

# Linear Functions and Relations

## Then

In Chapter 3, you graphed linear functions.

## Now

In Chapter 4, you will:


- Write and graph linear equations in various forms.
- Use scatter plots and lines of fit, and write equations of best-fit lines using linear regression.
- Identify and graph special functions.

### KY Program of Studies

**HS-AT-S-EI11** Students will relate the coefficients of a linear equation and the slope and  $x$ - and  $y$ -intercepts of its graph.

**HS-G-S-CG2** Students will describe a line by a linear equation.

## Why?

 **TRAVEL** The number of trips people take changes from year to year. From the yearly data, patterns emerge. Rate of change can be applied to these data to determine a linear model. This can be used to predict the number of trips taken in future years.

# Get Ready for Chapter 4

**Diagnose Readiness** You have two options for checking Prerequisite Skills.

## Text Option

Take the Quick Check below. Refer to the Quick Review for help.

### QuickCheck

Evaluate  $3a^2 - 2ab + c$  for the values given. (Lesson 1-5)

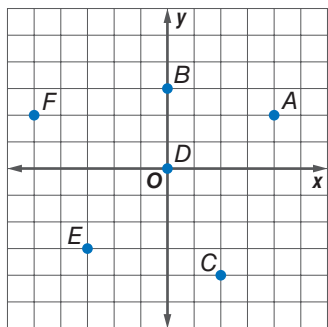
- $a = 2, b = 1, c = 5$
- $a = -3, b = -2, c = 3$
- $a = -1, b = 0, c = 11$
- $a = 5, b = -3, c = -9$
- CAR RENTAL** The cost of renting a car is given by  $49x + 0.3y$ .  $x$  represents the number of days rented, and  $y$  represents the number of miles driven. Find the cost for a five-day rental in which the car is driven 125 miles.

Solve each equation for the given variable. (Lesson 2-8)

- $x + y = 5$  for  $y$
- $2x - 4y = 6$  for  $x$
- $y - 2 = x + 3$  for  $y$
- $4x - 3y = 12$  for  $x$
- GEOMETRY** The formula for the perimeter of a rectangle is  $P = 2w + 2\ell$ , where  $w$  represents width and  $\ell$  represents length. Solve the formula for  $w$ .

Write the ordered pair for each point. (Lesson 1-6)

- A
- B
- C
- D
- E
- F



### QuickReview

#### EXAMPLE 1

Evaluate  $2(m - n)^2 + 3p$  for  $m = 5, n = 2,$  and  $p = -3$ .

$2(m - n)^2 + 3p$	<b>Original expression</b>
$= 2(5 - 2)^2 + 3(-3)$	<b>Substitute.</b>
$= 2(3)^2 + 3(-3)$	<b>Subtract.</b>
$= 2(9) + 3(-3)$	<b>Evaluate power.</b>
$= 18 + (-9)$	<b>Multiply.</b>
$= 9$	<b>Add.</b>

#### EXAMPLE 2

Solve  $5x + 15y = 9$  for  $x$ .

$5x + 15y = 9$	<b>Original equation</b>
$5x + 15y - 15y = 9 - 15y$	<b>Subtract 15y from each side.</b>
$5x = 9 - 15y$	<b>Simplify.</b>
$\frac{5x}{5} = \frac{9 - 15y}{5}$	<b>Divide each side by 5.</b>
$x = \frac{9}{5} - 3y$	<b>Simplify.</b>

#### EXAMPLE 3

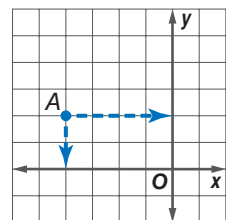
Write the ordered pair for A.

**Step 1** Begin at point A.

**Step 2** Follow along a vertical line to the  $x$ -axis. The  $x$ -coordinate is  $-4$ .

**Step 3** Follow along a horizontal line to the  $y$ -axis. The  $y$ -coordinate is 2.

The ordered pair for point A is  $(-4, 2)$ .



## Online Option



Take a self-check Chapter Readiness Quiz at [glencoe.com](http://glencoe.com).

# Get Started on Chapter 4

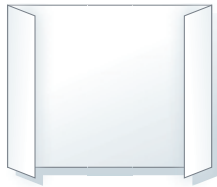
You will learn several new concepts, skills, and vocabulary terms as you study Chapter 4. To get ready, identify important terms and organize your resources. You may wish to refer to **Chapter 0** to review prerequisite skills.

## FOLDABLES®

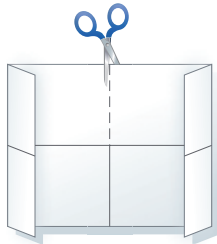
### Study Organizer

**Linear Functions** Make this Foldable to help you organize your Chapter 4 notes about linear functions. Begin with one sheet of 11" by 17" paper.

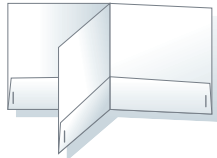
- 1 **Fold** each end of the paper in about 2 inches.



- 2 **Fold** along the width and the length. Unfold. Cut along the fold line from the top to the center.



- 3 **Fold** the top flaps down. Then fold in half and turn to form a folder. Staple the flaps down to form pockets.



- 4 **Label** the front with the chapter title.



**KY Math Online** [glencoe.com](http://glencoe.com)

- Study the chapter online
- Explore **Math in Motion**
- Get extra help from your own **Personal Tutor**
- Use **Extra Examples** for additional help
- Take a **Self-Check Quiz**
- Review Vocabulary** in fun ways

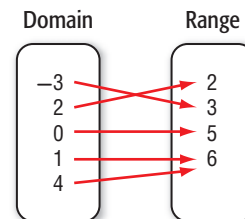
## New Vocabulary

English	Español
slope-intercept form	forma pendiente-intersección
linear extrapolation	extrapolación lineal
point-slope form	forma punto-pendiente
parallel lines	rectas paralelas
perpendicular lines	rectas perpendiculares
scatter plot	gráfica de dispersión
line of fit	recta de ajuste
linear interpolation	interpolación lineal
best-fit line	recta de ajuste óptimo
linear regression	retroceso lineal
correlation coefficient	coeficiente de correlación
median-fit line	línea de mediana-ataque
step function	función etapa
greatest integer function	función del máximo entero
piecewise function	función a intervalos

## Review Vocabulary

**coefficient** • p. 26 • **coeficiente** the numerical factor of a term

**function** • p. 45 • **función** a relation in which each element of the domain is paired with exactly one element of the range



**ratio** • p. 111 • **razón** a comparison of two numbers by division

**Multilingual eGlossary** [glencoe.com](http://glencoe.com)

# Graphing Technology Lab

## Investigating Slope-Intercept Form

### Objective

Use a graphing calculator to collect data and investigate slope-intercept form.



**HS-G-S-CG2** Students will describe a line by a linear equation. Also addresses HS-DAP-S-DR6.

### Set Up the Lab

- Cut a small hole in a top corner of a plastic sandwich bag. Hang the bag from the end of the force sensor.
- Connect the force sensor to your data collection device.



### ACTIVITY

- Step 1** Use the sensor to collect the weight with 0 washers in the bag. Record the data pair in the calculator.
- Step 2** Place one washer in the plastic bag. Wait for the bag to stop swinging, then measure and record the weight.
- Step 3** Repeat the experiment, adding different numbers of washers to the bag. Each time, record the number of washers and the weight.

### Analyze the Results

1. The domain contains values represented by the independent variable, washers. The range contains values represented by the dependent variable, weight. Use the graphing calculator to create a scatter plot using the ordered pairs (washers, weight).
2. Write a sentence that describes the points on the graph.
3. Describe the position of the point on the graph that represents the trial with no washers in the bag.
4. The rate of change can be found by using the formula for slope.

$$\frac{\text{rise}}{\text{run}} = \frac{\text{change in weight}}{\text{change in number of washers}}$$

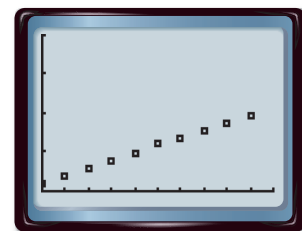
Find the rate of change in the weight as more washers are added.

5. Explain how the rate of change is shown on the graph.

### Make a Conjecture

The graph shows sample data from a washer experiment. Describe the graph for each situation.

6. A bag that hangs weighs 0.8 N when empty and increases in weight at the rate of the sample.
7. A bag that has the same weight when empty as the sample and increases in weight at a faster rate.
8. A bag that has the same weight when empty as the sample and increases in weight at a slower rate.



[0, 20] scl: 2 by [0, 1] scl: 0.25

# Graphing Equations in Slope-Intercept Form

## Then

You found rates of change and slopes. (Lesson 3-3)

## Now

- Write and graph linear equations in slope-intercept form.
- Model real-world data with equations in slope-intercept form.

### KY Program of Studies

HS-G-S-CG2 Students will describe a line by a linear equation.

HS-AT-S-EI11 Students will relate the coefficients of a linear equation and the slope and  $x$ - and  $y$ -intercepts of its graph. Also addresses HS-AT-S-PRF13 and HS-AT-S-EI15.

### New Vocabulary

slope-intercept form

### KY Math Online

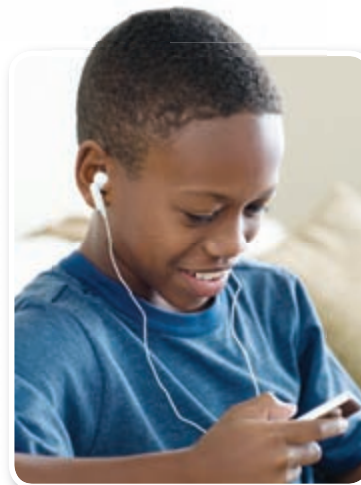
glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help
- Math in Motion

## Why?

Jamil has 500 songs on his MP3 player. He joins a music club that lets him download 30 songs per month for a monthly fee. The number of songs that Jamil could eventually have in his player if he does not delete any songs is represented by  $y = 30x + 500$ .

**Slope-Intercept Form** An equation of the form  $y = mx + b$ , where  $m$  is the slope and  $b$  is the  $y$ -intercept, is in **slope-intercept** form.  $m$  and  $b$  are called *parameters* of the equation. Changing either value changes the equation's graph.



### Key Concept

## Slope-Intercept Form

For Your

FOLDABLE

**Words** The slope-intercept form of a linear equation is  $y = mx + b$ , where  $m$  is the slope and  $b$  is the  $y$ -intercept.

**Example**

$$y = mx + b$$

$$y = 2x + 6$$

slope  $\nearrow$  y-intercept  $\nwarrow$

▶ Math in Motion, BrainPOP® glencoe.com

### EXAMPLE 1 Write and Graph an Equation

Write an equation in slope-intercept form for the line with a slope of  $\frac{3}{4}$  and a  $y$ -intercept of  $-2$ . Then graph the equation.

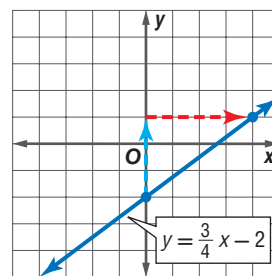
$$y = mx + b \quad \text{Slope-intercept form}$$

$$y = \frac{3}{4}x + (-2) \quad \text{Replace } m \text{ with } 4 \text{ and } b \text{ with } -2.$$

$$y = \frac{3}{4}x - 2 \quad \text{Simplify.}$$

Now graph the equation.

- Step 1** Plot the  $y$ -intercept  $(0, -2)$ .
- Step 2** The slope is  $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$ . From  $(0, -2)$ , move up 3 units and right 4 units. Plot the point.
- Step 3** Draw a line through the two points.



### Check Your Progress

Write an equation of a line in slope intercept form with the given slope and  $y$ -intercept. Then graph the equation.

1A. slope:  $-\frac{1}{2}$ ,  $y$ -intercept 3

1B. slope:  $-3$ ,  $y$ -intercept  $-8$

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Equations are often not written in slope-intercept form. When this occurs, it is often easier to rewrite the equation so that it is in slope-intercept form before you graph the equation.

### StudyTip

**Dependent Variables**  
 $y$  is the *dependent* variable because it *depends* on changes in  $x$ .

### StudyTip

**Counting and Direction** When counting rise and run, a negative sign may be associated with the value in the numerator or denominator. If with the numerator, begin by counting down for the rise. If with the denominator, count left when counting the run. The resulting line will be the same.

### EXAMPLE 2 Graph Linear Equations

Graph  $3x + 2y = 6$ .

Rewrite the equation in slope-intercept form.

$$3x + 2y = 6 \quad \text{Original equation}$$

$$3x + 2y - 3x = 6 - 3x \quad \text{Subtract } 3x \text{ from each side.}$$

$$2y = 6 - 3x \quad \text{Simplify.}$$

$$2y = -3x + 6 \quad 6 - 3x = 6 + (-3x) \text{ or } -3x + 6$$

$$\frac{2y}{2} = \frac{-3x + 6}{2} \quad \text{Divide each side by 2.}$$

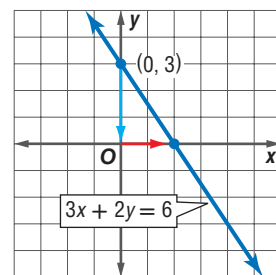
$$y = -\frac{3}{2}x + 3 \quad \text{Slope-intercept form}$$

Now graph the equation. The slope is  $-\frac{3}{2}$ , and the  $y$ -intercept is 3.

