - 8 = 12

59. HEALTH When exercising, a person's pulse rate should not exceed a certain limit. This maximum rate is represented by the expression 0.8(220 - a), where *a* is age in years. Find the age of a person whose maximum pulse rate is 152. (Lesson 2-4)

Solve each equation. Check your solution. (Lesson 2-3)

- **60.** 15 = 4a 5**61.** 7g 14 = -63**62.** $9 + \frac{y}{5} = 6$ **63.** $\frac{t}{8} 6 = -12$
- **64. GEOMETRY** Find the area of $\triangle ABC$ if each small triangle has a base of 5.2 inches and a height of 4.5 inches. (Lesson 1-3)

Evaluate each expression. (Lesson 1-2)

65. $3 + 16 \div 8 \cdot 5$

66. $4^2 \cdot 3 - 5(6+3)$

Skills Review

Solve each equation. (Lesson 2-2)

67. 4*p* = 22

68. 5*h* = 33

69. 1.25y = 4.375

- **53. GEOMETRY** If $\angle LMN$ is similar to $\angle LPO$, what is *z*? **F** 240 **G** 140 **H** 120 **J** 70 **G** 140 **H** 120 **J** 70 **G** 140 **H** 120 **J** 70
- **54.** Which equation below illustrates the Commutative Property?
 - A (3x + 4y) + 2z = 3x + (4y + 2z)
 - $\mathbf{B} \quad 7(x+y) = 7x + 7y$
 - **C** xyz = yxz
 - $\mathbf{D} \ x + 0 = x$



Spreadsheet Lab Financial Ratios

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Other Calculator Keystrokes



HS-NPO-S-NO12 Students will develop fluency in operations with real numbers and matrices, using mental computation or paper-and-pencil calculations for simple cases and calculators and/or computers for more complicated cases.

You can use a spreadsheet to investigate the debt-to-income ratio in mortgage lending.

ACTIVITY

Dorrie is thinking about buying a house. She has the following expenses: rent of \$650, credit card monthly bills of \$320, a car payment of \$410, and a student loan payment of \$115. Dorrie has a yearly salary of \$46,500. You can use a spreadsheet to find Dorrie's debt-toincome ratio.

- **Step 1** Enter Dorrie's debts in column B.
- **Step 2** Add her debts using a function in cell B6. Go to Insert and then Function. Then choose Sum. The resulting sum of 1495 should appear in B6.
- **Step 3** Now insert Dorrie's salary in column C. Remember to find her monthly salary by dividing the yearly salary by 12.

A mortgage company will use her debt-to-income ratio in part to determine if Dorrie qualifies for a mortgage loan. The **debt-to-income ratio** is calculated as *how much she owes per month* divided by *how much she earns each month*.

Step 4 Enter a formula to find the debt-to-income ratio in cell C6. In the formula bar, enter =B6/C2.

The ratio of about 0.39 appears. An ideal ratio would be 0.36 or less. A ratio higher than 0.36 would cause an increased interest rate or may require a higher down payment.

The spreadsheet shows a debt-to-income ratio of about 0.39. Dorrie should try to eliminate or reduce some debts or try to earn more money in order to lower her debt-to-income ratio.

Elery-	
1 de	
J.	

Lab 2-6 B Spreadsheet.xls 🛛 🖃 🔀							
\diamond	Α	В	С		~		
1	Type of Debt	Expenses	Salary				
2	Rent	650	3,875				
3	Credit Cards	320					
4	Car Payment	410					
5	Student Loan	115					
6		1495	0.385806				
7							
	▶ 📕 े Sheet 1 🖌	Sheet 2 / Sh	leet 3		$\mathbf{\mathbf{v}}$		
<	III			>			

Exercises

- 1. If Dorrie waits until she pays off her credit card bills to buy a house, what would be her new debt-to-income ratio?
- **2.** Dorrie decides to reduce her monthly credit card payments to \$160 per month, and she sells her car. How would her debt-to-income ratio change?
- 3. How could Dorrie improve her debt-to-income ratio?
- **4.** How would your spreadsheet be different if Dorrie had income other than her monthly salary?

Then Value calved prop

You solved proportions. (Lesson 2-6)

Now/

- Find the percent of change.
- Solve problems involving percent of change.

KY Program of Studies

HS-NPO-S-RP1 Students will calculate and apply ratios, proportions, rates and percentages to solve problems.

New Vocabulary

percent of change percent of increase percent of decrease

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Percent of Change

Why?

Every year, millions of people volunteer their time to improve their community. The difference in the number of volunteers from one year to the next can be used to determine a percent to represent the increase or decrease in volunteers.

Percent of Change Percent of change is the ratio of the change in an amount to the original amount expressed as a percent. If the new number is greater than the original number, the percent of change is a **percent of increase.** If the new number is less than the original number, the percent of change is a **percent of decrease.**





Source: Bureau of Labor and Statistics

EXAMPLE 1 Percent of Change

Determine whether each percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change.

a. original: 20 final: 23

Subtract the original amount from the final amount to find the amount of change: 23 - 20 = 3.

Since the new amount is greater than the original, this is a percent of increase.

Use the original number, 20, as the base.

change
$$\longrightarrow$$
 $\frac{3}{20} = \frac{r}{100}$
original amount \longrightarrow $\frac{3}{20} = r(20)$
 $3(100) = r(20)$
 $300 = 20r$
 $\frac{300}{20} = \frac{20r}{20}$
 $15 = r$

The percent of increase is 15%.

Check Your Progress

1A. original: 66 new: 30
1C. original: 24 new: 40

b. original: 25 final: 17

Subtract the original amount from the final amount to find the amount of change: 17 - 25 = -8.

Since the new amount is less than the original, this is a percent of decrease.

Use the original number, 25, as the base.

change
$$\rightarrow$$
 $\frac{-8}{25} = \frac{r}{100}$
original amount \rightarrow $\frac{-8}{25} = \frac{r}{100}$
 $-8(100) = r(25)$
 $-800 = 25r$
 $\frac{-800}{25} = \frac{25r}{25}$
 $-32 = r$

The percent of decrease is 32%.

1B. original: 9.8 new: 12.1 **1D.** original: 500 new: 131

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Real-World Link

Over 11.8 million people around the world went on a cruise in 2006. Source: M&L Research, Inc.

Real-World EXAMPLE 2 Percent of Change

CRUISE The number of cruise ships in North America increased 18% from 2000 to 2004. If there were 192 ships in 2005, how many were there in 2000?

Let c = the number of cruise ships in 2000. Since 18% is a percent of increase, the number of cruise ships in 2000 is less than the number of ships in 2005.

change \longrightarrow $\frac{192 - c}{c} = \frac{18}{100}$	Percent proportion
(192 - c)100 = 18c	Find the cross products.
19,200 - 100c = 18c	Distibutive Property
19,200 - 100c + 100c = 18c + 100c	Add 100c to each side.
19,200 = 118c	Simplify.
$\frac{19,200}{118} = \frac{118c}{118}$	Divide each side by 118.
163 = c	Simplify.

There were approximately 163 cruise ships in 2000.

Check Your Progress

2. TUITION A recent percent of increase in tuition at Northwestern University, in Evanston, Illinois, was 5.4%. If the new cost is \$29,940 per year, find the original cost per year.

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Solve Problems Two applications of percent of change are sales tax and discounts. Sales tax is an example of a percent of increase. Discount is an example of a percent of decrease.

EXAMPLE 3 Sales Tax									
SHOPP is \$28. for he	SHOPPING Marta is purchasing wire and beads to make jewelry. Her merchandise is \$28.62 before tax. If the tax is 7.25% of the total sales, what is the actual cost for her purchases?								
Step 1	Find the	e tax.							
	The tax	is 7.25% of the price of the tie	cket.						
	7.25% (of $$28.62 = 0.0725 \times 28.62$	7.25% = 0.0725						
		≈ 2.07495	Use a calculator.						
Step 2	Find the	e cost with tax.							
	Round Add thi	\$2.07495 to \$2.07 since tax is a size amount to the original price	always rounded to the nearest cent. e: \$28.62 + \$2.07 = \$30.69.						
The to	The total price of Marta's jewelry supplies is \$30.69.								
Check	Check Your Progress								
3. SHO	PPING A	new DVD costs \$24.99. If the	e sales tax is 6.85%, what is the total cost?						
			Personal Tutor glencoe.com						

To find a discounted amount, you will follow similar steps to those for sales tax.

StudyTip

Key Words

When translating a problem from word sentences to math sentences, the word "is" translates to =, and the word "of" translates to ×.

EXAMPLE 4 Discounts

DISCOUNT Since Tyrell has earned good grades in school, he qualifies for the Good Student Discount on his car insurance. His monthly payment without the discount is \$85. If the discount is 20%, what will he pay each month?

Step 1 Find the discount.

The discount is 20% of the original payment.

20% of $\$85 = 0.20 \times 85$ **20% = 0.20** = 17 **Use a calculator.**

Step 2 Find the cost after discount.

Subtract \$17 from the original payment: \$85.00 - \$17 = \$68.

With the Good Student Discount, Tyrell will pay only \$68 per month for car insurance.