**Step 1** Plot the  $y$ -intercept  $(0, 3)$ .

**Step 2** The slope is  $\frac{\text{rise}}{\text{run}} = -\frac{3}{2}$ . From  $(0, 3)$ , move down 3 units and right 2 units. Plot the point.

**Step 3** Draw a line through the two points.



### Check Your Progress

Graph each equation.

**2A.**  $3x - 4y = 12$

**2B.**  $-2x + 5y = 10$

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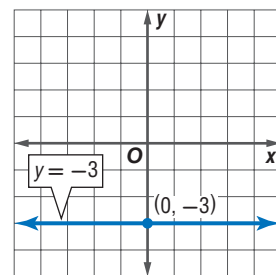
Horizontal lines have a slope of 0. So, equations of horizontal lines can be written in slope intercept form as  $y = 0x + b$  or  $y = b$ . Vertical lines have no slope. So, equations of vertical lines cannot be written in slope-intercept form.

### EXAMPLE 3 Graph Linear Equations

Graph  $y = -3$ .

**Step 1** Plot the  $y$ -intercept  $(0, -3)$ .

**Step 2** The slope is 0. Draw a line through the points with  $y$ -coordinate  $-3$ .



### Check Your Progress

Graph each equation.

**3A.**  $y = 5$

**3B.**  $2y = 1$

[glencoe.com](http://glencoe.com)

Notice that the equations of horizontal lines do not have an  $x$  variable. The graph of a horizontal line does not cross the  $x$ -axis. The equation  $y = 0$  lies on the  $x$ -axis.

There are times when you will need to write an equation when given a graph. To do this, locate the  $y$ -intercept and use the rise and run to find another point on the graph. Then write the equation in slope-intercept form.

### Test-Taking Tip

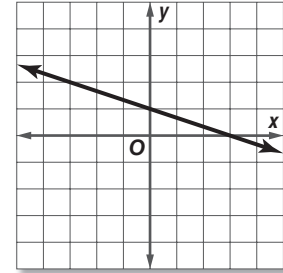
#### Eliminating Choices

Analyze the graph to determine the slope and the  $y$ -intercept. Then you can save time by eliminating answer choices that do not match the graph.

### KCCT EXAMPLE 4 MA-HS-5.1.5

Which of the following is an equation in slope-intercept form for the line shown in the graph?

- A  $y = -3x + 1$
- B  $y = -3x + 3$
- C  $y = -\frac{1}{3}x + 1$
- D  $y = -\frac{1}{3}x + 3$



#### Read the Test Item

You need to find the slope and  $y$ -intercept of the line to write the equation.

#### Solve the Test Item

**Step 1** The line crosses the  $y$ -axis at  $(0, 1)$ , so the  $y$ -intercept is 1. The answer is either A or C.

**Step 2** To get from  $(0, 1)$  to  $(3, 0)$ , go down 1 unit and 3 units to the right. The slope is  $-\frac{1}{3}$ .

**Step 3** Write the equation.

$$y = mx + b$$

$$y = -\frac{1}{3}x + 1$$

**CHECK** The graph also passes through  $(-3, 0)$ . If the equation is correct, this should be a solution.

$$y = -\frac{1}{3}x + 1$$

$$2 \stackrel{?}{=} -\frac{1}{3}(-3) + 1$$

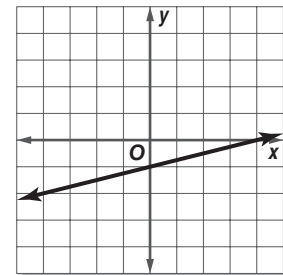
$$2 \stackrel{?}{=} 1 + 1$$

$$2 = 2 \checkmark \quad \text{The answer is C.}$$

#### Check Your Progress

4. Which of the following is an equation in slope-intercept form for the line shown in the graph?

- F  $y = \frac{1}{4}x - 1$
- G  $y = \frac{1}{4}x + 4$
- H  $y = 4x - 1$
- J  $y = 4x + 4$



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**Modeling Real-World Data** Real-world data can be modeled by a linear equation if there is a constant rate of change in the data. The rate of change represents the slope. The  $y$ -intercept is the point where the value of the independent variable is 0.



## Real-World EXAMPLE 5 Write and Graph a Linear Equation

**SPORTS** Use the information at the left about high school sports.

- a. Write a linear equation to find the number of girls in high school sports after 1997.

<b>Words</b>	Number of girls competing	equals	rate of change	times	number of years	plus	amount at start.
<b>Variable</b>	Let $G$ = number of girls competing.				Let $n$ = number of years since 1997.		
<b>Equation</b>	$G$	=	0.06	×	$n$	+	2.6

The equation is  $G = 0.06n + 2.6$ .

- b. Graph the equation.

The  $y$ -intercept is where the data begins. So, the graph passes through  $(0, 2.6)$ .

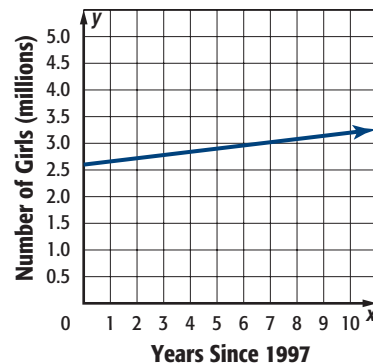
The rate of change is the slope, so the slope is 0.06.

- c. Estimate the number of girls competing in 2017.

The year 2017 is 20 years after 1997.

$$\begin{aligned}
 G &= 0.06n + 2.6 && \text{Write the equation.} \\
 &= 0.06(20) + 2.6 && \text{Replace } n \text{ with } 20. \\
 &= 3.8 && \text{Simplify.}
 \end{aligned}$$

There will be about 3.8 million girls competing in high school sports in 2017.



### Real-World Link

In 1997, about 2.6 million girls competed in high school sports. The number of girls competing in high school sports has increased by an average of 0.06 million per year since 1997.

Source: National Federation of High School Associations

### Check Your Progress

5. **FUNDRAISERS** The band boosters are selling submarine sandwiches for \$5 each. They bought \$1160 in ingredients to make the sandwiches.

- Write an equation for the profit  $P$  made on  $n$  sandwiches.
- Graph the equation.
- Find the total profit if 1400 sandwiches are sold.

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## Check Your Understanding

### Example 1 p. 214

Write an equation of a line in slope-intercept form with the given slope and  $y$ -intercept. Then graph the equation.

- slope: 2,  $y$ -intercept: 4
- slope:  $-5$ ,  $y$ -intercept: 3
- slope:  $\frac{3}{4}$ ,  $y$ -intercept:  $-1$
- slope:  $-\frac{5}{7}$ ,  $y$ -intercept:  $-\frac{2}{3}$

### Examples 2 and 3 p. 215

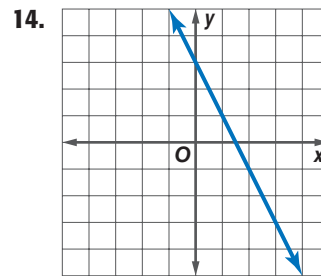
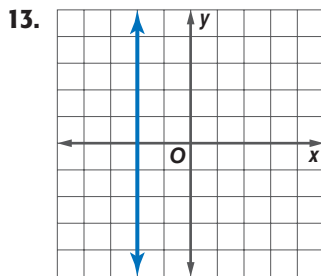
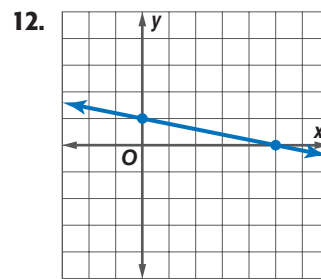
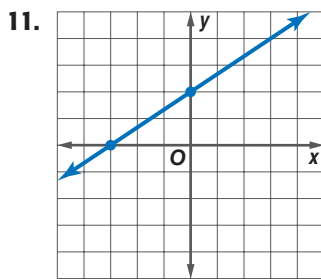
Graph each equation.

- $-4x + y = 2$
- $2x + y = -6$
- $-3x + 7y = 21$
- $6x - 4y = 16$
- $y = -1$
- $15y = 3$



**Example 4**  
p. 216

Write an equation in slope-intercept form for each graph shown.



**Example 5**  
p. 217

15. **SAVING** Rondell has saved \$75 toward a new stereo system for his car. He plans to save \$10 each week for the next few weeks.
- Write an equation for the total amount  $S$  that he has saved after  $w$  weeks.
  - Graph the equation.
  - Find out how much Rondell will have saved after 8 weeks.
16. **TRAVEL** Ana is driving from her home in Miami, Florida, to her grandmother's house in New York City. On the first day, she will travel 240 miles to Orlando, Florida, to pick up her cousin to travel with her. Then they will travel 350 miles each day.
- Write an equation for the total number of miles  $m$  that they have traveled after  $d$  days.
  - Graph the equation.
  - It is 1293 miles from Miami to New York City. How long will it take Ana and her cousin to reach New York City?

**Practice and Problem Solving**

= **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

**Example 1**  
p. 214

Write an equation of a line in slope-intercept form with the given slope and  $y$ -intercept. Then graph the equation.

17. slope: 5,  $y$ -intercept: 8  
19. slope:  $-4$ ,  $y$ -intercept: 6  
21. slope: 3,  $y$ -intercept:  $-4$

18. slope: 3,  $y$ -intercept: 10  
20. slope:  $-2$ ,  $y$ -intercept: 8  
22. slope: 4,  $y$ -intercept:  $-6$

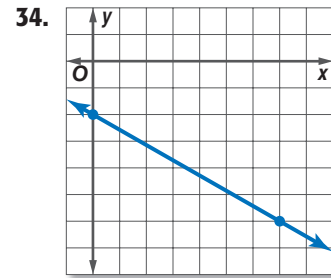
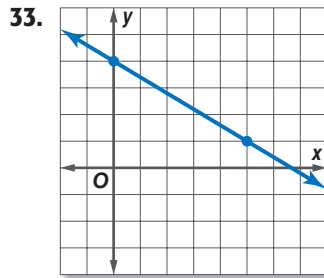
**Examples 2 and 3**  
p. 215

Graph each equation.

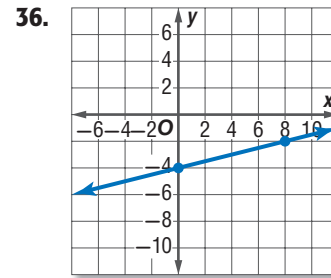
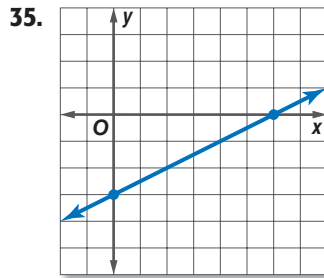
23.  $-3x + y = 6$   
25.  $-2x + y = -4$   
27.  $5x + 2y = 8$   
29.  $y = 7$   
31.  $21 = 7y$
24.  $-5x + y = 1$   
26.  $y = 8x - 7$   
28.  $4x + 9y = 27$   
30.  $y = -\frac{2}{3}$   
32.  $3y - 6 = 2x$

**Example 4**  
p. 216

Write an equation in slope-intercept form for each graph shown.



Write an equation in slope-intercept form for each graph shown.



**Example 5**  
p. 217

**37. MANATEES** In 1991, 1267 manatees inhabited Florida's waters. The manatee population has increased at a rate of 123 manatees per year.

- Write an equation for the manatee population,  $P$ ,  $t$  years since 1991.
- Graph this equation.
- In 2006, the manatee was removed from Florida's endangered species list. What was the manatee population in 2006?

Write an equation of a line in slope-intercept form with the given slope and  $y$ -intercept.

- |   |   |
|---|---|
| 38. slope: $\frac{1}{2}$ , $y$ -intercept: $-3$ | 39. slope: $\frac{2}{3}$ , $y$ -intercept: $-5$ |
| 40. slope: $-\frac{5}{6}$ , $y$ -intercept: $5$ | 41. slope: $-\frac{3}{7}$ , $y$ -intercept: $2$ |
| 42. slope: $1$ , $y$ -intercept: $4$            | 43. slope: $0$ , $y$ -intercept: $5$            |

Graph each equation.

- |                            |                               |                             |
|----------------------------|-------------------------------|-----------------------------|
| 44. $y = \frac{3}{4}x - 2$ | 45. $y = \frac{5}{3}x + 4$    | 46. $3x + 8y = 32$          |
| 47. $5x - 6y = 36$         | 48. $-4x + \frac{1}{2}y = -1$ | 49. $3x - \frac{1}{4}y = 2$ |

**50. TRAVEL** A rental company charges \$8 per hour for a mountain bike plus a \$5 fee for a helmet.

- Write an equation in slope-intercept form for the total rental cost  $C$  for a helmet and a bicycle for  $t$  hours.
- Graph the equation.
- What would the cost be for two helmets and 2 bicycles for 8 hours?



**Real-World Link**

More than 3 million teens participate in mountain biking each year.

Source: *Statistical Abstract of the United States*

**51. COLLEGE TUITION** For Illinois residents, the average tuition at Chicago State University is \$157 per credit hour. Fees cost \$218 per year.

- Write an equation in slope-intercept form for the tuition  $T$  for  $c$  credit hours.
- Find the cost for a student who is taking 32 credit hours.

Write an equation of a line in slope-intercept form with the given slope and  $y$ -intercept.

52. slope:  $-1$ ,  $y$ -intercept:  $0$                       53. slope:  $0.5$ ,  $y$ -intercept:  $7.5$   
54. slope:  $0$ ,  $y$ -intercept:  $7$                       55. slope:  $-1.5$ ,  $y$ -intercept:  $-0.25$   
56. Write an equation of a horizontal line that crosses the  $y$ -axis at  $(0, -5)$ .  
57. Write an equation of a line that passes through the origin with a slope of  $3$ .  
58. **TEMPERATURE** The temperature dropped rapidly overnight. Starting at  $80^\circ\text{F}$ , the temperature dropped  $3^\circ$  per minute.

- a. Draw a graph that represents this constant temperature drop from  $0$  to  $8$  minutes.  
b. Write an equation that describes this situation. Describe the meaning of each variable as well as the slope and  $y$ -intercept.

59. **FITNESS** Refer to the information at the right.

- a. Write an equation that represents the cost  $C$  of a membership for  $m$  months.  
b. What does the slope represent in this situation?  
c. What does the  $y$ -intercept represent in this situation?  
d. What is the cost of a two-year membership?

**GET FIT GYM**  
startup fee \$145



\$45 monthly fee

60. **MAGAZINES** A teen magazine began with a circulation of  $500,000$  in its first year. Since then, each year the circulation has increased an average of  $33,388$  per year.

- a. Write an equation that represents the circulation  $c$  after  $y$  years.  
b. What does the slope represent in this situation?  
c. What does the  $y$ -intercept represent in this situation?  
d. If the magazine began in  $1944$ , and this trend continues, in what year will the circulation reach  $3,000,000$ ?

61. **CELL PHONES** In  $2007$ ,  $3.25$  billion customers worldwide used a cell phone, and  $1000$  new customers signed up for a cell phone every minute.

- a. Write an equation for the number of cell phone customers in billions,  $C$ ,  $t$  years since  $2007$ . Graph this equation.  
b. How many cell phone customers will there be in  $2011$ ?  
c. During what year will the number of customers reach  $10$  billion?



### Real-World Link

*Seventeen* magazine debuted in  $1944$ , during World War II. Today, the magazine has a circulation of over  $2,000,000$ .

Source: *Chicago-Sun Times*

## H.O.T. Problems

Use Higher-Order Thinking Skills

62. **OPEN ENDED** Draw a graph representing a real-world linear function and write an equation for the graph. Describe what the graph represents.  
63. **REASONING** Determine whether the equation of a vertical line can be written in slope-intercept form. Explain your reasoning.  
64. **CHALLENGE** Summarize the characteristics that the graphs  $y = 2x + 3$ ,  $y = 4x + 3$ ,  $y = -x + 3$ , and  $y = -10x + 3$  have in common.  
65. **WRITING IN MATH** If given an equation in standard form, explain how to determine the rate of change.  
66. **WRITING IN MATH** Explain how you would use a given  $y$ -intercept and the slope to make predictions about what the  $y$ -value will be for a given  $x$ -value without graphing.



**67.** A music store has  $x$  CDs in stock. If 350 are sold and  $3y$  are added to stock, which expression represents the number of CDs in stock?

- A  $350 + 3y - x$       C  $x + 350 + 3y$   
 B  $x - 350 + 3y$       D  $3y - 350 - x$

**68. PROBABILITY** The table shows the result of a survey of students' favorite extracurricular activity. What is the probability that a student's favorite activity is sports or drama club?

Extracurricular Activity	Students
art club	24
band	134
choir	37
drama club	46
mock trial	19
school paper	26
sports	314

- F  $\frac{3}{8}$       G  $\frac{4}{9}$       H  $\frac{3}{5}$       J  $\frac{2}{3}$

**69.** A recipe for fruit punch calls for 2 ounces of orange juice for every 8 ounces of lemonade. If Jennifer uses 64 ounces of lemonade, which proportion can she use to find  $x$ , the number of ounces of orange juice she should add to make the fruit punch?

- A  $\frac{2}{x} = \frac{64}{6}$       C  $\frac{2}{8} = \frac{x}{64}$   
 B  $\frac{8}{x} = \frac{64}{2}$       D  $\frac{6}{2} = \frac{x}{64}$

**70. EXTENDED RESPONSE** The table shows the results of a canned food drive 1225 cans were collected, and the 12<sup>th</sup>-grade class collected 55 more cans than the 10<sup>th</sup>-grade class. How many cans each did the 10<sup>th</sup>- and 12<sup>th</sup>-grade classes collect? Show your work.

Grade	Cans
9	340
10	$x$
11	280
12	$y$

### Spiral Review

Find the next three terms in the sequence. (Lesson 3-6)

- 71.** 1, 3, 7, 13, 21, ...      **72.** 8, 6, 9, 7, 10, 8, 11, ...      **73.** 14, 16, 11, 13, 8, ...      **74.** 1, 4, 9, 16, ...

**75. GAME SHOWS** Contestants on a game show win money by answering 10 questions. The value of each question increases by \$2500. (Lesson 3-5)

- a. If the value of the first question is \$3000, find the value of the 10th question.  
 b. If the contestant answers all 10 questions correctly, how much will he or she win?

Suppose  $y$  varies directly as  $x$ . Write a direct variation equation that relates  $x$  and  $y$ . Then solve. (Lesson 3-4)

- 76.** If  $y = 10$  when  $x = 5$ , find  $y$  when  $x = 6$ .  
**77.** If  $y = -16$  when  $x = 4$ , find  $x$  when  $y = 20$ .  
**78.** If  $y = 6$  when  $x = 18$ , find  $y$  when  $x = -12$ .  
**79.** If  $y = 12$  when  $x = 15$ , find  $x$  when  $y = -6$ .

### Skills Review

Find the slope of the line that passes through each pair of points. (Lesson 3-3)

- 80.** (2, 3), (9, 7)      **81.** (-3, 6), (2, 4)      **82.** (2, 6), (-1, 3)      **83.** (-3, 3), (1, 3)

# Graphing Technology Lab

## The Family of Linear Graphs

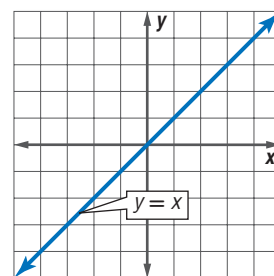


**HS-AT-S-EI11** Students will relate the coefficients of a linear equation and the slope and  $x$ - and  $y$ -intercepts of its graph. Also addresses HS-DAP-S-DR6.

A family of people is related by birth, marriage, or adoption. Often people in families share characteristics. The graphs in a family share at least one characteristic. Graphs in the linear family are all lines, with the simplest graph in the family being that of the parent function  $y = x$ .

You can use a graphing calculator to investigate how changing the parameters  $m$  and  $b$  in  $y = mx + b$  affects the graphs in the family of linear functions.

Parent Graph



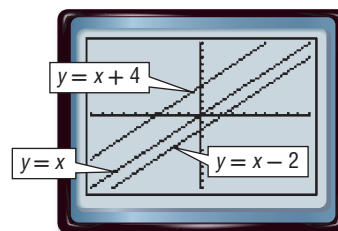
### ACTIVITY 1 Changing $b$ in $y = mx + b$

Graph  $y = x$ ,  $y = x + 4$ , and  $y = x - 2$  in the standard viewing window.

Enter the equations in the  $Y=$  list as  $Y_1$ ,  $Y_2$ , and  $Y_3$ . Then graph the equations.

**KEYSTROKES:** Review graphing on pages 167 and 168.

- How do the slopes of the graphs compare?
- Compare the graph of  $y = x + 4$  and the graph of  $y = x$ . How would you obtain the graph of  $y = x + 4$  from the graph of  $y = x$ ?
- How would you obtain the graph of  $y = x - 2$  from the graph of  $y = x$ ?



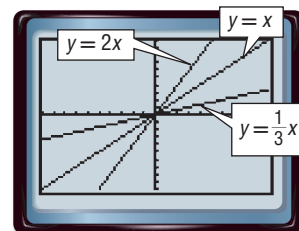
Changing  $m$  in  $y = mx + b$  affects the graphs in a different way than changing  $b$ . First, investigate positive values of  $m$ .

### ACTIVITY 2 Changing $m$ in $y = mx + b$ , Positive Values

Graph  $y = x$ ,  $y = 2x$ , and  $y = \frac{1}{3}x$  in the standard viewing window.

Enter the equations in the  $Y=$  list and graph.

- How do the  $y$ -intercepts of the graphs compare?
- Compare the graph of  $y = 2x$  and the graph of  $y = x$ .
- Which is steeper, the graph of  $y = \frac{1}{3}x$  or the graph of  $y = x$ ?



Does changing  $m$  to a negative value affect the graph differently than changing it to a positive value?

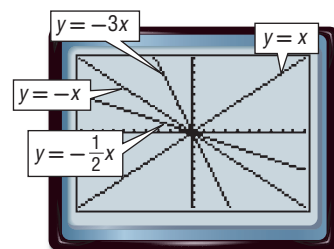
### ACTIVITY 3 Changing $m$ in $y = mx + b$ , Negative Values

Graph  $y = x$ ,  $y = -x$ ,  $y = -3x$ , and  $y = -\frac{1}{2}x$  in the standard viewing window.

Enter the equations in the Y= list and graph.

**3A.** How are the graphs with negative values of  $m$  different than graphs with a positive  $m$ ?

**3B.** Compare the graphs of  $y = -x$ ,  $y = -3x$ , and  $y = -\frac{1}{2}x$ . Which is steepest?



### Analyze the Results

Graph each set of equations on the same screen. Describe the similarities or differences among the graphs.

- $y = 2x$   
 $y = 2x + 3$   
 $y = 2x - 7$
- $y = x + 1$   
 $y = 2x + 1$   
 $y = \frac{1}{4}x + 1$
- $y = x + 4$   
 $y = 2x + 4$   
 $y = \frac{3}{4}x + 4$
- $y = 0.5x + 2$   
 $y = 0.5x - 5$   
 $y = 0.5x + 4$
- $y = -2x - 2$   
 $y = -4.2x - 2$   
 $y = -\frac{1}{3}x - 2$
- $y = 3x$   
 $y = 3x + 6$   
 $y = 3x - 7$
- Families of graphs have common characteristics. What do the graphs of all equations of the form  $y = mx + b$  have in common?
- How does the value of  $b$  affect the graph of  $y = mx + b$ ?
- What is the result of changing the value of  $m$  on the graph of  $y = mx + b$  if  $m$  is positive?
- How can you determine which graph is steepest by examining the following equations?  
 $y = 3x$ ,  $y = -4x - 7$ ,  $y = \frac{1}{2}x + 4$
- Explain how knowing about the effects of  $m$  and  $b$  can help you sketch the graph of an equation.
- The equation  $y = k$  can also be a parent graph. Graph  $y = 5$ ,  $y = 2$ , and  $y = -4$  on the same screen. Describe the similarities or differences among the graphs.

### Extension

Nonlinear functions can also be defined in terms of a family of graphs. Graph each set of equations on the same screen. Describe the similarities or differences among the graphs.

- $y = x^2$   
 $y = -3x^2$   
 $y = (-3x)^2$
- $y = x^2$   
 $y = x^2 + 3$   
 $y = (x - 2)^2$
- $y = x^2$   
 $y = 2x^2 + 4$   
 $y = (3x)^2 - 5$
- Describe the similarities and differences in the classes of functions  $f(x) = x^2 + c$  and  $f(x) = (x + c)^2$ , where  $c$  is any real number.

# Writing Equations in Slope-Intercept Form

## Then

You graphed lines given the slope and the  $y$ -intercept. (Lesson 4-1)

## Now

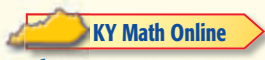
- Write an equation of a line in slope-intercept form given the slope and one point.
- Write an equation of a line in slope-intercept form given two points.



**HS-AT-S-EI11** Students will relate the coefficients of a linear equation and the slope and  $x$ - and  $y$ -intercepts of its graph.  
**HS-AT-S-EI17** Students will write and solve linear sentences, describing real-world situations by using and relating formulas, tables, graphs and equations. Also addresses HS-AT-S-VEO2.