Check Your Progress

4. SALES A picture frame originally priced at \$14.89 is on sale for 40% off. What is the discounted price?

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🧭 Check Your Understanding

Example 1 p. 119	State whether each percent of change is a percent of <i>increase</i> or a percent of <i>decrease</i> . Then find the percent of change. Round to the nearest whole percent.						
	original: 78 new: 125	2. original: 41 new: 24					
	3. original: 6 candles new: 8 candles	4. original: 35 computers new: 32 computers					
Example 2 p. 120	5. GEOGRAPHY The distance Phoenix to Flagstaff is abo distance from Phoenix to I	From Phoenix to Tucson is 120 miles. The distance from ut 21.7% longer. To the nearest mile, what is the Flagstaff?					
Example 3	Find the total price of each it	em.					
p. 120	6. dress: \$22.50 sales tax: 7.5%	7. video game: \$35.99 sales tax: 6.75%					
	8. PROM A limo costs \$85 to rent for 3 hours plus a 7% sales tax. What is the total cost to rent a limo for 6 hours?						
	9. GAMES A computer game cost of the game?	costs \$49.95 plus a 6.25% sales tax. What is the total					
Example 4	Find the discounted price of	each item.					
p. 121	10. guitar: \$95.00 discount: 15% 11. DVD: \$22.95 discount: 25%						
	12. SKATEBOARD A skateboard costs \$99.99. If you have a coupon for 20% off, how much will you save?						
	13. TICKETS Tickets to the cou	nty fair are \$8 for an adult and \$5 for a child. If you					

3. TICKETS Tickets to the county fair are \$8 for an adult and \$5 for a child. If you have a 15% discount for every ticket you purchase, how much will it cost for 2 adult tickets and 2 child tickets?

Practice and Problem Solving

State whether each percent of change is a percent of *increase* or a percent of **Example 1** decrease. Then find the percent of change. Round to the nearest whole percent. p. 119

- 14. original: 35 new: 40
- **16.** original: 27 new: 73
- **18.** original: 21.2 grams new: 10.8 grams
- **20.** original: \$68 new: \$76
- new: 40 hours
- Example 2 **22.** GASOLINE The average cost of regular gasoline in North Carolina increased by p. 120 73% from 2006 to 2007. If the average cost of gas in 2006 was \$2.069, what was the average cost in 2007? Round to the nearest cent.
 - **23.** CARS Beng is shopping for a car. The cost of a new car is \$15,500. This is 25% greater than the cost of a used car. What is the cost of the used car?

Example 3 p. 120

Find the total price of each item.

24. messenger bag: \$28.00 **25.** software: \$45.00 **26.** vase: \$5.50 tax: 7.25% tax: 5.5% tax: 6.25% **27.** book: \$25.95 **28.** magazine: \$3.50 29. pillow: \$9.99 tax: 5.25% tax: 5.75% tax: 6.75%

Example 4

p. 121



Real-World Link

Softball became an Olympic event in 1996. The American women, winning the gold medal in 2004, capped a 79-game winning streak. Source: USA TODAY



Find the discounted price of each item.

Find the final price of each item.

36.	sweater: \$14.99	37.	printer: \$60.00	38 .	board game: \$25.00
	discount: 12%		discount: 25%		discount: 15%
	tax: 6.25%		tax: 6.75%		tax: 7.5%

- **39.** CONSUMER PRICE INDEX An index measures the percent change of a value from a base year. An index of 115 means that there was a 15% increase from the base year. In 2000, the consumer price index of dairy products was 160.7. In 2005, it was 182.4. Determine the percent of change.
- **40. INVESTING** The current price of each share of a technology company is \$135. If this represents a 16.2% increase over the past year, what was the price per share a year ago?
- **41. SHOPPING** A group of girls are shopping for dresses to wear to the spring dance. One finds a dress priced \$75 with a 20% discount. A second girl finds a dress priced \$85 with a 30% discount.
 - **a.** Find the amount of discount for each dress.
 - **b.** Which girl is getting the better price for the dress?
- **42. RECREATIONAL SPORTS** In 2001, there were 80 million youth softball teams. By 2005, there were 85 million. Determine the percent of increase.

32. athletic shoes: \$59.99

discount: 40%

discount: 20%

35. belt: \$14.99



- new: 21 **19.** original: 11 feet new: 25 feet
- **21.** original: 21 hours

GROCERIES Analyze the costs of the grocery items listed in the table below. Which item had the greatest percent increase in cost from 2000 to 2005?

Average Retail Prices of Selected Grocery Items					
Grocery Item	Cost in 2000 (\$ per pound)	Cost in 2005 (\$ per pound)			
milk (gallon)	2.79	3.24			
eggs (dozen)	0.96	1.35			
chicken (whole)	1.08	1.06			
ground beef	1.63	2.30			
apples	0.82	0.97			
iceberg lettuce	0.85	0.85			
peanut butter	1.96	1.70			

Source: Statistical Abstract of the United States

- **44. MULTIPLE REPRESENTATIONS** In this problem, you will explore patterns in percentages.
 - **a. TABULAR** Copy and complete the following table.

1% of	500	is 5.	100% of	is 20.	% of 80 is 20.
2% of		is 5.	50% of	is 20.	% of 40 is 20.
4% of		is 5.	25% of	is 20.	% of 20 is 20.
8% of		is 5.	12.5% of	is 20.	% of 10 is 20.

- **b. VERBAL** Describe the patterns in the second and fifth columns.
- **c. ANALYTICAL** Use the patterns to write the fifth row of the table.



Real-World Link

In a recent year, the amount of sales tax varied across the United States from 0% to 7.25%.

Source: Federation of Tax Administrators

H.O.T. Problems

Use Higher-Order Thinking Skills

- **45. OPEN ENDED** Write a real-world problem to find the total price of an item including sales tax.
- **46. REASONING** If you have 75% of a number *n*, what percent of decrease is it from the number *n*? If you have 40% of a number *a*, what percent of decrease do you have from the number *a*? What pattern do you notice? Is this always true?
- **47. FIND THE ERROR** Maddie and Xavier are solving for the percent change if the original amount was \$25 and the new amount is \$28. Is either of them correct? Explain your reasoning.

Maddie	Xavier
$\frac{3}{28} = \frac{v}{100} \\ 3(100) = 28v$	$\frac{3}{25} = \frac{r}{100} \\ 3(100) = 25r$
300 = 28r	300 = 25r
10.7 = r	12 = r

- **48. CHALLENGE** Determine whether the following statement is *sometimes, always*, or *never* true. *The percent of change is less than* 100%.
- **49.** WRITING IN MATH Explain how to find a percent of change between two values and how to determine whether the change is a percent of increase or decrease.

50. GEOMETRY The rectangle has a perimeter of *P* centimeters. Which equation could be used to find the length ℓ of the rectangle?

KCCT PRACTICE MA-HS-1.4.1



- **B** $P = 4.8 + \ell$ **D** $P = 4.8 + 2\ell$
- **51. SHORT RESPONSE** Henry is painting a wall that has dimensions of 12 feet by 14 feet. If the paint costs \$1.25 per square foot, how much will it cost him to paint the wall?

- **52.** The number of students at Franklin High School increased from 840 to 910 over a 5-year period. What was the percent of increase?
 - **F** 8.3%
 - **G** 14.0%
 - J 18.5%
 - H 92.3%

A $\frac{1}{6}$

53. PROBABILITY Two six-sided dice are rolled. The sum of the numbers of dots on the faces of the two dice is recorded. What is the probability that the sum is 10?

B $\frac{1}{3}$ **C** $\frac{1}{12}$

 $D\frac{1}{36}$

Spiral Review

54. TRAVEL The Chan's minivan requires 5 gallons of gasoline to travel 120 miles. How many gallons of gasoline will they need to travel 360 miles? (Lesson 2-6)

Evaluate each expression if x = -2, y = 6, and z = 4. (Lesson 2-5)

55. |3 - x| + 7 **56.** 12 - |z + 9| **57.** |y + x| - z + 4

Solve each equation. Round to the nearest hundredth. Check your solution. (Lesson 2-4)

58.	1.03p - 4 = -2.15p + 8.72	59 .	18 - 3.8t = 7.36 - 1.9t
60.	5.4w + 8.2 = 9.8w - 2.8	61.	2[d + 3(d - 1)] = 18

Solve each equation. Check your solution. (Lesson 2-3)

62 .	5n + 6 = -4	63.	-11 = 7 + 3c
64.	15 = 4a - 5	65.	-14 + 7g = -63

66. RIVERS The Congo River in Africa is 2900 miles long. That is 310 miles longer than the Niger River, which is also in Africa. (Lesson 2-2)

a. Write an equation you could use to find the length of the Niger River.

- **b.** What is the length of the Niger River?
- **67. GEOMETRY** Two perpendicular lines meet to form four right angles. Write two different if-then statements for this definition. (Lesson 1-8)

Skills Review

Translate each equation into a sentence. (Lesson 2-1)

68.	d - 14 = 5	69.	2f + 6 = 19
71.	3a + 5 = 27 - 2a	72.	$-6c^2 - 4c = 25$

70. *y* − 12 = *y* + 8
73. *d*⁴ + 64 = 3*d*³ + 77

124 Chapter 2 Linear Equations

Algebra Lab Percentiles

Objective

You will use percentiles to represent data.

A **percentile** is a measure that is often used to report test data, such as standardized test scores. It tells us what percent of the total scores were at or below the given percentile.

- Percentiles measure rank from the bottom.
- There is no 0 percentile rank. The lowest score is at the 1st percentile.
- There is no 100th percentile rank. The highest score is at the 99th percentile.

ACTIVITY

A talent show was held for the fifteen finalists in the Teen Idol contest. Each performer received a score from 0 through 30 with 30 being the highest. The table shows the names and scores for the finalists.

Name	Score	Name	Score
Arnold	17	Malik	10
Benito	9	Natalie	26
Carmen	21	Pearl	4
Delia	29	Twyla	6
Fernando	15	Victor	28
Horatio	5	Warren	22
Ingrid	11	Yolanda	18
Ishi	27		

- Step 1 Write one score on each of 15 slips of paper.
- **Step 2** Arrange the slips vertically from greatest to least score.
- **Step 3** Find Victor's percentile rank.