## New Vocabulary

linear extrapolation



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- Personal Tutor
- Self-Check Quiz
- Homework Help

## Why?

Americans are taking more vacations. In 2000, there were 337.1 million vacations taken. In 2004, there were 375.4 million vacations taken. You can find the average rate of change for these data. Then you can write an equation that would model the average number of vacations taken per year.



**Write an Equation Given the Slope and a Point** The next example shows how to write an equation of a line if you are given a slope and a point other than the  $y$ -intercept.

### EXAMPLE 1 Write an Equation Given the Slope and a Point

Write an equation of the line that passes through  $(2, 1)$  with a slope of 3.

You are given the slope but not the  $y$ -intercept.

**Step 1** Find the  $y$ -intercept.

$$y = mx + b \quad \text{Slope-intercept form}$$

$$1 = 3(2) + b \quad \text{Replace } m \text{ with } 3, y \text{ with } 1, \text{ and } x \text{ with } 2.$$

$$1 = 6 + b \quad \text{Simplify.}$$

$$1 - 6 = 6 + b - 6 \quad \text{Subtract } 6 \text{ from each side.}$$

$$-5 = b \quad \text{Simplify.}$$

**Step 2** Write the equation in slope-intercept form.

$$y = mx + b \quad \text{Slope-intercept form}$$

$$y = 3x - 5 \quad \text{Replace } m \text{ with } 3 \text{ and } b \text{ with } -5.$$

Therefore, the equation of the line is  $y = 3x - 5$ .

### Check Your Progress

Write an equation of a line that passes through the given point and has the given slope.

1A.  $(-2, 5)$ , slope 3

1B.  $(4, -7)$ , slope  $-1$

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**Write an Equation Given Two Points** If you are given two points through which a line passes, you can use them to find the slope first. Then follow the steps in Example 1 to write the equation.

**EXAMPLE 2 Write an Equation Given Two Points**

Write an equation of the line that passes through each pair of points.

**a.** (3, 1) and (2, 4)**Step 1** Find the slope of the line containing the given points.

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope Formula} \\
 &= \frac{4 - 1}{2 - 3} && (x_1, y_1) = (3, 1) \text{ and } (x_2, y_2) = (2, 4) \\
 &= \frac{3}{-1} \text{ or } -3 && \text{Simplify.}
 \end{aligned}$$

**Step 2** Use the slope and either of the two points to find the  $y$ -intercept.

$$\begin{aligned}
 y &= mx + b && \text{Slope-intercept form} \\
 4 &= (-3)(2) + b && \text{Replace } m \text{ with } -3, x \text{ with } 2, \text{ and } y \text{ with } 4. \\
 4 &= -6 + b && \text{Simplify.} \\
 4 - (-6) &= -6 + b - (-6) && \text{Subtract } -6 \text{ from each side.} \\
 10 &= b && \text{Simplify.}
 \end{aligned}$$

**Step 3** Write the equation in slope-intercept form.

$$\begin{aligned}
 y &= mx + b && \text{Slope-intercept form} \\
 y &= -3x + 10 && \text{Replace } m \text{ with } -3 \text{ and } b \text{ with } 10.
 \end{aligned}$$

Therefore, the equation of the line is  $y = -3x + 10$ .**b.** (-4, -2) and (-5, -6)**Step 1** Find the slope of the line containing the given points.

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope Formula} \\
 &= \frac{-6 - (-2)}{-5 - (-4)} && (x_1, y_1) = (-4, -2) \text{ and } (x_2, y_2) = (-5, -6) \\
 &= \frac{-4}{-1} \text{ or } 4 && \text{Simplify.}
 \end{aligned}$$

**Step 2** Use the slope and either of the two points to find the  $y$ -intercept.

$$\begin{aligned}
 y &= mx + b && \text{Slope-intercept form} \\
 -2 &= 4(-4) + b && \text{Replace } m \text{ with } 4, x \text{ with } -4, \text{ and } y \text{ with } -2. \\
 -2 &= -16 + b && \text{Simplify.} \\
 -2 - (-16) &= -16 + b - (-16) && \text{Subtract } -16 \text{ from each side.} \\
 14 &= b && \text{Simplify.}
 \end{aligned}$$

**Step 3** Write the equation in slope-intercept form.

$$\begin{aligned}
 y &= mx + b && \text{Slope-intercept form} \\
 y &= 4x + 14 && \text{Replace } m \text{ with } 4 \text{ and } b \text{ with } 14.
 \end{aligned}$$

Therefore, the equation of the line is  $y = 4x + 14$ .**StudyTip****Choosing a point**

Given two points on a line, you may select either point to be  $(x_1, y_1)$ . Be sure to remain consistent throughout the problem.

**StudyTip**

**Slope** If the  $(x_1, y_1)$  coordinates are negative, be sure to account for both the negative signs and the subtraction symbols in the Slope Formula.

**Check Your Progress**

Write an equation of the line that passes through each pair of points.

**2A.** (-1, 12), (4, -8)**2B.** (5, -8), (-7, 0)





### Real-World Career

#### Baggage Handler

Airline ground crew responsibilities include checking tickets, helping passengers with luggage, and making sure that baggage is secure in the baggage compartment of the plane. This job usually requires a high school diploma or GED.

Source: Airline Jobs

## Real-World EXAMPLE 3 Use Slope-Intercept Form

**AIR FARES** The table shows the average fares for domestic flights in certain years. Write an equation that could be used to predict air fares if fares continue to increase at this rate.

Year	Cost (\$)
2004	354
2005	366
2006	378
2007	390

**Understand** You know the air fares for the years listed.

**Plan** Let  $x$  represent the number of years since 2000, and let  $y$  represent the air fare. Write an equation of the line that passes through  $(5, 366)$  and  $(6, 378)$ .

**Solve** Find the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{378 - 366}{6 - 5} \\ &= \frac{12}{1} \text{ or } 12 \end{aligned}$$

**Slope Formula**

Let  $(x_1, y_1) = (5, 366)$  and  $(x_2, y_2) = (6, 378)$ .

**Simplify.**

Choose  $(6, 378)$  and find the  $y$ -intercept of the line.

$$\begin{aligned} y &= mx + b \\ 378 &= 12(6) + b \\ 378 &= 72 + b \\ 306 &= b \end{aligned}$$

**Slope-intercept form**

Replace  $m$  with 12,  $x$  with 6, and  $y$  with 378.

**Simplify.**

**Subtract 72 from each side.**

Write the equation using  $m = 12$  and  $b = 306$ .

$$\begin{aligned} y &= mx + b \\ y &= 12x + 306 \end{aligned}$$

**Slope-intercept form**

Replace  $m$  with 12 and  $b$  with 306.

**Check** Check your result by using the coordinates of the other point.

$$\begin{aligned} y &= 12x + 306 \\ 366 &\stackrel{?}{=} 12(5) + 306 \\ 366 &= 366 \checkmark \end{aligned}$$

**Original equation**

Replace  $x$  with 5 and  $y$  with 366.

**Simplify.**

### Check Your Progress

3. **MONEY** In addition to his weekly salary, Ethan is paid \$16 per delivery. Last week, he made 5 deliveries, and his total pay was \$215. Write a linear equation to find Ethan's total weekly pay  $T$  if he makes  $d$  deliveries.

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You can use a linear equation to make predictions about values that are beyond the range of the data. This process is called **linear extrapolation**.

## Real-World EXAMPLE 4 Predict from Slope-Intercept Form

**AIR FARES** Use the equation from Example 3 to estimate the cost of airfares in 2010.

$$\begin{aligned} y &= 12x + 306 \\ &= 12(10) + 306 \\ &= 426 \end{aligned}$$

**Original equation**

Replace  $x$  with 10.

An estimate of the average air fares is \$426.

### Check Your Progress

4. **MONEY** Use the equation in Check Your Progress 3 to predict how much money Ethan will earn in a week if he makes 8 deliveries.

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## Check Your Understanding

### Example 1 p. 224

Write an equation of the line that passes through the given point and has the given slope.

- $(3, -3)$ , slope 3
- $(2, 4)$ , slope 2
- $(1, 5)$ , slope  $-1$
- $(-4, 6)$ , slope  $-2$

### Example 2 p. 225

Write an equation of the line that passes through each pair of points.

- $(4, -3)$ ,  $(2, 3)$
- $(-7, -3)$ ,  $(-3, 5)$
- $(-1, 3)$ ,  $(0, 8)$
- $(-2, 6)$ ,  $(0, 0)$

### Examples 3 and 4 p. 226

9. **WHITewater RAFTING** Ten people from a local youth group went to Black Hills Whitewater Rafting Tour Company for a one-day rafting trip. The group paid \$425.
- Write an equation in slope-intercept form to find the total cost  $C$  for  $p$  people.
  - How much would it cost for 15 people?



## Practice and Problem Solving



= **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

### Example 1 p. 224

Write an equation of the line that passes through the given point and has the given slope.

- $(3, 1)$ , slope 2
- $(-1, 4)$ , slope  $-1$
- $(1, 0)$ , slope 1
- $(7, 1)$ , slope 8
- $(2, 5)$ , slope  $-2$
- $(2, 6)$ , slope 2

### Example 2 p. 225

Write an equation of the line that passes through each pair of points.

- $(9, -2)$ ,  $(4, 3)$
- $(-2, 5)$ ,  $(5, -2)$
- $(-5, 3)$ ,  $(0, -7)$
- $(3, 5)$ ,  $(2, -2)$
- $(-1, -3)$ ,  $(-2, 3)$
- $(-2, -4)$ ,  $(2, 4)$

### Examples 3 and 4 p. 226

22. **RC CAR** Greg is driving a remote control car at a constant speed. He starts the timer when the car is 5 feet away. After 2 seconds the car is 35 feet away.

- Write a linear equation to find the distance  $d$  of the car from Greg.
- Estimate the distance the car has traveled after 10 seconds.

23. **TRAVEL** Refer to the beginning of the lesson.

- Write a linear equation to find the number of vacations (in millions)  $y$  after  $x$  years. Let  $x$  be the number of years since 2000.
- Estimate the number of vacations that will be taken in 2012.

24. **BOOKS** In 1904, a dictionary cost 30¢. Since then the cost of a dictionary has risen an average of 6¢ per year.

- Write a linear equation to find the cost  $C$  of a dictionary after  $y$  years.
- If this trend continues, what will the cost of a dictionary be in 2020?

Write an equation of the line that passes through the given point and has the given slope.

- $(4, 2)$ , slope  $\frac{1}{2}$
- $(3, -2)$ , slope  $\frac{1}{3}$
- $(6, 4)$ , slope  $-\frac{3}{4}$
- $(2, -3)$ , slope  $\frac{2}{3}$
- $(2, -2)$ , slope  $\frac{2}{7}$
- $(-4, -2)$ , slope  $-\frac{3}{5}$

### Problem-Solving Tip

#### Determine Reasonable Answers

Deciding whether an answer is reasonable is useful when an exact answer is not necessary.



### Real-World Link

There are approximately 73 million dogs kept as pets in the United States. Thirty-nine percent of households in the United States own at least one dog.

Source: The Humane Society of the United States

31. **DOGS** In 2001, there were about 56.1 thousand golden retrievers registered in the United States. In 2002, the number was 62.5 thousand.
- Write a linear equation to find the number of golden retrievers  $G$  that will be registered in year  $t$ , where  $t = 0$  is the year 2000.
  - Graph the equation.
  - Estimate the number of golden retrievers that will be registered in 2012.
32. **GYM MEMBERSHIPS** A local recreation center offers a yearly membership for \$265. The center offers aerobics classes for an additional \$5 per class.
- Write an equation that represents the total cost of the membership.
  - Carly spent \$500 at the recreation center in one year. For how many aerobics classes did she pay?
33. **SUBSCRIPTION** A magazine offers an online subscription that allows you to view 25 archived articles. For more than 25, there is a fee for each article. To view 30 archived articles, you pay \$49.15. To view 33 archived articles, you pay \$57.40.
- What is the cost of each archived article?
  - What is the cost of the magazine subscription?

Write an equation of the line that passes through the given points.

34.  $(5, -2), (7, 1)$     35.  $(5, -3), (2, 5)$     36.  $(\frac{5}{4}, 1), (-\frac{1}{4}, \frac{3}{4})$     37.  $(\frac{5}{12}, -1), (-\frac{3}{4}, \frac{1}{6})$

Determine whether the given point is on the line. Explain why or why not.

38.  $(3, -1); y = \frac{1}{3}x + 5$     39.  $(6, -2); y = \frac{1}{2}x - 5$

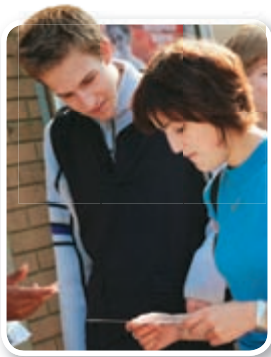
For Exercises 40–42, determine which equation best represents each situation. Explain the meaning of each variable.