Victor had a score of 28. There are 13 scores below his score. To find his percentile rank, use the following formula:

 $\frac{\text{number of scores below 28}}{\text{total number of scores}} \cdot 100 = \frac{13}{15} \cdot 100 \text{ or about 87.}$

Victor scored at the 87th percentile in the contest.

Analyze the Results

- 1. Find the median, lower quartile, and upper quartile of the scores.
- **2.** Which performer was at the 50th percentile? Which performer was at the 25th percentile? the 75th percentile?
- **3.** Compare and contrast the values for the median, lower quartile, and upper quartile and the scores for the 25th, 50th, and 75th percentiles.
- **4.** While Victor scored at the 87th percentile, what percent of the 30 possible points did he score?
- 5. Compare and contrast the percentile rank and the percent score.
- **6.** Are there any outliers in the data that could alter the results of our computations?

Then

You solved equations with variables on each side. (Lesson 2-4)

Now

- Solve equations for given variables.
- Use formulas to solve real-world problems.

KY Program of Studies

HS-M-S-SM4 Students will use unit analysis to check measurement computations. HS-AT-S-EI5 Students will solve an equation involving several variables for one variable in terms of the others. Also addresses HS-NPO-S-NO9, HS-M-S-MPA1, and HS-AT-S-EI17.

New Vocabulary literal equation

dimensional analysis unit analysis

KY Math Online

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Literal Equations and Dimensional Analysis

Why?

Each year, more people use credit cards to make everyday purchases. If the entire balance is not paid by the due date, compound interest is applied. The formula for computing the balance of an account with compound interest added annually is A = P(1 + r).

- *A* represents the amount of money on the account including the interest,
- *P* is the amount in the account before interest is added,
- *r* is the interest rate written as a decimal.



Solve for a Specific Variable Some equations such as the one above contain more than one variable. At times, you will need to solve these equations for one of the variables.

EXAMPLE 1 Solve for a Specific Variable

Solve 4m - 3n = 8 for *m*.

4m - 3n = 8 Original equation 4m - 3n + 3n = 8 + 3n Add 3n to each side. 4m = 8 + 3n Simplify. $\frac{4m}{4} = \frac{8 + 3n}{4}$ Divide each side by 4. $m = \frac{8}{4} + \frac{3}{4}n$ Simplify. $m = 2 + \frac{3}{4}n$ Simplify.

Check Your Progress

Solve each equation for the variable indicated.

1A. 15 = 3n + 6p, for n **1B.** $\frac{k-2}{5} = 11j$, for k

 1C. 28 = t(r + 4), for t **1D.** a(q - 8) = 23, for q

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Sometimes we need to solve equations for a variable that is on both sides of the equation. When this happens, you must get all terms with that variable onto one side of the equation. It is then helpful to use the Distributive Property to isolate the variable for which you are solving.

StudyTip

Solving for a Specific Variable When an equation has more than one variable, it can be helpful to highlight the variable for which you are solving on your paper.

EXAMPLE 2 Solve for a Specific Variable



Since division by 0 is undefined, $3 - z \neq 0$ so $z \neq 3$.

Check Your Progress

Solve each equation for the variable indicated.

2A. d + 5c = 3d - 1, for d

2B. 6q - 18 = qr + t, for qPersonal Tutor glencoe.com

Use Formulas A formula or equation that involves several variables is called a **literal** equation. To solve a literal equation, apply the process of solving for a specific variable.

Real-World EXAMPLE 3 Use Literal Equations

YO-YOS Use the information about the largest yo-yo at the left. The formula for the circumference of a circle is $C = 2\pi r$, where *C* represents circumference and *r* represents radius.

a. Solve the formula for *r*.

$C = 2\pi r$	Formula for circumference
$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$	Divide each side by 2π .
$\frac{C}{2\pi} = r$	Simplify.

b. Find the radius of the yo-yo.

$\frac{C}{2\pi} = r$	Formula for radius
$\frac{32.7}{2\pi} = r$	C = 32.7
$5.2 \approx r$	Use a calculator.

The yo-yo has a radius of about 5.2 feet.

Check Your Progress

- **3. GEOMETRY** The formula for the volume of a rectangular prism is $V = \ell wh$, where ℓ is the length, w is the width, and h is the height.
 - **A.** Solve the formula for *w*.
 - **B.** Find the width of a rectangular prism that has a volume of 79.04 cubic centimeters, a length of 5.2 centimeters, and a height of 4 centimeters.

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Real-World Link

The largest yo-yo in the world is 32.7 feet in circumference. It was launched by crane from a height of 189 feet.

Source: Guinness Book of World Records When using formulas, you may want to use dimensional analysis. Dimensional analysis or **unit analysis** is the process of carrying units throughout a computation.

StudyTip

Rewriting Equations with Exponents When rewriting an equation to solve for a specific variable, the exponents of a variable are moved with that variable. They are not separated.

EXAMPLE 4 **Use Dimensional Analysis**

RUNNING A 10K run is 10 kilometers long. If 1 meter = 1.094 yards, use dimensional analysis to find the length of the race in miles. (*Hint*: 1 mi = 1760 yd)

Since the given conversion relates meters to yards, first convert 10 kilometers to meters. Then multiply by the conversion factor such that the unit meters

are divided out. To convert from yards to miles, multiply by $\frac{1 \text{ mi}}{1760 \text{ vd}}$

length of run	×	kilometers to meters	×	meters to yards	×	yards to miles
10 km	×	<u>1000 m</u> 1 km	×	1.094 yd 1 m	×	<u>1 mi</u> 1760 yd

Notice how the units cancel, leaving the unit to which you are converting.

$$10 \text{ km} \times \frac{1000 \text{ sm}}{1 \text{ km}} \times \frac{1.094 \text{ yd}}{1 \text{ sm}} \times \frac{1 \text{ mi}}{1760 \text{ yd}} = \frac{10940}{1760} \text{ mi}$$
$$= \frac{10,940}{1760}$$
$$\approx 6.2 \text{ mi}$$

A 10K race is approximately 6.2 miles.

Check Your Progress

4. A car travels a distance of 100 feet in about 2.8 seconds. What is the velocity of the car in miles per hour? Round to the nearest whole number.

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h

 $V = \pi r^2 h$

Check Your Understanding

Examples 1 and 2 Solve each equation or formula for the variable indicated. pp. 126-127 1) 5a + c = -8a, for a **2.** 7h + f = 2h + g, for g **3.** $\frac{k+m}{-7} = n$, for k **4.** q = p(r + s), for p Example 3 5. PACKAGING A hand soap company wants to use a cylindrical p. 127 container to hold their new liquid soap. Use the formula for the volume of a cylinder. **a.** Solve the formula for *h*. **b.** What is the height of a container if the volume is 56.52 cubic inches and the radius is 1.5 inches? Round to the nearest tenth. **Example 4 6. SHOPPING** Scott found a rare video game on an online auction site priced at p. 128 35 Australian dollars. If the exchange rate is \$1 U.S. = \$1.24 Australian, find the cost of the game in United States dollars. Round to the nearest cent.

7. PHOTOGRAPHY A fisheye lens has a minimum focus range of 13.5 centimeters. If 1 centimeter is equal in length to 0.39 inches, what is the minimum focus range of the lens in feet?

Practice and Problem Solving

Examples 1 and 2 Solve each equation or formula for the variable indicated.

8.
$$u = vw + z$$
, for v
 9 $x = b - cd$, for c

 10. $fg - 9h = 10j$, for g
 11. $10m - p = -n$, for m

 12. $r = \frac{2}{3}t + v$, for t
 13. $\frac{5}{9}v + w = z$, for v

 14. $\frac{10ac - x}{11} = -3$, for a
 15. $\frac{df + 10}{6} = g$, for f

Example 3 p. 127

pp. 126-127

16. FITNESS The formula to compute a person's body mass index is $B = 703 \cdot \frac{w}{h^2}$. *B* represents the body mass index, *w* is the person's weight in pounds, and *h* represents the person's height in inches.

- **a.** Solve the formula for *w*.
- **b.** What is the weight to the nearest pound of a person who is 64 inches tall and has a body mass index of 21.45?
- 17. PHYSICS Acceleration is the measure of how fast a velocity is changing. The

formula for acceleration is $a = \frac{v_f - v_i}{t}$. *a* represents the acceleration rate,

 v_{f} is the final velocity, v_{i} is the initial velocity, and *t* represents the time in seconds.

- **a.** Solve the formula for v_{f} .
- **b.** What is the final velocity of a runner who is accelerating at 2 feet per second squared for 3 seconds with an initial velocity of 4 feet per second?

Example 4 p. 128

- **18. SWIMMING** A swimmer swims laps in an Olympic-sized pool 50 meters long. If each lap is 100 meters long, how many laps equal one mile? Round to the nearest tenth. (*Hint*: 1 foot ≈ 0.3048 meter)
- **19. TRAVEL** In Canada, gasoline is sold by the liter. How many liters of gasoline are needed to fill a 13.2-gallon tank? There are 1.06 quarts per 1 liter. Round to the nearest tenth.

Solve each equation or formula for the variable indicated.

20. -14n + q = rt - 4n, for n

22. ax + z = aw - y, for a

Select an appropriate unit from the choices below and convert the rate to that unit.



mpr
mpr

mm/s

21. 18t + 11v = w - 13t, for t

23. 10c - f = -13 + cd, for *c*

25. a snail moving at 3.6 m/h

27. a satellite moving at 234,000 m/min

km/s

- **24.** a car traveling at 36 ft/s
- **26.** a person walking at 3.4 mph
- **28. DANCING** The formula $P = \frac{1.2W}{H^2}$ represents the amount of pressure exerted on the floor by a ballroom dancer's heel. In this formula, *P* is the pressure in pounds per
 - square inch, *W* is the weight of a person wearing the shoe in pounds, and *H* is the width of the heel of the shoe in inches.
 - **a.** Solve the formula for *W*.
 - **b.** Find the weight of the dancer if the heel is 3 inches wide and the pressure exerted is 30 pounds per square inch.



Real-World Link

In most dance competitions, dancers compete as couples, or formation teams. At "Jack & Jill" competitions, individuals are paired by random draw.

Source: USA Dance

Write an equation and solve for the variable indicated.

H.O.T. Problems

a. $n = \frac{x+1}{2}$

- **29.** Seven less than a number *t* equals another number *r* plus 6. Solve for *t*.
- **30.** Ten plus eight times a number *a* equals eleven times another number *d* minus six. Solve for *a*.
- **31.** Nine tenths of a number g is the same as seven plus two thirds of another number *k*. Solve for *k*.
- **32.** Three fourths of a number *p* less two is five sixths of another number *r* plus five. Solve for *r*.
- **GIFTS** Ashley has 214 square inches of paper to wrap a gift box. The surface area S of the box can be found by using the formula $S = 2w(\ell + h) + 2\ell h$, where w is the width of the box, ℓ is the length of the box, and *h* is the height. If the length of the box is 7 inches and the width is 6 inches, how tall can Ashley's box be?
- **34. MULTIPLE REPRESENTATIONS** In this problem, you will investigate cylinders. The surface area of cylinder can be found by the formula $S = 2\pi rh + 2\pi r^2$.
 - **a. ALGEBRAIC** Solve the formula for *h*. Rewrite your solution using 3.14 for π and 2500 for S.
 - **b. TABULAR** Make a table of values using your new formula to find h if r = 20, 15, 10, 5, and 0. Round to the nearest hundredth.
 - **c. VERBAL** What do we know about the domain (possible values of *r*) in this situation?

Use Higher-Order Thinking Skills

35. CHALLENGE The circumference of an NCAA women's basketball is 29 inches, and the rubber coating is $\frac{3}{4}$ inch thick. Use the formula $v = \frac{4}{3}\pi r^3$, where v represents the volume and *r* is the radius of the inside of the ball, to determine the volume

36. REASONING Select an appropriate unit to describe the speed of a car traveling on the highway and the speed of a caterpillar crawling on a tree. Can the same unit

of the air inside the ball. Round to the nearest hundred.

San

4a - 5b =

-56

b =

be used for each situation? Explain.



Real-World Link

In 2007, Louisiana State University lost to the University of Connecticut by a score of 71 to 72, ending a 43-game winning streak, an NCAA Women's Basketball record.

Source: National Collegiate Athletic Association

37. FIND THE ERROR Sandrea and Fernando are solving the equation
$$4a - 5b = 7$$
 for *b*.
Is either of them correct? Explain.
$$\begin{array}{r} \textbf{Sandrea} \\ \textbf{4a - 5b = 7} \\ \textbf{-5b = 7 - 4a} \\ \frac{-5b}{-5} = \frac{7 - 4a}{-5} \\ \textbf{b} = \frac{7 - 4a}{-5} \end{array}$$

- **38. OPEN ENDED** Write a formula for A, the area of a geometric figure such as a triangle or rectangle. Then solve the formula for a variable other than A.
- **39.** CHALLENGE Solve each equation or formula for the variable indicated.

$$\frac{y-1}{xy}$$
 for x **b.** $\frac{x+y}{x-y} = \frac{1}{2}$ for y

40. WRITING IN MATH Explain what a literal equation is and how to solve one.

KCCT PRACTICE MA-HS-1.4.1

41. Eula is investing \$6000, part at 4.5% interest and the rest at 6% interest. If *d* represents the amount invested at 4.5%, which expression represents the amount of interest earned in one year by the amount paying 6%?

Α	0.06 <i>d</i>	С	0.06(d + 6000)
В	0.06(d - 6000)	D	0.06(6000 - d)

- **42.** Todd drove from Boston to Cleveland, a distance of 616 miles. His breaks, gasoline, and food stops took 2 hours. If his trip took 16 hours altogether, what was Todd's average speed?
 - F
 38.5 mph
 H
 44 mph

 G
 40 mph
 J
 47.5 mph

- **43. SHORT RESPONSE** Brian has 3 more books than Erika. Jasmine has triple the number of books that Brian has. Altogether Brian, Erika, and Jasmine have 22 books. How many books does Jasmine have?
- **44. GEOMETRY** Which of the following best describes a plane?
 - A a location having neither size nor shape
 - **B** a flat surface made up of points having no depth
 - C made up of points and has no thickness or width
 - **D** a boundless, three-dimensional set of all points

Spiral Review

Find the final price of each item. (Lesson 2-7)

45.	lamp: \$120.00 discount: 20% tax: 6%	46.	dress: \$70.00 4 discount: 30% tax: 7%	7.	camera: \$58.00 discount: 25% tax: 6.5%
48.	jacket: \$82.00 discount: 15% tax: 6%	49.	comforter: \$67.00 5 discount: 20% tax: 6.25%	0.	lawnmower: \$720.00 discount: 35% tax: 7%

Solve each proportion. If necessary, round to the nearest hundredth. (Lesson 2-6)

51.
$$\frac{3}{4.5} = \frac{x}{2.5}$$
 52. $\frac{2}{0.36} = \frac{7}{p}$ **53.** $\frac{m}{9} = \frac{2.8}{4.9}$

- **54. JOBS** Laurie mows lawns to earn extra money. She can mow at most 30 lawns in one week. She profits \$15 on each lawn she mows. Identify a reasonable domain and range for this situation and draw a graph. (Lesson 1-6)
- **55. ENTERTAINMENT** Each member of the pit orchestra is selling tickets for the school musical. The trombone section sold 50 floor tickets and 90 balcony tickets. Write and evaluate an expression to find how much money the trombone section collected. (Lesson 1-2)



Skills Review

Solve each equation. (Lesson 2-5)

56. 8k + 9 = 7k + 6

59. $\frac{1}{4} - \frac{2}{3}y = \frac{3}{4} - \frac{1}{3}y$

57. 3 - 4q = 10q + 10

60. 4(2a-1) = -10(a-5)

58. $\frac{3}{4}n + 16 = 2 - \frac{1}{8}n$ **61.** 2(w - 3) + 5 = 3(w - 1)

Then

You translated sentences into equations. (Lesson 2-1)

Now/

- Solve mixture problems.
- Solve uniform motion problems.



HS-M-S-MPA1 Students will apply units of measurements of physical quantities correctly in expressions, equations and problem solutions that involve measurement.

New Vocabulary

weighted average mixture problem uniform motion problem rate problem



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Weighted Averages

Why?

Baseball players' performance is measured in large part by statistics. Slugging average (SLG) is a weighted average that measures the power of a hitter. It assigns values based on the number of bases reached after each hit. The slugging average is calculated by using the following formula.

 $SLG = \frac{1B + (2 \times 2B) + (3 \times 3B) + (4 \times HR)}{at \ bats}$



Weighted Averages The batter's slugging percentage is an example of a weighted average. The **weighted average** *M* of a set of data is the sum of the product of the number of units and the value per unit divided by the sum of the number of units.

Mixture problems are problems in which two or more parts are combined into a whole. They are solved using weighted averages. In a mixture problem, the units are usually the number of gallons or pounds and the value is the cost, value, or concentration per unit.

Real-World EXAMPLE 1 Mixture Problem

RETAIL A tea company sells blended tea for \$25 per pound. To make blackberry tea, dried blackberries that cost \$10.50 per pound are blended with black tea that costs \$35 per pound. How many pounds of black tea should be added to 5 pounds of dried blackberries to make blackberry tea?

Step 1 Let *w* be the weight of the black tea. Make a table to organize the information.

	Number of Units (lb)	Price per Unit (\$)	Total Price (price)(units)
Dried Blackberries	5	10.50	10.50(5)
Black Tea	W	35	35 <i>w</i>
Blackberry Tea	5 + W	25	25(5 + <i>w</i>)

Write an equation using the information in the table.

Price of				price of
blackberries	plus	price of tea	equals	blackberry tea.
10.50(5)	+	35w	=	25(5+w)

Step 2 Solve the equation.

10.50(5) + 35w = 25(5 + w) 52.5 + 35w = 125 + 25w 52.5 + 35w - 25w = 125 + 25w - 25w 52.5 + 10w = 125 52.5 - 52.5 + 10w = 125 - 52.5 10w = 72.5w = 7.25

Original equation Distributive Property Subtract 25*w* from each side. Simplify. Subtract 52.5 from each side. Simplify. Divide each side by 10.

StudyTip

Mixture Problems

When you organize the information in mixture problems, remember that the final mixture must contain the sum of the parts in the correct quantities and at the correct percents. To make the blackberry tea, 7.25 pounds of black tea will need to be added to the dried blackberries.

Check Your Progress

1. COFFEE How many pounds of coffee beans that sell for \$9.50 per pound should be mixed with 2 pounds of coffee beans that sell for \$11.75 per pound to obtain a mix that sells for \$10 per pound?

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Sometimes mixture problems are expressed in terms of percents.

Real-World EXAMPLE 2 Percent Mixture Problem

FRUIT PUNCH Mrs. Matthews has 16 cups of punch that is 3% pineapple juice. She also has a punch that is 33% pineapple juice. How many cups of the 33% punch will she need to add to the 3% punch to obtain a punch that is 20% pineapple juice?