**A**  $y = -\frac{1}{3}x + 72$

**B**  $y = 2x + 225$

**C**  $y = 8x + 4$

40. **CONCERTS** Tickets to a concert cost \$8 each plus a processing fee of \$4 per order.
41. **FUNDRAISING** The freshman class has \$225 in their treasury. They sell raffle tickets at \$2 each to raise money for a field trip.
42. **POOLS** At the beginning of summer, the water level of a swimming pool in Tucson, Arizona, is 6 feet. The rate of evaporation in Tucson is  $\frac{1}{3}$  inch per day.
43. **ENVIRONMENT** A manufacturer implemented a program to reduce the volume of waste sent to landfills. In 1998 they sent 946 tons of waste to landfills. Each year after that, they reduced their waste by an average 28.4 tons.
- How many tons were sent to the landfill in 2010?
  - In what year will it become impossible for this trend to continue? Explain.
44. **MULTIPLE REPRESENTATIONS** In this problem, you will explore the slopes of perpendicular lines.
- GRAPHICAL** On a coordinate plane, graph  $y = \frac{3}{4}x + 1$ .
  - PICTORIAL** Use a straightedge and a protractor to draw a line that is perpendicular to the line you graphed.
  - ALGEBRAIC** Find the equation of the line that is perpendicular to the original line. Describe which method you used to write the equation.
  - ANALYTICAL** Compare the slopes of each line. Describe the relationship, if any, between the two values.



### Real-World Link

55% of people who buy tickets for events use online ticket agents.

Source: Pew Internet & American Life Project

**45. CONCERT TICKETS** Jackson is ordering tickets for a concert through an online ticket service. There is a processing fee for each order, and the tickets are \$52 each. Jackson ordered 5 tickets and the cost was \$275.

- Determine the amount of the processing fee. Write a linear equation to represent the total cost  $C$  for  $t$  tickets.
- Make a table of values for at least three other numbers of tickets.
- Graph this equation. Predict how much it would cost to order 8 tickets.

**46. MUSIC** A music store is offering a Frequent Buyers Club membership to its customers. The membership requires a \$22 annual fee, and then a member can buy CDs at a reduced price. After a purchase of 17 CDs in one year, the cost is \$111.25.

- Determine the cost of each CD for a member.
- Write a linear equation to represent the total cost  $y$  of a one year membership, if  $x$  CDs are purchased.
- Graph this equation.

### H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

**47. FIND THE ERROR** Tess and Jacinta are writing an equation of the line through  $(3, -2)$  and  $(6, 4)$ . Is either of them correct? Explain your reasoning.

*Tess*

$$m = \frac{4 - (-2)}{6 - 3} = \frac{6}{3} \text{ or } 2$$

$$y = mx + b$$

$$6 = 2(4) + b$$

$$6 = 8 + b$$

$$-2 = b$$

$$y = 2x - 2$$

*Jacinta*

$$m = \frac{4 - (-2)}{6 - 3} = \frac{6}{3} \text{ or } 2$$

$$y = mx + b$$

$$-2 = 2(3) + b$$

$$-2 = 6 + b$$

$$-8 = b$$

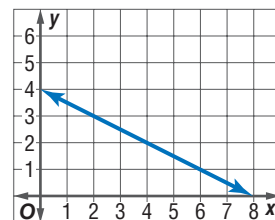
$$y = 2x - 8$$

**48. CHALLENGE** Consider three points,  $(3, 7)$ ,  $(-6, 1)$  and  $(9, p)$ , on the same line. Find the value of  $p$  and explain your steps.

**49. REASONING** Consider the standard form of a linear equation,  $Ax + By = C$ .

- Rewrite the standard form of a linear equation into slope-intercept form.
- What is the slope?
- What is the  $y$ -intercept?
- Is this true for any real value of  $A$ ,  $B$ , and  $C$ ?

**50. OPEN ENDED** Create a real-world situation that fits the graph at the right. Define the two quantities and describe the functional relationship between them. Write an equation to represent this relationship and describe what the slope and  $y$ -intercept mean.

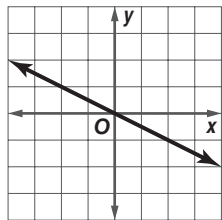


**51. WRITING IN MATH** Linear equations are useful in predicting future events. Describe some factors in real-world situations that might affect the reliability of the graph in making any predictions.

**52. WRITING IN MATH** What information is needed to write the equation of a line? Explain.

53. Which equation *best* represents the graph?

- A  $y = 2x$
- B  $y = -2x$
- C  $y = \frac{1}{2}x$
- D  $y = -\frac{1}{2}x$

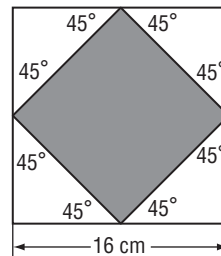


54. Roberto receives an employee discount of 12%. If he spends \$355 at the store, how much is his discount to the nearest dollar?

- F \$3
- G \$4
- H \$30
- J \$43

55. **GEOMETRY** The midpoints of the sides of the square below are joined to form a smaller square. What is the area of the smaller square?

- A  $64 \text{ cm}^2$
- B  $128 \text{ cm}^2$
- C  $248 \text{ cm}^2$
- D  $256 \text{ cm}^2$



56. **SHORT RESPONSE** If  $\frac{5(x+4)}{2} + 7 = 37$ , what is the value of  $3x - 9$ ?

### Spiral Review

Graph each equation. (Lesson 4-1)

57.  $y = 3x + 2$

58.  $y = -4x + 2$

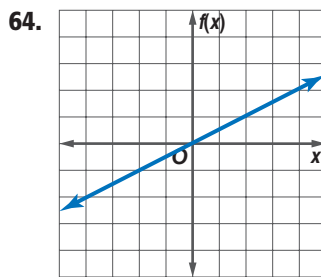
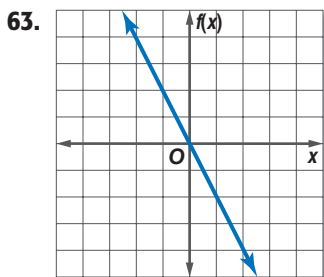
59.  $3y = 2x + 6$

60.  $y = \frac{1}{2}x + 6$

61.  $3x + y = -1$

62.  $2x + 3y = 6$

Write an equation in function notation for each relation. (Lesson 3-6)



65. **METEOROLOGY** The distance  $d$  in miles that the sound of thunder travels in  $t$  seconds is given by the equation  $d = 0.21t$ . (Lesson 3-1)

- a. Graph the equation.
- b. Use the graph to estimate how long it will take you to hear thunder from a storm 3 miles away.

Solve each equation. Check your solution. (Lesson 2-3)

66.  $-5t - 2.2 = 2.9$

67.  $-5.5a - 43.9 = 77.1$

68.  $4.2r + 7.14 = 12.6$

69.  $-14 - \frac{n}{9} = 9$

70.  $\frac{-8b - (-9)}{-10} = 17$

71.  $9.5x + 11 - 7.5x = 14$

### Skills Review

Find the value of  $r$  so the line through each pair of points has the given slope. (Lesson 3-3)

72.  $(6, -2), (r, -6), m = 4$

73.  $(8, 10), (r, 4), m = 6$

74.  $(7, -10), (r, 4), m = -3$

75.  $(6, 2), (9, r), m = -1$

76.  $(9, r), (6, 3), m = -\frac{1}{3}$

77.  $(5, r), (2, -3), m = \frac{4}{3}$

## Writing Equations in Point-Slope Form

### Then

You wrote linear equations given either one point and the slope or two points.  
(Lesson 4 2)

### Now

- Write equations of lines in point-slope form.
- Write linear equations in different forms.



**HS-AT-S-EI11** Students will relate the coefficients of a linear equation and the slope and  $x$ - and  $y$ -intercepts of its graph.  
**HS-AT-S-EI17** Students will write and solve linear sentences, describing real-world situations by using and relating formulas, tables, graphs and equations.

### New Vocabulary

point-slope form



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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

### Why?

Most humane societies have foster homes for newborn puppies, kittens, and injured or ill animals. During the spring and summer, a large shelter can place 3000 animals in homes each month.

If a shelter had 200 animals in foster homes at the beginning of spring, the number of animals in foster homes at the end of the summer could be represented by the equation  $y = 3000x + 200$ , where  $x$  is the number of months and  $y$  is the number of animals.



**Point-Slope Form** An equation of a line can be written in **point-slope form** when given the coordinates of one known point on a line and the slope of that line.



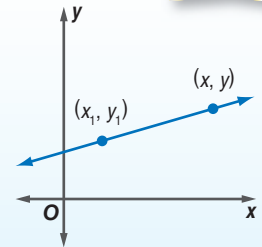
### Key Concept

### Point-Slope Form

For Your FOLDABLE

**Words** The linear equation  $y - y_1 = m(x - x_1)$  is written in point-slope form, where  $(x_1, y_1)$  is a given point on a non-vertical line and  $m$  is the slope of the line.

**Symbols**  $y - y_1 = m(x - x_1)$



### EXAMPLE 1 Write and Graph an Equation in Point-Slope Form

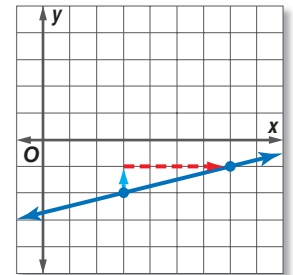
Write an equation in point-slope form for the line that passes through  $(3, -2)$  with a slope of  $\frac{1}{4}$ . Then graph the equation.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

$$y - (-2) = \frac{1}{4}(x - 3) \quad (x_1, y_1) = (3, -2), m = \frac{1}{4}$$

$$y + 2 = \frac{1}{4}(x - 3) \quad \text{Simplify.}$$

Plot the point at  $(3, -2)$  and use the slope to find another point on the line. Draw a line through the two points.



### Check Your Progress

- Write an equation in point-slope form for the line that passes through  $(-2, 1)$  with a slope of  $-6$ . Then graph the equation.

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**Forms of Linear Equations** If you are given the slope and the coordinates of one or two points, you can write the linear equation in the following ways.

### StudyTip

**Slope** The slope of the line remains unchanged throughout the line. You can go in either direction along the line using the same rise over run and you will always end at a point on the line.

### Concept Summary

### Writing Equations

For Your  
**FOLDABLE**

#### Given the Slope and One Point

- Step 1** Substitute the value of  $m$  and let the  $x$  and  $y$  coordinates be  $(x_1, y_1)$ . Or, substitute the values of  $m$ ,  $x$ , and  $y$  into the slope-intercept form and solve for  $b$ .
- Step 2** Rewrite the equation in the needed form.

#### Given Two Points

- Step 1** Find the slope.
- Step 2** Choose one of the two points to use.
- Step 3** Follow the steps for writing an equation given the slope and one point.

### Review Vocabulary

#### standard form of a linear equation

$Ax + By = C$ , where  $A \geq 0$ ,  $A$  and  $B$  are not both zero, and  $A$ ,  $B$ , and  $C$  are integers with a greatest common factor of 1 (Lesson 3-1)

### EXAMPLE 2 Writing an Equation in Standard Form

Write  $y - 1 = -\frac{2}{3}(x - 5)$  in standard form.

Remember, an equation in standard form looks like  $Ax + By = C$ .

$$y - 1 = -\frac{2}{3}(x - 5) \quad \text{Original equation}$$

$$3(y - 1) = 3\left(-\frac{2}{3}\right)(x - 5) \quad \text{Multiply each side by 3 to eliminate the fraction.}$$

$$3(y - 1) = -2(x - 5) \quad \text{Simplify.}$$

$$3y - 3 = -2x + 10 \quad \text{Distributive Property}$$

$$3y = -2x + 13 \quad \text{Add 3 to each side.}$$

$$2x + 3y = 13 \quad \text{Add 2x to each side.}$$

#### Check Your Progress

2. Write  $y - 1 = 7(x + 5)$  in standard form.

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To find the  $y$ -intercept of an equation, rewrite the equation in slope-intercept form.

### EXAMPLE 3 Writing an Equation in Slope-Intercept Form

Write  $y + 3 = \frac{3}{2}(x + 1)$  in slope-intercept form.

$$y + 3 = \frac{3}{2}(x + 1) \quad \text{Original equation}$$

$$y + 3 = \frac{3}{2}x + \frac{3}{2} \quad \text{Distributive Property}$$

$$y = \frac{3}{2}x - \frac{3}{2} \quad \text{Subtract 3 from each side.}$$

#### Check Your Progress

3. Write  $y + 6 = -3(x - 4)$  in slope-intercept form.

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Being able to use a variety of forms of linear equations can be useful in other subjects as well.

### StudyTip

Opposite sides of a square have equal slopes. This may be useful if the coordinates for one of the vertices is unavailable. You could use the slope of the opposite side.