Step 1 Let x = the amount of 33% solution to be added. Make a table.

	Amount of Punch (cups)	Amount of Pineapple Juice
3% Punch	16	0.03(16)
33% Punch	X	0.33 <i>x</i>
20% Punch	16 + <i>x</i>	0.20(16 + x)

Write an equation using the information in the table.

Amount of pineapple		amount of pineapple		amount of pineapple
juice in 3% punch	plus	juice in 33% punch	equals	juice in 20% punch.
0.03(16)	+	0.33 <i>x</i>	=	0.20(16 + x)

Step 2 Solve the equation.

0.03(16) + 0.33x = 0.20(16 + x)	Original equation
0.48 + 0.33x = 3.2 + 0.20x	Simplify.
0.48 + 0.33x - 0.20x = 3.2 + 0.20x - 0.20x	Subtract 0.20x from each side.
0.48 + 0.13x = 3.2	Simplify.
0.48 - 0.48 + 0.13x = 3.2 - 0.48	Subtract 0.48 from each side.
0.13x = 2.72	Simplify.
$\frac{0.13x}{0.13} = \frac{2.72}{0.13}$	Divide each side by 0.13.
x = 20.9	Round to the nearest tenth.

Mrs. Matthews should add 20.9 cups of the 33% punch to the 16 cups of the 3% punch.

Check Your Progress

2. ANTIFREEZE One type of antifreeze is 40% glycol, and another type of antifreeze is 60% glycol. How much of each kind should be used to make 100 gallons of antifreeze that is 48% glycol?

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Problem-SolvingTip

Make a Table Using a table is a great way to organize the given information. It also helps you understand how to write an equation to solve for the missing value. **Uniform Motion Problems** Motion problems are another application of weighted averages. Uniform motion problems or rate problems are problems in which an object moves at a certain speed or rate. The formula d = rt is used to solve these problems. In the formula, *d* represents distance, *r* represents rate, and *t* represents time.



Real-World Link

In-line skating is the fourth most popular recreational activity in the U.S.

Source: Statistical Abstract of the United States

Real-World EXAMPLE 3 Speed of One Vehicle

INLINE SKATING Travis and Tony skate in the park. It took them 40 minutes to skate 5 miles. The return trip took them 30 minutes. What was their average speed for the entire trip?

- **Understand** We know that the boys did not travel the same amount of time on each portion of their trip. So, we will need to find the weighted average of their speeds. We are asked to find their average speed for both portions of the trip.
 - **Plan** First find the rate of the going portion, and then the return portion of the trip. Because the rate is in miles per hour we convert 40 minutes to 0.667 hours and 30 minutes to 0.5 hours

Going

$r = \frac{d}{t}$	Formula for rate
$=\frac{5 \text{ miles}}{0.667 \text{ hours}}$ or 7.5 miles per hour	Substitution $d = 5$ miles, $t = 0.667$ hours
Return	
$r = \frac{d}{t}$	Formula for rate
$=\frac{5 \text{ miles}}{0.5 \text{ hours}}$ or 10 miles per hour	Substitution $d = 5$ miles, $t = 0.5$ hours

Because we are looking for a weighted average we cannot just average their speeds. We need to find the weighted average for the round trip

Solve
$$M = \frac{(\text{rate of going})(\text{time of going}) + (\text{rate of return})(\text{time of return})}{\text{time of going} + \text{time of return}}$$

= $\frac{(7.5)(0.667) + (10)(0.5)}{0.667 + 0.5}$ Substitution

 $=\frac{10.0025}{1.167}$ or about 8.6

Simplify.

Their average speed was about 8.6 miles per hour.

Check Our solution of 8.6 miles per hour is between the going portion rate, 7.5 miles per hour, and the return rate, 10 miles per hour. So, we know that our answer is reasonable.

Check Your Progress

3. EXERCISE Austin jogged 2.5 miles in 16 minutes and then walked 1 mile in 10 minutes. What was his average speed?

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The formula d = rt can also be used to solve real-world problems involving two vehicles in motion.

StudyTip

Draw a Diagram

Drawing a diagram is not just for geometry problems. You can use diagrams to visualize many problem situations that can be represented by equations.

Real-World EXAMPLE 4 Speeds of Two Vehicles

FREIGHT TRAINS Two trains are 550 miles apart heading toward each other on parallel tracks. Train A is traveling east at 35 miles per hour, while Train B travels west at 45 miles per hour. When will the trains pass each other?

Step 1 Draw a diagram.



Step 2 Let t = the number of hours until the trains pass each other. Make a table.

	r	t	<i>d</i> = <i>rt</i>
Train A	35	t	35t
Train B	45	t	45 <i>t</i>

Step 3 Write and solve an equation.

Distance traveled by		distance traveled		
Train A	plus	by Train B	equals	550 miles
35 <i>t</i>	+	45t	=	550
35t + 45t = 550	Original equ	ation		
80t = 550	Simplify.			
$\frac{80t}{80} = \frac{550}{80}$	Divide each	side by 80.		
t = 6.875	Simplify.			

The trains will pass each other in about 6.875 hours.

Check Your Progress

4. CYCLING Two cyclists begin traveling in opposite directions on a circular bike trail that is 5 miles long. One cyclist travels 12 miles per hour, and the other travels 18 miles per hour. How long will it be before they meet?

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🗹 Check Your Understanding

Example 1 p. 132	FOOD Tasha ordered soup and salad for lunch. The soup cost 15 cents per ounce, and the salad cost 20 cents per ounce. If Tasha ordered 10 ounces of soup for lunch and the total cost was \$3.30, how many ounces of salad did Tasha order?
Example 2 p. 133	2. CHEMISTRY A chemistry experiment calls for a 30% solution of sodium chloride. Margo has 40 milliliters of 25% solution. How many milliliters of 60% solution should she add to obtain the required 30% solution?
Example 3 p. 134	3. TRAVEL A boat travels 16 miles due north in 2 hours and 24 miles due west in 2 hours. What is the average speed of the boat?
	4. EXERCISE Felisa jogged 3 miles in 25 minutes and then jogged 3 more miles in 30 minutes. What was her average speed in miles per minute?
Example 4 p. 135	5. CYCLING A cyclist begins traveling 18 miles per hour. At the same time and at the same starting point, an inline skater follows the cyclist's path and begins traveling 6 miles per hour. After how much time will they be 24 miles apart?

Practice and Problem Solving

6. CANDY A candy store wants to create a mix using two hard candies. One is priced at \$5.45 per pound, and the other is priced at \$7.33 per pound. How many pounds of the \$7.33 candy should be mixed with 11 pounds of the \$5.45 candy to sell the mixture for \$6.14 per pound?

BUSINESS Party Supplies Inc. sells metallic balloons for \$2 each and helium balloons for \$3.50 per bunch. Yesterday, they sold 36 more metallic balloons than the number of bunches of helium balloons. The total sales for both types of balloons were \$281. Let *b* represent the number of metallic balloons sold.

a. Copy and complete the table representing the problem.

	Number	Price	Total Price
Metallic Balloons	Ь		
Bunches of Helium Balloons	<i>b</i> — 36		

- **b.** Write an equation to represent the problem.
- **c.** How many metallic balloons were sold?
- d. How many bunches of helium balloons were sold?
- **8. MONEY** Lakeisha spent \$4.57 on color copies and black-and-white copies for her project. She made 7 more black-and-white copies than color copies. How many color copies did she make?

Type of Copy	Cost per Page
color	\$0.44
black-and-white	\$0.07

= Step-by-Step Solutions begin on page R12.

Extra Practice begins on page 815.

- **Example 2 P.** 133
 9. FISH Rosamaria is setting up a 20-gallon saltwater fish tank that needs to have a salt content of 3.5%. If Rosamaria has water that has 2.5% salt and water that has 3.7% salt, how many gallons of the water with 3.7% salt content should Rosamaria use?
 - **10. CHEMISTRY** Hector is performing a chemistry experiment that requires 160 milliliters of 40% sulfuric acid solution. He has a 25% sulfuric acid solution and a 50% sulfuric acid solution. How many millimeters of each solution should he mix to obtain the needed solution?
- **Example 3 p. 134 11. TRAVEL** A boat travels 36 miles in 1.5 hours and then 14 miles in 0.75 hour. What is the average speed of the boat?
 - **12. RUNNING** A runner ran 1.5 miles in 28 minutes and then 1.2 more miles in 10 minutes. What was the average speed in miles per minute?
 - **13. AIRLINERS** Two airliners are 1600 miles apart and heading toward each other at different altitudes. The first plane is traveling north at 620 miles per hour, while the second is traveling south at 780 miles per hour. When will the planes pass each other?

Example 4 P. 135 14. SAILING A ship is sailing due east at 20 miles per hour when it passes the lighthouse. At the same time a ship is sailing due west at 15 miles per hour when it passes a point. The lighthouse and the point are 175 miles apart. When will these ships pass each other?

15. CHEMISTRY A lab technician has 40 gallons of a 15% iodine solution. How gallons of a 40% iodine solution must he add to make a 20% iodine solution?



Real-World Link

Different countries have individual grading scales. For example, French schools give number grades ranging from 0 to 20, rather than letter grades like those in the U.S.

Source: Morris

16. GRADES At Westbridge High School, a student's grade point average (GPA) is based on the student's grade and the class credit rating. Brittany's grades for this quarter are shown. Find Brittany's GPA if a grade of A equals 4 and a B equals 3.