### EXAMPLE 4 Point-Slope Form and Standard Form

**GEOMETRY** The figure shows square  $RSTU$ .

**a.** Write an equation in point-slope form for the line containing side  $\overline{TU}$ .

**Step 1** Find the slope of  $\overline{TU}$ .

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope Formula}$$

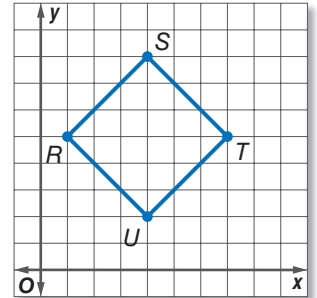
$$= \frac{5 - 2}{7 - 4} \text{ or } 1 \quad (x_1, y_1) = (4, 2) \text{ and } (x_2, y_2) = (7, 5)$$

**Step 2** You can select either point for  $(x_1, y_1)$  in the point-slope form.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

$$y - 2 = 1(x - 4) \quad (x_1, y_1) = (4, 2)$$

$$y - 5 = 1(x - 7) \quad (x_1, y_1) = (7, 5)$$



**b.** Write an equation in standard form for the same line.

$y - 2 = 1(x - 4)$	<b>Original equation</b>	$y - 5 = 1(x - 7)$
$y - 2 = 1x - 4$	<b>Distributive Property</b>	$y - 5 = 1x - 7$
$y = 1x - 2$	<b>Add to each side.</b>	$y = 1x - 2$
$-1x + y = -2$	<b>Subtract 1x from each side.</b>	$-1x + y = -2$
$x - y = 2$	<b>Multiply each side by <math>-1</math>.</b>	$x - y = 2$

### Check Your Progress

**4A.** Write an equation in point-slope form of the line containing side  $\overline{ST}$ .

**4B.** Write an equation in standard form of the line containing  $\overline{ST}$ .

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### Check Your Understanding

**Example 1**  
p. 231

Write an equation in point-slope form for the line that passes through the given point with the slope provided. Then graph the equation.

1.  $(-2, 5)$ , slope  $-6$       2.  $(-2, -8)$ , slope  $\frac{5}{6}$       3.  $(4, 3)$ , slope  $-\frac{1}{2}$

**Example 2**  
p. 232

Write each equation in standard form.

4.  $y + 2 = \frac{7}{8}(x - 3)$       5.  $y + 7 = -5(x + 3)$       6.  $y + 2 = \frac{5}{3}(x + 6)$

**Example 3**  
p. 232

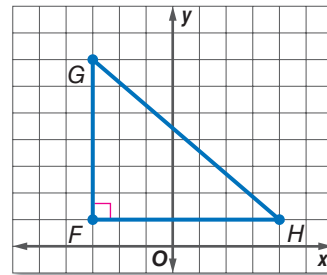
Write each equation in slope-intercept form.

7.  $y - 10 = 4(x + 6)$       8.  $y - 7 = -\frac{3}{4}(x + 5)$       9.  $y - 9 = x + 4$



**Example 4**  
p. 233

10. **GEOMETRY** Use right triangle  $FGH$ .
- Write an equation in point-slope form for the line containing  $\overline{GH}$ .
  - Write the standard form of the line containing  $\overline{GH}$ .



## Practice and Problem Solving

 = **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

**Example 1**  
p. 231

Write an equation in point-slope form for the line that passes through each point with the given slope. Then graph the equation.

- $(5, 3)$ ,  $m = 7$
- $(2, -1)$ ,  $m = -3$
- $(-6, -3)$ ,  $m = -1$
- $(-7, 6)$ ,  $m = 0$
- $(-2, 11)$ ,  $m = \frac{4}{3}$
- $(-7, -8)$ ,  $m = -\frac{3}{7}$
- Write an equation in point-slope form for a line that passes through  $(-2, -9)$  with a slope of  $-\frac{7}{5}$ .
- Write an equation in point-slope form for a horizontal line that passes through  $(-6, 0)$ .

**Example 2**  
p. 232

Write each equation in standard form.

- $y - 10 = 2(x - 8)$
- $y - 6 = -3(x + 2)$
- $y - 9 = -6(x + 9)$
- $y + 4 = \frac{2}{3}(x + 7)$
- $y + 7 = \frac{9}{10}(x + 3)$
- $y + 7 = -\frac{3}{2}(x + 1)$
- $2y + 3 = -\frac{1}{3}(x - 2)$
- $4y - 5x = 3(4x - 2y + 1)$

**Example 3**  
p. 232

Write each equation in slope-intercept form.

- $y - 6 = -2(x - 7)$
- $y - 11 = 3(x + 4)$
- $y + 5 = -6(x + 7)$
- $y - 1 = \frac{4}{5}(x + 5)$
- $y + 2 = \frac{1}{6}(x - 4)$
- $y + 6 = -\frac{3}{4}(x + 8)$
- $y + 3 = -\frac{1}{3}(2x + 6)$
- $y + 4 = 3(3x + 3)$

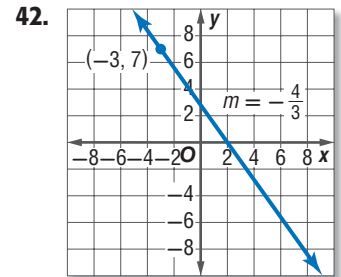
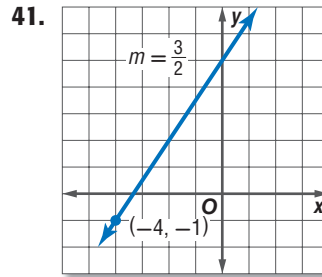
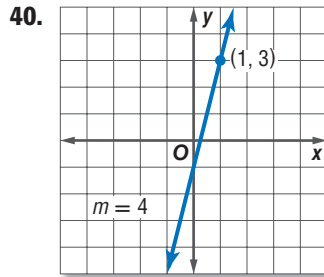
**Example 4**  
p. 233

- 35. MOVIE RENTALS** The number of copies of a movie rented at a video store decreased at a constant rate of 5 copies per week. The 6th week after the movie was released, 4 copies were rented. How many copies were rented during the second week?
- 36. TELEVISION** A cable company offers their premium service for \$39.95 per month plus a one-time setup fee. The total cost for setup and 6 months of service is \$264.70.
- Write an equation in point-slope form to find the total price  $y$  for any number of months  $x$ . (*Hint:* The point  $(6, 264.70)$  is a solution to the equation.)
  - Write the equation in slope-intercept form.
  - What is the setup fee?

Write each equation in standard form.

- $y + 8 = -\frac{11}{12}(x - 14)$
- $y - 3 = 2.5(x + 1)$
- $y + 2.1 = 1.4(x - 5)$

Write an equation in point-slope form for the line that passes through each point with the given slope.



Write each equation in slope-intercept form.

43.  $y + \frac{3}{5} = x - \frac{2}{5}$

44.  $y - \frac{7}{2} = \frac{1}{2}(x - 4)$

45.  $y + \frac{1}{3} = \frac{5}{6}(x + \frac{2}{5})$

46. Write an equation in point-slope form, slope-intercept form, and standard form for a line that passes through  $(-2, 8)$  with slope  $\frac{8}{5}$ .

47. Line  $\ell$  passes through  $(-9, 4)$  with slope  $\frac{4}{7}$ . Write an equation in point-slope form, slope-intercept form, and standard form for line  $\ell$ .

48. **WEATHER** Barometric pressure is a linear function of altitude. The barometric pressure is 598 millimeters of mercury (mmHg) at an altitude of 1.8 kilometers. The barometric pressure is 577 millimeters of mercury at an altitude of 2.1 kilometers.

- Write a formula for the barometric pressure as a function of the altitude.
- What is the altitude if the pressure is 657 millimeters of mercury?



**Real-World Link**

At higher altitudes, air is at a lower pressure and contains less oxygen. Prolonged exposure to low air pressure causes altitude sickness. Experienced mountain climbers take precautions to prevent altitude sickness.

Source: Altitude Physiology Expeditions Charity

**H.O.T. Problems**

Use **H**igher-**O**rders **T**hinking Skills

49. **WHICH ONE DOESN'T BELONG?** Identify the equation that does not belong with the other three. Explain your reasoning.

$y - 5 = 3(x - 1)$     $y + 1 = 3(x + 1)$     $y + 4 = 3(x + 1)$     $y - 8 = 3(x - 2)$

50. **FIND THE ERROR** Juana and Sabrina wrote an equation in point-slope form for the line that passes through  $(3, -7)$  and  $(-6, 4)$ . Is either of them correct? Explain your reasoning.

*Juana*  
 $y - 7 = -\frac{11}{9}(x + 3)$

*Sabrina*  
 $y - 4 = -\frac{9}{11}(x + 6)$

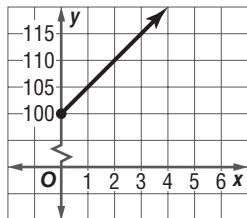
51. **OPEN ENDED** Describe a real-life scenario that has a constant rate of change and a value at a particular time of  $(x, y)$ . Represent this situation using an equation in slope-intercept form and an equation in point-slope form.

52. **REASONING** Write an equation for the line that passes through  $(-4, 8)$  and  $(3, -7)$ . What is the slope? Where does the line intersect the  $x$ -axis? the  $y$ -axis?

53. **CHALLENGE** Write an equation in point-slope form for the line that passes through the points  $(f, g)$  and  $(h, j)$ .

54. **WRITING IN MATH** Demonstrate how you can use the Slope Formula to write the point-slope form of an equation of a line.

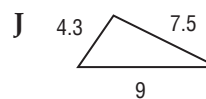
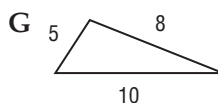
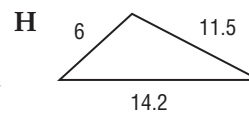
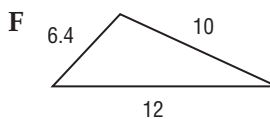
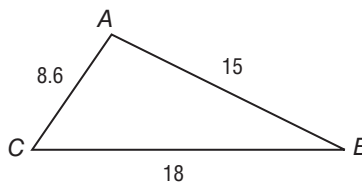
55. Which statement is *most* strongly supported by the graph?



- A You have \$100 and plan to spend \$5 each week.
- B You have \$100 and plan to save \$5 each week.
- C You need \$100 for a new CD player and plan to save \$5 each week.
- D You need \$100 for a new CD player and plan to spend \$5 each week.

56. **SHORT RESPONSE** A store offers a promotion in which customers can earn a \$5 gift certificate for every \$75 they spend. How much would a customer have to spend to earn \$35 worth of gift certificates?

57. **GEOMETRY** Which triangle is similar to  $\triangle ABC$ ?



58. In a class of 25 students, 6 have blue eyes, 15 have brown hair, and 3 have blue eyes and brown hair. How many students have neither blue eyes nor brown hair?
- A 4
  - B 7
  - C 10
  - D 22

### Spiral Review

Write an equation of the line that passes through each pair of points. (Lesson 4-2)

- 59. (4, 2), (-2, -4)
- 60. (3, -2), (6, 4)
- 61. (-1, 3), (2, -3)
- 62. (2, -2), (3, 2)
- 63. (7, -2), (-4, -2)
- 64. (0, 5), (-3, 5)

Write an equation in slope-intercept form of the line with the given slope and *y*-intercept. (Lesson 4-1)

- 65. slope: -2, *y*-intercept: 6
- 66. slope: 3, *y*-intercept: -5
- 67. slope:  $\frac{1}{2}$ , *y*-intercept: 3
- 68. slope:  $-\frac{3}{5}$ , *y*-intercept: 12
- 69. slope: 0, *y*-intercept: 3
- 70. slope: -1, *y*-intercept: 0

71. **THEATER** The Coral Gables Actors' Playhouse had 7 rows of seats in the orchestra section. The number of seats in each row forms an arithmetic sequence, as shown in the table. On opening night, 368 tickets were sold for the orchestra section. Was the section oversold? (Lesson 3-5)

Rows	Number of Seats
7	76
6	68
5	60

### Skills Review

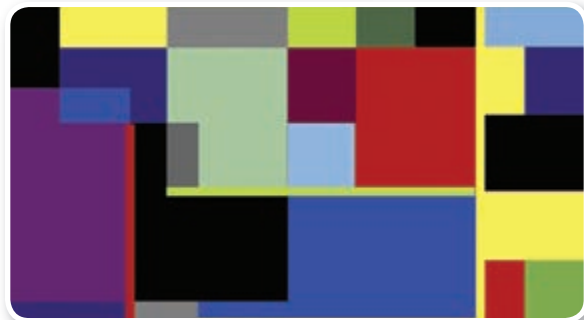
Solve each equation or formula for the variable specified. (Lesson 2-7)

- 72.  $y = mx + b$ , for *m*
- 73.  $v = r + at$ , for *a*
- 74.  $km + 5x = 6y$ , for *m*
- 75.  $4b - 5 = -t$ , for *b*

# Parallel and Perpendicular Lines

## Why?

Notice the squares, rectangles and lines in the piece of art shown at the right. Some of the lines intersect forming right angles. Other lines do not intersect at all.



## Then

You wrote equations in slope-intercept form.

(Lesson 4-2)

## Now

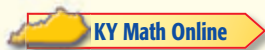
- Write an equation of the line that passes through a given point, parallel to a given line.
- Write an equation of the lines that passes through a given point, perpendicular to a given line.



**HS-G-S-CG1** Students will express the intuitive concept of the “slant” of a line as slope, use the coordinates of two points on a line to determine its slope and use slope to express the parallelism and perpendicularity of lines.  
**HS-AT-S-EI11** Students will relate the coefficients of a linear equation and the slope and  $x$ - and  $y$ -intercepts of its graph. Also addresses HS-AT-S-EI15.

## New Vocabulary

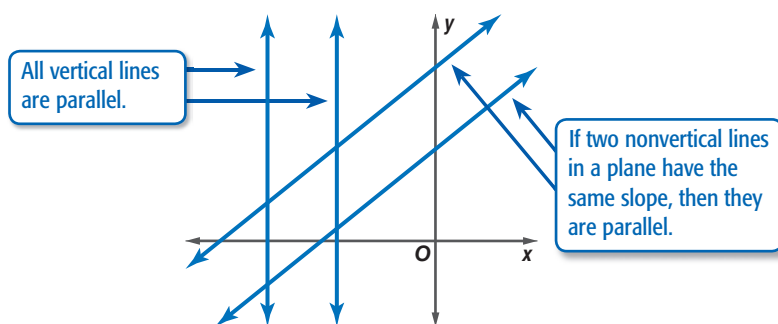
parallel lines  
perpendicular lines



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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

**Parallel Lines** Lines in the same plane that do not intersect are called **parallel lines**. Parallel lines have the same slope.



You can write an equation of a line parallel to a given line if you know a point on the line and an equation of the given line. First find the slope of the given line. Then, substitute the point provided and the slope from the given line into the point-slope form.

### EXAMPLE 1 Parallel Line Through a Given Point

Write an equation in slope-intercept form for the line that passes through  $(-3, 5)$  and is parallel to the graph of  $y = 2x - 4$ .

**Step 1** The slope of the line with equation  $y = 2x - 4$  is 2. The line parallel to  $y = 2x - 4$  has the same slope, 2.

**Step 2** Find the equation in slope-intercept form.

$y - y_1 = m(x - x_1)$	<b>Point-slope form</b>
$y - 5 = 2[x - (-3)]$	<b>Replace <math>m</math> with 2 and <math>(x_1, y_1)</math> with <math>(-3, 5)</math>.</b>
$y - 5 = 2(x + 3)$	<b>Simplify.</b>
$y - 5 = 2x + 6$	<b>Distributive Property</b>
$y - 5 + 5 = 2x + 6 + 5$	<b>Add 5 to each side.</b>
$y = 2x + 11$	<b>Write the equation in slope-intercept form.</b>

### Check Your Progress

- Write an equation in point-slope form for the line that passes through  $(4, -1)$  and is parallel to the graph of  $y = \frac{1}{4}x + 7$ .

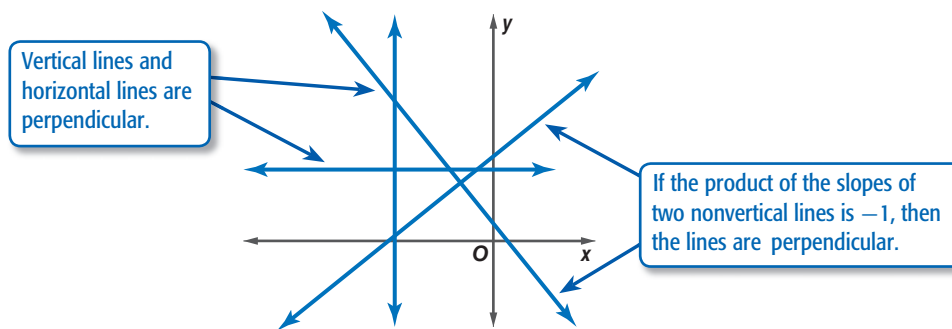
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## Review Vocabulary

### opposite reciprocals

The opposite reciprocal of  $\frac{a}{b}$  is  $-\frac{b}{a}$ . Their product is  $-1$ .

**Perpendicular Lines** Lines that intersect at right angles are called **perpendicular lines**. The slopes of perpendicular lines are opposite reciprocals. That is, if the slope of a line is 4, the slope of the line perpendicular to it is  $-\frac{1}{4}$ .



You can use the properties of the slopes of perpendicular lines to determine whether two lines are perpendicular.

## Real-World EXAMPLE 2 Slopes of Perpendicular Lines

**DESIGN** The outline of a company's new logo is shown on a coordinate plane.

a. Is  $\angle DFE$  a right angle in the logo?

If  $\overline{BE}$  and  $\overline{AD}$  are perpendicular, then  $\angle DFE$  is a right angle. Find the slopes of  $\overline{BE}$  and  $\overline{AD}$ .

$$\text{slope of } \overline{BE}: m = \frac{1-3}{7-2} \text{ or } -\frac{2}{5}$$

$$\text{slope of } \overline{AD}: m = \frac{6-1}{4-2} \text{ or } \frac{5}{2}$$

The line segments are perpendicular because  $-\frac{2}{5} \times \frac{5}{2} = -1$ . Therefore,  $\angle DFE$  is a right angle.

b. Is each pair of opposite sides parallel?

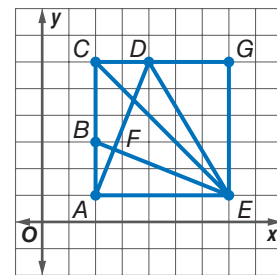
If a pair of opposite sides are parallel, then they have the same slope.

$$\text{slope of } \overline{AC}: m = \frac{6-1}{2-2} \text{ or undefined}$$

Since  $\overline{AC}$  and  $\overline{GE}$  are both parallel to the  $y$ -axis, they are vertical and are therefore parallel.

$$\text{slope of } \overline{CG}: m = \frac{6-6}{7-2} \text{ or } 0$$

Since  $\overline{CG}$  and  $\overline{AE}$  are both parallel to the  $x$ -axis, they are horizontal and are therefore parallel.



## Real-World Link

The oldest treehouse still in existence was built in England in the 1700s.

Source: The Treehouse Company

## Check Your Progress

2. **CONSTRUCTION** On the plans for a treehouse, a beam represented by  $\overline{QR}$  has endpoints  $Q(-6, 2)$  and  $R(-1, 8)$ . A connecting beam represented by  $\overline{ST}$  has endpoints  $S(-3, 6)$  and  $T(-8, 5)$ . Are the beams perpendicular? Explain.

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You can determine whether two linear equations are parallel or perpendicular by comparing the slopes of each line.

## ReadingMath

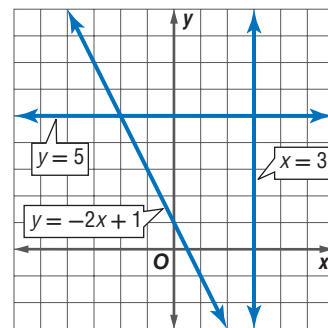
**Parallel and perpendicular lines** The symbol for parallel is  $\parallel$ . The symbol for perpendicular is  $\perp$ .

### EXAMPLE 3 Parallel or Perpendicular Lines

Determine whether the graphs of  $y = 5$ ,  $x = 3$ , and  $y = -2x + 1$  are *parallel* or *perpendicular*. Explain.

Graph each line on a coordinate plane.

From the graph, you can see that  $y = 5$  is parallel to the  $x$ -axis and  $x = 3$  is parallel to the  $y$ -axis. Therefore, they are perpendicular. None of the lines are parallel.



#### Check Your Progress

3. Determine whether the graphs of  $6x - 2y = -2$ ,  $y = 3x - 4$ , and  $y = 4$  are *parallel* or *perpendicular*. Explain.

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You can write the equation of a line perpendicular to a given line if you know a point on the line and the equation of the given line.

## StudyTip

**Graphing Perpendicular Lines** To visualize the problem, graph the given equation on a coordinate grid and plot the given point. Using a ruler, draw a line perpendicular to the given line that passes through the point.

### EXAMPLE 4 Parallel Line Through a Given Point

Write an equation in slope-intercept form for the line that passes through  $(-4, 6)$  and is perpendicular to the graph of  $2x + 3y = 12$ .

- Step 1** Find the slope of the given line by solving the equation for  $y$ .

$$2x + 3y = 12 \quad \text{Original equation}$$

$$2x - 2x + 3y = -2x + 12 \quad \text{Subtract } 2x \text{ from each side.}$$

$$3y = -2x + 12 \quad \text{Simplify.}$$

$$\frac{3y}{3} = \frac{-2x + 12}{3} \quad \text{Divide each side by 3.}$$

$$y = -\frac{2}{3}x + 4 \quad \text{Simplify.}$$

The slope is  $-\frac{2}{3}$ .

- Step 2** The slope of the perpendicular line is the opposite reciprocal of  $-\frac{2}{3}$  or  $\frac{3}{2}$ . Find the equation of the perpendicular line.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

$$y - 6 = \frac{3}{2}(x - (-4)) \quad (x_1, y_1) = (-4, 6) \text{ and } m = \frac{3}{2}$$

$$y - 6 = \frac{3}{2}(x + 4) \quad \text{Simplify.}$$

$$y - 6 = \frac{3}{2}x + 6 \quad \text{Distributive Property}$$

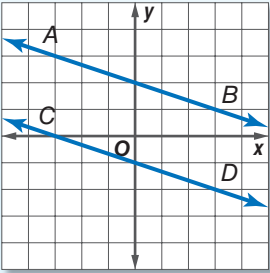
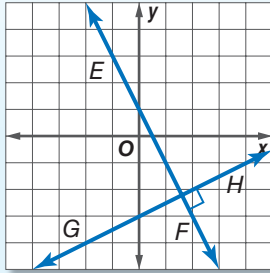
$$y - 6 + 6 = \frac{3}{2}x + 6 + 6 \quad \text{Add 6 to each side.}$$

$$y = \frac{3}{2}x + 12 \quad \text{Simplify.}$$

#### Check Your Progress

4. Write an equation in slope-intercept form for the line that passes through  $(4, 7)$  and is perpendicular to the graph of  $y = \frac{2}{3}x - 1$ .

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	Parallel Lines	Perpendicular Lines
<b>Words</b>	Two nonvertical lines are parallel if they have the same slope.	Two nonvertical lines are perpendicular if the product of their slopes is $-1$ .
<b>Symbols</b>	$\overline{AB} \parallel \overline{CD}$	$\overline{EF} \perp \overline{GH}$
<b>Models</b>		

 Check Your Understanding

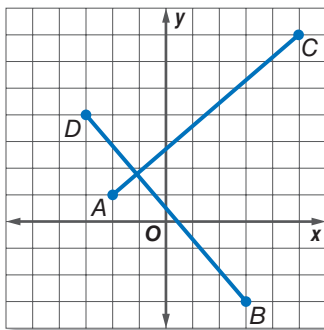
**Example 1**  
p. 237

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of each equation.

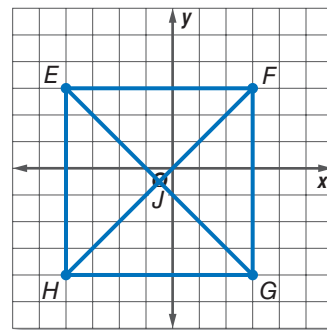
- $(-1, 2), y = \frac{1}{2}x - 3$
- $(0, 4), y = -4x + 5$

**Example 2**  
p. 238

**3. GARDENS** A garden is in the shape of a quadrilateral with vertices  $A(-2, 1)$ ,  $B(3, -3)$ ,  $C(5, 7)$ , and  $D(-3, 4)$ . Two paths represented by  $\overline{AC}$  and  $\overline{BD}$  cut across the garden. Are the paths perpendicular? Explain.



**4. GEOMETRY** A square is a quadrilateral that has opposite sides parallel, consecutive sides that are perpendicular, and diagonals that are perpendicular. Determine whether the quadrilateral is a square. Explain.



**Example 3**  
p. 239

Determine whether the graphs of the following equations are *parallel* or *perpendicular*. Explain.

- $y = -2x, 2y = x, 4y = 2x + 4$
- $y = \frac{1}{2}x, 3y = x, y = -\frac{1}{2}x$

**Example 4**  
p. 239

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation.

- $(-2, 3), y = -\frac{1}{2}x - 4$
- $(-1, 4), y = 3x + 5$
- $(2, 3), 2x + 3y = 4$
- $(3, 6), 3x - 4y = -2$

# Practice and Problem Solving

 = **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

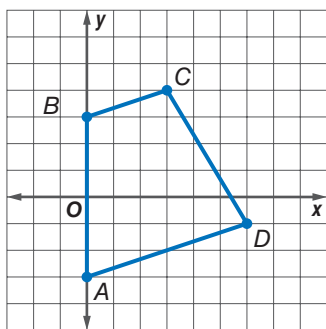
**Example 1**  
p. 237

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of each equation.