Class	Credit Rating	Grade
Algebra 1	1	А
Science	1	А
English	1	В
Spanish	1	А
Music	$\frac{1}{2}$	В

17. SPORTS Steve participated in a triathlon that included swimming, biking, and

running. In the triathlon, Steve swam 0.5 mile in 15 minutes, biked 20 miles in 90 minutes, and ran 4 miles in 30 minutes. What was Steve's average speed for the triathlon in miles per hour?

- **18. MUSIC** Amalia has 10 songs on his MP3 player. If 3 of those songs are 5 minutes long, 3 are 4 minutes long, 2 are 2 minutes long, and 2 are 3.5 minutes long, what is the average length of the songs in his collection?
- **9 DISTANCE** Garcia is driving to Florida for vacation. The trip is a total of 625 miles.
 - **a.** How far can he drive in 6 hours if he is driving at 65 miles per hour?
 - **b.** If Garcia maintains a speed of 65 miles per hour, how long will it take him to drive to Florida?
- **20. TRAVEL** Two buses leave Smithville at the same time, one traveling north and the other traveling south. The northbound bus travels at 50 miles per hour, and the southbound bus travels at 65 miles per hour. Let *t* represent the amount of time since their departure.
 - **a.** Copy and complete the table representing the situation.

	r	t	<i>d</i> = <i>rt</i>
Northbound bus	?	?	?
Southbound bus	?	?	?

- **b.** Write an equation that could be used to determine when the trains will be 345 miles apart.
- **c.** In how many hours will the trains be 345 miles apart? Explain how you found your answer.
- **21. TRAVEL** A subway travels 60 miles per hour from Glendale to Midtown. Another subway, traveling at 45 miles per hour, takes 11 minutes longer for the same trip. How far apart are Glendale and Midtown?

H.O.T. Problems

Use Higher-Order Thinking Skills

- **22. OPEN ENDED** Write a problem that depicts motion in opposite directions.
- **23. REASONING** Describe the conditions so that adding a 50% solution to a 100% solution would produce a 75% solution.
- **24. CHALLENGE** Find five consecutive odd integers in which the sum of the first and the fifth is one less than three times the fourth.
- **25. CHALLENGE** Describe a situation involving mixtures that could be represented by the equation 1.00x + 0.15(36) = 0.50(x + 36).
- **26.** WRITING IN MATH Describe how a gallon of 25% solution is added to an unknown amount of 10% solution to get a 15% solution.

KCCT PRACTICE MA-HS-1.4.1

27. If 2x + y = 5, what is the value of 4x?

- **A** 10 y **B** 10 - 2y**C** $\frac{5 - y}{2}$
- D $\frac{10-y}{2}$

28. Which expression is equivalent to $7x^23x^{-4}$?

- **F** $21x^{-8}$
- **G** $21x^2$
- **H** $21x^{-6}$
- J $21x^{-2}$

29. GEOMETRY What is the base of the triangle if the area is 56 square meters?







30. SHORT RESPONSE Brianne makes blankets for a baby store. She works on the blankets 30 hours per week. The store pays her \$9.50 per hour plus 30% of the profit. If her hourly rate is increased by \$0.75 and her commission is raised to 40%, how much will she earn in dollars if a total of \$300 profit is made from the blankets sold?

Spiral Review

Solve each equation or formula for *x*. (Lesson 2-8)

31. 2bx - b = -5

32. 3x - r = r(-3 + x)

33.
$$A = 2\pi r^2 + 2\pi rx$$

37. $2x^2 + 3 = 21$

44. 18 - 2k = 24

47. $\frac{n}{6} + 1 = 5$

34. SKIING Yuji is registering for a ski camp. The cost of the camp is \$1254, but there is a sales tax of 7%. What is the total cost of the camp including tax? (Lesson 2-7)

Translate each equation into a sentence. (Lesson 2-1)

35.	$\frac{n}{-6} = 2n + 1$	36. $18 - 5h = 13h$
-----	-------------------------	----------------------------

Refer to the graph.

- **38.** Name the ordered pair at point *A* and explain what it represents. (Lesson 1-6)
- **39.** Name the ordered pair at point *B* and explain what it represents. (Lesson 1-6)
- **40.** Identify the independent and dependent variables for the function. (Lesson 1-6)
- **41. BASEBALL** Tickets to a baseball game cost \$18.95, \$12.95, or \$9.95. A hot dog and soda combo costs \$5.50. The Madison family is having a reunion. They buy 10 tickets in each price category and plan to buy 30 combos. What is the total cost for the tickets and meals? (Lesson 1-3)

43. 9m - 11 = -29

46. $7 = \frac{h}{2} + 3$



Skills Review

Solve each equation. (Lesson 2-4)

42. a - 8 = 15

45. 5 - 8y = 61

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Study Guide and Review

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- STUDYTO GO
- Vocabulary Review

Chapter Summary

Key Concepts

Writing Equations (Lesson 2-1)

• Identify the unknown you are looking for and assign a variable to it. Then, write the sentence as an equation.

Solving Equations (Lessons 2-2 to 2-4)

- Addition and Subtraction Properties of Equality: If an equation is true and the same number is added to or subtracted from each side, the resulting equation is true.
- Multiplication and Division Properties of Equality: If an equation is true and each side is multiplied or divided by the same nonzero number, the resulting equation is true.
- Steps for Solving Equations:
 - **Step 1** Simplify the expression on each side. Use the Distributive Property as needed.
 - **Step 2** Use the Addition and/or Subtraction Properties of Equality to get the variables on one side and the numbers without variables on the other side.
 - **Step 3** Use the Multiplication or Division Property of Equality to solve.

Absolute Value Equations (Lesson 2-5)

• For any real numbers a and b, if |a| = b, then a = b or a = -b.

Ratios and Proportions (Lesson 2-6)

• The Means-Extremes Property of Proportion states that in a proportion, the product of the extremes is equal to the product of the means.

FOLDABLES[®] Study Organizer

Be sure the Key Concepts are noted in your Foldable.



Key Vocabulary 🧀

consecutive integers (p. 92)	percent of decrease (p. 119)
limensional analysis (p. 128)	percent of increase (p. 119)
equivalent equations (p. 83)	proportion (p. 111)
extremes (p. 112)	rate (p. 113)
ormula (p. 76)	ratio (p. 111)
<mark>dentities</mark> (p. 98)	scale (p. 114)
iteral equation (p. 127)	scale model (p. 114)
neans (p. 112)	solve an equation (p. 83)
nulti-step equations (p. 91)	unit analysis (p. 128)
number theory (p. 92)	unit rate (p. 113)
percent of change (p. 119)	weighted average (p. 132)

Vocabulary Check 🧀

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

- In order to write an equation, identify the unknown for which you are looking and assign a(n) <u>number</u> to it.
- **2.** To <u>solve an equation</u> means to find the value of the variable that makes the equation true.
- **3.** The numbers 10, 12, and 14 are an example of <u>consecutive even integers</u>.
- **4.** The <u>absolute value</u> of any number is simply the distance the number is away from zero on a number line.
- **5.** A(n) <u>equation</u> is a comparison of two numbers by division.
- **6.** An equation stating that two ratios are equal is called a(n) <u>proportion</u>.
- **7.** If the new number is less than the original number, the percent of change is a percent of <u>increase</u>.
- **8.** The <u>weighted average</u> of a set of data is the sum of the product of the number of units and the value per unit divided by the sum of the number of units.

Lesson-by-Lesson Review

2-1

CHAPTER

Writing Equations (pp. 75–80)

Translate each sentence into an equation.

- **9.** The sum of five times a number *x* and three is the same as fifteen.
- **10.** Four times the difference of *b* and six is equal to *b* squared.
- **11.** One half of *m* cubed is the same as four times *m* minus nine.

Translate each equation into a sentence.

12.
$$3p + 8 = 20$$

13.
$$h^2 - 5h + 6 = 0$$

- $14. \ \frac{3}{4}w^2 + \frac{2}{3}w \frac{1}{5} = 2$
- **15. FENCING** Adrianne wants to create an outdoor rectangular kennel. The length will be three feet more than twice the width. Write and use an equation to find the length and the width of the kennel if Adrianne has 54 feet of fencing.

EXAMPLE 1

Translate the following sentence into an equation.

Six times the sum of a number n and four is the same as the difference between two times n to the second power and ten.

HS-AT-S-VE01,

HS-NPO-S-PN04,

HS-AT-S-EI3

HS-AT-S-EI1

 $6(n+4) = 2n^2 - 10$

EXAMPLE 2

Translate $3d^2 - 9d + 8 = 4(d + 2)$ into a sentence.

Three times a number d squared minus nine times d increased by eight is equal to four times the sum of d and two.

2-2 Solving One-Step Equations (pp. 81–89)

Solve each equation. Check your solution.

-11

16.
$$x - 9 = 4$$

17. $-6 + g = 1$

18.
$$\frac{5}{9} + w = \frac{7}{9}$$

19.
$$3.8 = m + 1.7$$

20.
$$\frac{a}{12} = 5$$

21.
$$8y = 48$$

22.
$$\frac{2}{5}b = -4$$

23.
$$-\frac{t}{16} = -\frac{7}{8}$$

24. AGE Max is four years younger than his sister Brenda. The total of their ages is 16. Write and solve an equation to find their ages.

EXAMPLE 3

Solve $x - 13 = 9$. Check your solution.		
x - 13 = 9	Original equation	
x - 13 + 13 = 9 + 13	Add 13 to each side.	
x = 22	-13 + 13 = 0 and $9 + 13 = 22$	
To check that 22 is the solution, substitute 22 for x in the original equation.		
CHECK $x - 13 = 9$	Original equation	

22 − 13 ² 9	Substitute 22 for x.
9 = 9	Subtract.





HS-NPO-S-PN04, HS-AT-S-EI3

Solving Multi-Step Equations (pp. 90–96)

2-3

Solve each equation. Check your solution.

25. $2d - 4 = 8$	26. $-9 = 3t + 6$
27. $14 = -8 - 2k$	28. $\frac{n}{4} - 7 = -2$
29. $\frac{r+4}{3} = 7$	30. $-18 = \frac{9-a}{2}$
31. $6g - 3.5 = 8.5$	32. $0.2c + 4 = 6$
33. $\frac{f}{3} - 9.2 = 3.5$	34. $4 = \frac{-3u - (-7)}{-8}$

- **35. CONSECUTIVE INTEGERS** Find three consecutive odd integers such that their sum is 63.
- **36. CONSECUTIVE INTEGERS** Find three consecutive integers such that their sum is -39.

EXAMPLE 4

Solve $7y - 9 = 33$. Check your solution.	
7y - 9 = 33	Original equation
7y - 9 + 9 = 33 + 9	Add 9 to each side.
7y = 42	Simplify.
$\frac{7y}{7} = \frac{42}{7}$	Divide each side by 7.
y = 6	Simplify.
CHECK $7y - 9 = 33$	Original equation
7 (6) − 9 ≟ 33	Substitute 6 for <i>y</i> .
$42 - 9 \stackrel{?}{=} 33$	Multiply.
33 = 33 ✓	Subtract.

2-4	Solving Equations with the Variable on Each Side	(pp. 97–102)
-----	--	--------------

Solve each equation. Check your solution.

37.
$$8m + 7 = 5m + 16$$

38.
$$2h - 14 = -5h$$

39.
$$21 + 3j = 9 - 3j$$

40.
$$\frac{x-3}{4} = \frac{x}{2}$$

41.
$$\frac{6r-7}{10} = \frac{r}{4}$$

$$10^{-1}$$
 10^{-1} $10^{$

43.
$$-2(b-3) - 4 = 18$$

44.
$$4(3w-2) = 8(2w+3)$$

Write an equation and solve each problem.

- **45.** Find the sum of three consecutive odd integers if the sum of the first two integers is equal to twenty-four less than four times the third integer.
- **46. TRAVEL** Mr. Jones drove to a business meeting that was 480 miles away. His travel time to the meeting was 8 hours and from the meeting was 6 hours. Find his rate of travel for each leg of the trip.

EXAMPLE 5	
Solve $9w - 24 = 6w + 18$	
9w - 24 = 6w + 18	Original equation
9w - 24 - 6w = 6w + 18 - 6w	Subtract 6 <i>w</i> from each side.
3w - 24 = 18	Simplify.
3w - 24 + 24 = 18 + 24	Add 24 to each side.
3w = 42	Simplify.
$\frac{3w}{3} = \frac{42}{3}$	Divide each side by 3.
w = 14	Simplify.

EXAMPLE 6

Write an equation to find three consecutive integers such that three times the sum of the first two integers is the same as thirteen more than four times the third integer.

Let x, x + 1, and x + 2 represent the three consecutive integers.

3(x + x + 1) = 4(x + 2) + 13

Study Guide and Review

2-5 Solving Equations Involving Absolute Value (pp. 103–109) HS-AT-S-E HS-AT-S-E		
Evaluate each expression if $m = -8$, $n = 4$, and	EXAMPLE 7	
p = -12. 47. $ 3m - n $	Solve $ y - 9 = 16$. The set.	en graph the solution
48. $ -2p + m - 3n$	Case 1	
49. $-3 6n-2p $	y - 9 = 16	Original equation
50. $4 7m+3p +4n$	y - 9 + 9 = 16 + 9	Add 9 to each side.
Solve each equation. Then graph the solution set.	y = 25	Simplify.
51. $ x - 6 = 11$	Case 2	
52. $ -4w+2 = 14$	y - 9 = -16	Original equation
53. $\left \frac{1}{2}d - 6\right = 15$	y - 9 + 9 = -16 + 9	Add 9 to each side.
3 2h	y = -7	Simplify.
54. $\left \frac{2b}{3} + 8\right = 20$	The solution set is $\{-7, $	25}.
	Graph the points on a n	umber line.
	<mark>→ ● </mark> -10 -5 0 5	+ + + ◆ + ► 10 15 20 25 30

2-6 Ratios and Proportions (pp. 111–118)

Determine whether each pair of ratios are equivalent ratios. Write *yes* or *no*.

55. $\frac{27}{45}, \frac{3}{5}$ **56.** $\frac{18}{32}, \frac{3}{4}$

CHAPTER

Solve each proportion. If necessary, round to the nearest hundredth.

57.
$$\frac{4}{9} = \frac{a}{45}$$

58. $\frac{3}{8} = \frac{21}{t}$
59. $\frac{9}{12} = \frac{g}{16}$

60. CONSTRUCTION A new gym is being built at Greenfield Middle School. The length of the gym on the blueprints is 12 inches. The scale is $\frac{3}{4}$ inch = 5 feet. Find the actual length of the new gym.

EXAMPLE 8

Determine whether $\frac{7}{9}$ and $\frac{42}{54}$ are equivalent ratios. Write *yes* or *no*. Justify your answer.

First, simplify each ratio. $\frac{7}{9}$ is already in simplest form.

HS-NPO-S-RP1,

HS-NPO-S-N09

$$\frac{42}{54} = \frac{42 \div 6}{54 \div 6} = \frac{7}{9}$$

When expressed in simplest form, the ratios are equivalent. The answer is yes.

EXAMPLE 9

1

Solve $\frac{r}{8} = \frac{3}{4}$. If necessary, round to the nearest hundredth.

$\frac{r}{8} = \frac{3}{4}$	Original equation
(4) = 3(8)	Find the cross products.
4r = 24	Simplify.
$\frac{4r}{4} = \frac{24}{4}$	Divide each side by 4.
r = 6	Simplify.



HS-M-S-SM4,

2-7 Percent of Change (pp. 119–125)

State whether each percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change. Round to the nearest whole percent.

- **61.** original: 40, new: 50
- 62. original: 36, new: 24
- **63.** original: \$72, new: \$60

Find the total price of each item.

- **64.** boots: \$64, tax: 7%
- **65.** video game: \$49, tax: 6.5%
- **66.** hockey skates: \$199, tax: 5.25%

Find the discounted price of each item.

- **67.** MP3 player: \$69.00, discount: 20%
- **68.** jacket: \$129, discount: 15%
- **69.** backpack: \$45, discount: 25%
- **70. ATTENDANCE** An amusement park recorded attendance of 825,000 one year. The next year, the attendance increased to 975,000. Determine the percent of increase in attendance.

EXAMPLE 10

State whether the percent of change is a percent of *increase* or a percent of *decrease*. Then find the percent of change. Round to the nearest whole percent.

original: 80 new: 60

Since the new amount is less than the original, this is a percent of decrease. Subtract to find the amount of change: 80 - 60 = 20.

Use the original number, 80, as the base.



The percent of decrease is 25%.

2-8 Literal Equations and Dimensional Analysis (pp. 126–131) HS-AT-S-E15 Solve each equation or formula for the variable indicated. **71.** 3x + 2y = 9, for y

- **72.** $P = 2\ell + 2w$, for ℓ
- **73.** -5m + 9n = 15, for m
- **74.** 14w + 15x = y 21w, for w

75.
$$m = \frac{2}{5}y + n$$
, for y

76. 7d - 3c = f + 2d, for d

77. GEOMETRY The formula for the area of a trapezoid is $A = \frac{1}{2}h(a + b)$, where *h* represents the height and *a* and *b* represent the lengths of the bases. Solve the formula for *h*.

EXAMPLE 11	
Solve $6p - 8n = 12$ for <i>p</i> .	
6p - 8n = 12	Original equation
6p - 8n + 8n = 12 + 8n	Add 8 <i>n</i> to each side.
6p = 12 + 8n	Simplify.
$\frac{6p}{6} = \frac{12 + 8n}{6}$	Divide each side by 6.
$\frac{6p}{6} = \frac{12}{6} + \frac{8}{6}n$	Simplify.
$p = 2 + \frac{4}{3}n$	Simplify.


- **78. CANDY** Michael is mixing two types of candy for a party. The chocolate pieces cost \$0.40 per ounce, and the hard candy costs \$0.20 per ounce. Michael purchases 20 ounces of the chocolate pieces, and the total cost of his candy was \$11. How many ounces of hard candy did he purchase?
- **79. TRAVEL** A car travels 100 miles east in 2 hours and 30 miles north in half an hour. What is the average speed of the car?
- **80. BUSINESS** A candle supply store sells votive wax for \$0.90 a pound and low-shrink wax for \$1.04 a pound. How many pounds of low-shrink wax should be mixed with 8 pounds of votive wax to obtain a blend that sells for \$0.98 a pound?

EXAMPLE 12

METALS An alloy of metals is 25% copper. Another alloy is 50% copper. How much of each should be used to make 1000 grams of an alloy that is 45% copper?

HS-M-S-MPA1

Let x = the amount of the 25% copper alloy. Write and solve an equation.

0.25x + 0.50(1000 - x) = 0.45(1000)	Original Equation
0.25x + 500 - 0.50x = 450	Distributive Property
-0.25x + 500 = 450	Simplify.
-0.25x + 500 - 500 = 450 - 500	Subtract 500 from each side.
-0.25x = -50	Simplify.
$\frac{-0.25x}{-0.25} = \frac{-50}{-0.25}$	Divide each side by –0.25.
x = 200	Simplify.

200~grams of the 25% alloy and 800~grams of the 50% alloy should be used.

Translate each sentence into an equation.

- **1.** The sum of six and four times *d* is the same as *d* minus nine.
- **2.** Three times the difference of two *m* and five is equal to eight times *m* to the second power increased by four.

Solve each equation. Check your solutions.

- **3.** x 5 = -11**4.** $\frac{2}{3} = w + \frac{1}{4}$ **5.** $\frac{t}{6} = -3$

Solve each equation. Check your solution.

6.
$$2a - 5 = 13$$
 7. $\frac{p}{4} -$

8. MULTIPLE CHOICE At the Mama Mia Pizza Parlor, the price of a large pizza is determined by the equation 9 + 1.5x, where *x* represents the number of toppings added to a cheese pizza. Daniel spent \$13.50 on a large pizza. How many toppings did he select?

3 = 9

- **A** 0
- **B** 1
- **C** 3
- **D** 5

Solve each equation. Check your solution.

- **9.** 5y 4 = 9y + 8
- **10.** 3(2k-2) = -2(4k-11)
- **11. GEOMETRY** Find the value of *x* so that the figures have the same perimeter.



12. Evaluate the expression |3t - 2u| + 5v if t = 2, u = -5, and v = -3.

Solve each equation. Then graph the solution set.

- **13.** |p-4| = 6
- 14. |2b + 5| = 9

Solve each proportion. If necessary, round to the nearest hundredth.

15.
$$\frac{a}{3} = \frac{16}{24}$$

16. $\frac{9}{k+3} = \frac{3}{5}$

- 17. MULTIPLE CHOICE Akiko uses 2 feet of thread for every three squares that she sews for her quilt. How many squares can she sew if she has 38 feet of thread?
 - **F** 19
 - **G** 57
 - **H** 76
 - I 228
- **18.** State whether the percent of change is a percent of increase or a percent of decrease. Then find the percent of change. Round to the nearest whole percent.

original: 54 new: 45

- **19.** Find the total price of a sweatshirt that is priced at \$48 and taxed at 6.5%.
- **20. SHOPPING** Kirk wants to purchase a wide-screen TV. He sees an advertisement for a TV that was originally priced at \$3200 and is 20% off. Find the discounted price of the TV.
- **21.** Solve 5x 3y = 9 for *y*.
- **22.** Solve $A = \frac{1}{2}bh$ for *h*.
- **23.** CHEMISTRY Deon has 12 milliliters of a 5% solution. He also has a solution that has a concentration of 30%. How many milliliters of the 30% solution does Dave need to add to the 5% solution to obtain a 20% solution?
- **24. BICYCLING** Shanee bikes 5 miles to the park in 30 minutes. She then continues on and bikes 3 miles to the library in 45 minutes. What was her average speed?
- **25.** MAPS On a map of North Carolina, the distance between Charlotte and Wilmington is 14.75 inches. If 2 inches equals 24 miles, what is the approximate distance between the two cities?



Gridded Response Questions

In addition to multiple-choice, short-answer, and extended-response questions, you will likely encounter gridded-response questions on standardized tests. For gridded-response questions, you must print your answer on an answer sheet and mark in the correct circles on the grid to match your answer.

Strategies for Solving Gridded Response Questions

Step 1

Read the problem carefully.

- Ask yourself: "What information is given?" "What do I need to find?" "How do I solve this type of problem?"
- Solve the Problem: Use the information given in the problem to solve.
- **Check your answer:** If time permits, check your answer to make sure you have solved the problem correctly.

Step 2

Write your answer in the answer boxes.

- Print only one digit or symbol in each answer box.
- Do not write any digits or symbols outside the answer boxes.
- You may write your answer with the first digit in the left answer box, or with the last digit in the right answer box. You may leave blank any boxes you do not need on the right or the left side of your answer.

Step 3

Fill in the grid.

- Fill in only one bubble for every answer box that you have written in. Be sure not to fill in a bubble under a blank answer box.
- Fill in each bubble completely and clearly.

EXAMPLE

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

GRIDDED RESPONSE Ashley is 3 years older than her sister, Tina. Combined, the sum of their ages is 27 years. How old is Ashley?





Read the problem carefully. You are told that Ashley is 3 years older than her sister and that their ages combined equal 27 years. You need to find Ashley's age.



Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve. Copy and complete an answer grid on your paper.

- 1. Orlando has \$1350 in the bank. He wants to increase his balance to a total of \$2550 by depositing \$40 each week from his paycheck. How many weeks will he need to save in order to reach his goal?
- **2.** Fourteen less than three times a number is equal to 40. Find the number.
- 3. The table shows the regular prices and sale prices of certain items at a department store this week. What is the percent of discount during the sale?

ltem	Regular Price (\$)	Sale Price (\$)
pillows	25	20
sweaters	30	24
entertainment center	125	100

4. Maureen is driving from Raleigh, North Carolina, to Charlotte, North Carolina, to visit her brother at college. If she averages 65 miles per hour on the trip, then the equation $\frac{d}{2.65} = 65$ can be solved for the distance *d*. What is the distance, to the nearest mile from Raleigh to Charlotte?

 \bigcirc

6

(5)5

5. Find the value of *x* so that the figures below have the same area.



6. The sum of three consecutive whole numbers is 18. What is the greatest of the numbers?

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. Which point on the number line best represents the position of $\sqrt{8}$? (Lesson 0-2)



2. Find the value of *x* so that the figures have the same area. (Lesson 2-5)



- **3.** The elevation of Black Mountain is 27 feet more than 16 times the lowest point in the state. If the elevation of the lowest point in the state is 257 feet, what is the elevation of Black Mountain? (Lesson 2-2)
 - A 4,085 feet C 4,139 feet
 - **B** 4,103 feet **D** 4,215 feet
- **4.** The expression $(3x^2 + 5x 12) 2(x^2 + 4x + 9)$ is equivalent to which of the following? (Lesson 1-4)
 - **A** $x^2 3x 30$
 - **B** $x^2 + 13x + 6$
 - **C** $5x^2 + x 18$
 - **D** $x^2 + 3x 21$

5. The amount of soda, in fluid ounces, dispensed from a machine must satisfy the equation |a - 0.4| = 20. Which of the following graphs shows the acceptable minimum and maximum amounts that can be dispensed from the machine? (Lesson 2-5)



- **6.** If *a* and *b* represent integers, ab = ba is an example of which property? (Lesson 1-3)
 - A Associative Property
 - B Commutative Property
 - C Distributive Property
 - D Closure Property
- **7.** The sum of one fifth of a number and three is equal to half of the number. What is the number? (Lesson 2-4)

Α	5			C	15

- **B** 10 **D** 20
- **8.** Aaron charges \$15 to mow the lawn and \$10 per hour for other gardening work. Which expression represents his earnings? (Lesson 1-1)
 - **A** 10*h*

B 15h

- **C** 15h + 10
- **D** 15 + 10h

Test-TakingTip

Question 2 Use the figures and the formula for area to set up an equation. The product of the length and width of each figure should be equal.



Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. The formula for the lateral area of a cylinder is $A = 2\pi rh$, where *r* is the radius and *h* is the height. Solve the equation for *h*. (Lesson 2-9)



- **10. GRIDDED RESPONSE** Solve the proportion $\frac{x}{18} = \frac{7}{21}$. (Lesson 2-6)
- **11. GRIDDED RESPONSE** The table shows the cost of renting a moving van. If Miguel budgeted \$75, how many miles could he drive the van and maintain his budget? (Lesson 2-3)

Moving Van Rentals						
Flat Fee	\$50 for up to 300 miles					
Variable Fee	\$0.20 per mile over 300					

- **12.** Find the height of a soup can if the area of the label is 302 square centimeters and the radius of the can is 4 centimeters. Round to the nearest whole number. (Lesson 2-8)
- **13. GRIDDED RESPONSE** Lara's car needed a particular part that costs \$75. The mechanic charges \$50 per hour to install the part. If the total cost was \$350, how many hours did it take to install the part?

14. Lucinda is buying a set of patio furniture that is on sale for $\frac{4}{5}$ of the original price. After she uses a \$50 gift certificate, the total cost before sales tax is \$222. What was the original price of the patio furniture? (Lesson 2-3)

Extended Response

Record your answers on a sheet of paper. Show your work.

- 15. The city zoo offers a yearly membership that costs \$120. A yearly membership includes free parking. Members can also purchase a ride pass for an additional \$2 per day that allows them unlimited access to the rides in the park. Nonmembers pay \$12 for admission to the park, \$5 for parking, and \$5 for a ride pass. (Lesson 2-4)
 - **a.** Write an equation that could be solved for the number of visits it would take for the total cost to be the same for a member and a nonmember if they both purchase a ride pass each day. Solve the equation.
 - **b.** What would the total cost be for members and nonmembers after this number of visits?
 - **c.** Georgena is deciding whether or not to purchase a yearly membership. Explain how she could use the results above to help make her decision.

Need Extra Help?														
If you missed Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Go to Lesson or Page	0-2	2-5	2-2	1-4	2-5	1-3	2-4	1-1	2-8	2-6	2-3	2-8	2-3	2-3
KY Program of Studies	MA- HS- 1.1.1	MA- HS- 5.2.1	MA- HS- 1.5.1	MA- HS- 1.5.1	MA- HS- 5.2.1	MA- HS- 1.5.1	MA- HS- 1.5.1	MA- HS- 5.2.1	MA- HS- 5.3.2	MA- HS- 1.4.1	MA- HS- 5.3.1	MA- HS- 5.3.2	MA- HS- 5.3.1	MA- HS- 5.3.1