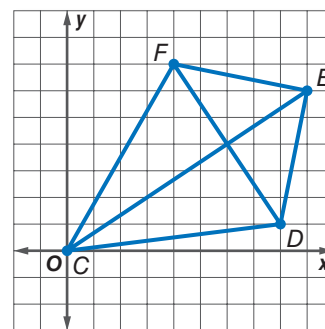
11.  $(3, -2)$ ,  $y = x + 4$       12.  $(4, -3)$ ,  $y = 3x - 5$       13.  $(0, 2)$ ,  $y = -5x + 8$   
 14.  $(-4, 2)$ ,  $y = -\frac{1}{2}x + 6$       15.  $(-2, 3)$ ,  $y = -\frac{3}{4}x + 4$       16.  $(9, 12)$ ,  $y = 13x - 4$

**Example 2**  
p. 238

17. **GEOMETRY** A trapezoid is a quadrilateral that has exactly one pair of parallel opposite sides. Determine whether  $ABCD$  is a trapezoid. Explain your reasoning.



18. **GEOMETRY** The quadrilateral  $CDEF$  is a kite. Determine whether the diagonals of the kite are perpendicular. Explain your reasoning.



19. Determine whether  $y = -6x + 4$  and  $y = \frac{1}{6}x$  are perpendicular. Explain.  
 20. **MAPS** On a map, Elmwood Drive passes through  $R(4, -11)$  and  $S(0, -9)$ , and Taylor Road passes through  $J(6, -2)$  and  $K(4, -5)$ . If they are straight lines, are the two streets perpendicular? Explain.

**Example 3**  
p. 239

Determine whether the graphs of the following equations are *parallel* or *perpendicular*. Explain.

21.  $2x - 8y = -24$ ,  $4x + y = -2$ ,  $x - 4y = 4$   
 22.  $3x - 9y = 9$ ,  $3y = x + 12$ ,  $2x - 6y = 12$

**Example 4**  
p. 239

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation.

23.  $(-3, -2)$ ,  $y = -2x + 4$       24.  $(-5, 2)$ ,  $y = \frac{1}{2}x - 3$       25.  $(-4, 5)$ ,  $y = \frac{1}{3}x + 6$   
 26.  $(2, 6)$ ,  $y = -\frac{1}{4}x + 3$       27.  $(3, 8)$ ,  $y = 5x - 3$       28.  $(4, -2)$ ,  $y = 3x + 5$

Write an equation in slope-intercept form for a line perpendicular to the graph of the equation that passes through the  $x$ -intercept of that line.

29.  $y = -\frac{1}{2}x - 4$       30.  $y = \frac{2}{3}x - 6$       31.  $y = 5x + 3$   
 32. Write an equation in slope-intercept form for the line that is perpendicular to the graph of  $3x + 2y = 8$  and passes through the  $y$ -intercept of that line.

Determine whether the graphs of each pair of equations are *parallel*, *perpendicular*, or *neither*.

33.  $y = 4x + 3$       34.  $y = -2x$       35.  $3x + 5y = 10$   
 $4x + y = 3$        $2x + y = 3$        $5x - 3y = -6$   
 36.  $-3x + 4y = 8$       37.  $2x + 5y = 15$       38.  $2x + 7y = -35$   
 $-4x + 3y = -6$        $3x + 5y = 15$        $4x + 14y = -42$





### Real-World Career

#### Archaeologist

An archaeologist studies artifacts of ancient civilizations to piece together information about ancient societies.

Archaeology is considered a subset of anthropology. Most archaeologists have a master's degree at least.

39. Write an equation of the line that is parallel to the graph of  $y = 7x - 3$  and passes through the origin.
40. **EXCAVATION** Scientists excavating a dinosaur mapped the site on a coordinate plane. If one bone lies from  $(-5, 8)$  to  $(10, -1)$  and a second bone lies from  $(-10, -3)$  to  $(-5, -6)$ , do the bones lie parallel to one another? Explain.
41. **ARCHAEOLOGY** In the ruins of an ancient civilization, an archaeologist found pottery at  $(2, 6)$  and hair accessories at  $(4, -1)$ . A pole is found with one end at  $(7, 10)$  and the other end at  $(14, 12)$ . Is the pole perpendicular to the line formed by the pottery and the hair accessories? Explain.
42. **GRAPHICS** To create a design on a computer, Andeana must enter the coordinates for points on the design. One line segment she drew has endpoints of  $(-2, 1)$  and  $(4, 3)$ . The other coordinates that Andeana entered are  $(2, -7)$  and  $(8, -3)$ . Could these points be the vertices of a rectangle? Explain.
43. **MULTIPLE REPRESENTATIONS** In this problem, you will explore parallel and perpendicular lines.
- GRAPHICAL** Graph the points  $A(-3, 3)$ ,  $B(3, 5)$ , and  $C(-4, 0)$  on a coordinate plane.
  - ANALYTICAL** Determine the coordinates of a fourth point  $D$  that would form a parallelogram. Explain your reasoning.
  - ANALYTICAL** What is the minimum number of points that would be moved in order to make the parallelogram into a rectangle? Describe which points should be moved and explain why.

### H.O.T. Problems

Use Higher-Order Thinking Skills

44. **CHALLENGE** If the line through  $(-2, 4)$  and  $(5, d)$  is parallel to the graph of  $y = 3x + 4$ , what is the value of  $d$ ?
45. **REASONING** Is a horizontal line perpendicular to a vertical line *sometimes*, *always*, or *never*? Explain your reasoning.
46. **OPEN ENDED** Graph a line that is parallel and a line that is perpendicular to  $y = 2x - 1$ .
47. **FIND THE ERROR** Carmen and Chase are finding an equation of the line that is perpendicular to the graph of  $y = \frac{1}{3}x + 2$  and passes through the point  $(-3, 5)$ . Is either of them correct? Explain your reasoning.

Carmen

$$y - 5 = -3[x - (-3)]$$

$$y - 5 = -3(x + 3)$$

$$y = -3x - 9 + 5$$

$$y = -3x - 4$$

Chase

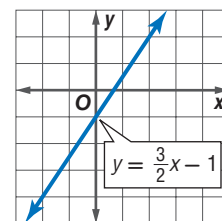
$$y - 5 = 3[x - (-3)]$$

$$y - 5 = 3(x + 3)$$

$$y = 3x + 9 + 5$$

$$y = -3x + 14$$

48. **WRITING IN MATH** Illustrate how you can determine whether two lines are parallel or perpendicular. Write an equation whose graph is parallel to the line shown in the graph at the right, and an equation whose graph is perpendicular to the line shown in the graph. Explain your reasoning.



## KCCT PRACTICE MA-HS-5.3.3

49. Which of the following is an algebraic translation of the following phrase?

5 less than the quotient of a number and 8

- A  $5 - \frac{n}{8}$                       C  $5 - \frac{8}{n}$   
 B  $\frac{n}{8} - 5$                       D  $\frac{8}{n} - 5$

50. A line through which two points would be parallel to a line with a slope of  $\frac{3}{4}$ ?

- F (0, 5) and (-4, 2)    H (0, 0) and (0, -2)  
 G (0, 2) and (-4, 1)    J (0, -2) and (-4, -2)

51. Which equation best fits the data in the table?

- A  $y = x + 4$   
 B  $y = 2x + 3$   
 C  $y = 7$   
 D  $y = 4x - 5$

x	y
1	5
2	7
3	9
4	11

52. **SHORT RESPONSE** Tyler is filling his 6000-gallon pool at a constant rate. After 4 hours, the pool contained 800 gal. How many total hours will it take to completely fill the pool?

## Spiral Review

Write each equation in standard form. (Lesson 4-3)

53.  $y - 13 = 4(x - 2)$

54.  $y - 5 = -2(x + 2)$

55.  $y + 3 = -5(x + 1)$

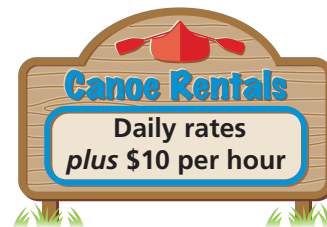
56.  $y + 7 = \frac{1}{2}(x + 2)$

57.  $y - 1 = \frac{5}{6}(x - 4)$

58.  $y - 2 = -\frac{2}{5}(x - 8)$

59. **CANOE RENTAL** Latanya and her friends rented a canoe for 3 hours and paid a total of \$45. (Lesson 4-2)

- a. Write a linear equation to find the total cost  $C$  of renting the canoe for  $h$  hours.  
 b. How much would it cost to rent the canoe for 8 hours?



Write an equation of the line that passes through each point with the given slope. (Lesson 4-2)

60. (5, -2),  $m = 3$

61. (-5, 4),  $m = -5$

62. (3, 0),  $m = -2$

63. (3, 5),  $m = 2$

64. (-3, -1),  $m = -3$

65. (-2, 4),  $m = -5$

Simplify each expression. If not possible, write *simplified*. (Lesson 1-4)

66.  $13m + m$

67.  $14a^2 + 13b^2 + 27$

68.  $3(x + 2x)$

69. **SALES** At a Farmers' Market, merchants can rent a small table for \$5.00 and a large table for \$8.50. One time, 25 small and 10 large tables were rented. Another time, 35 small and 12 large were rented. (Lesson 1-2)

- a. Write an expression to show the total amount of money collected.  
 b. Evaluate the expression.

## Skills Review

Express each relation as a graph. Then determine the domain and range. (Lesson 1-6)

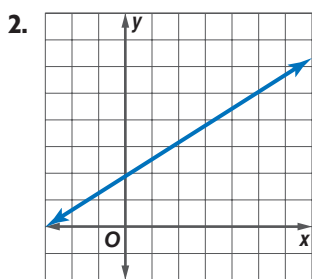
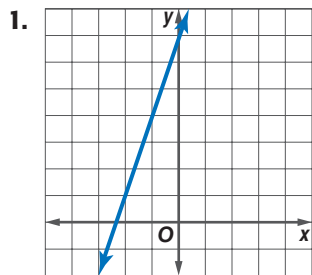
70.  $\{(3, 8), (3, 7), (2, -9), (1, -9), (-5, -3)\}$

71.  $\{(3, 4), (4, 3), (2, 2), (5, -4), (-4, 5)\}$

72.  $\{(0, 2), (-5, 1), (0, 6), (-1, 9), (-4, -5)\}$

73.  $\{(7, 6), (3, 4), (4, 5), (-2, 6), (-3, 2)\}$

Write an equation in slope-intercept form for each graph shown. (Lesson 4-1)



Graph each equation. (Lesson 4-1)

3.  $y = 2x + 3$

4.  $y = \frac{1}{3}x - 2$

5. **BOATS** The sign below describes the charge to rent a pontoon boat. Write an equation in slope-intercept form for the total rental cost  $C$  for a boat used for  $t$  hours. (Lesson 4-1)



Write an equation of a line with the given conditions. (Lesson 4-2)

6.  $(2, 5)$ ; slope 3  
 7.  $(-3, -1)$ , slope  $\frac{1}{2}$   
 8.  $(-3, 4)$ ,  $(1, 12)$   
 9.  $(-1, 6)$ ,  $(2, 4)$   
 10.  $(2, 1)$ , slope 0

11. **MULTIPLE CHOICE** Write an equation of a line that passes through the point  $(0, 0)$  and has slope  $-4$ . (Lesson 4-2)

A  $y = x - 4$

C  $y = -4x$

B  $y = x + 4$

D  $y = 4 - x$

Write an equation in point-slope form for the line that passes through each point with the given slope. (Lesson 4-3)

12.  $(1, 4)$ ,  $m = 6$

13.  $(-2, -1)$ ,  $m = -3$

14. Write an equation in point-slope form for the line that passes through the point  $(8, 3)$  and has slope  $-2$ . (Lesson 4-3)

15. Write the equation  $y + 3 = \frac{1}{2}(x - 5)$  in standard form. (Lesson 4-3)

16. Write the equation  $y + 4 = -7(x - 3)$  in slope-intercept form. (Lesson 4-3)

Write each equation in standard form. (Lesson 4-3)

17.  $y - 5 = -2(x - 3)$

18.  $y + 4 = \frac{2}{3}(x - 3)$

Write each equation in slope-intercept form. (Lesson 4-3)

19.  $y - 3 = 4(x + 3)$

20.  $y + 1 = \frac{1}{2}(x - 8)$

21. **MULTIPLE CHOICE** Determine whether the graphs of the pair of equations are *parallel*, *perpendicular*, or *neither*. (Lesson 4-4)

$$y = -6x + 8$$

$$3x + \frac{1}{2}y = -3$$

F parallel

G perpendicular

H neither

J not enough information

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation. (Lesson 4-4)

22.  $(3, -4)$ ;  $y = -\frac{1}{3}x - 5$

23.  $(0, -3)$ ;  $y = -2x + 4$

24.  $(-4, -5)$ ;  $-4x + 5y = -6$

25.  $(-1, -4)$ ;  $-x - 2y = 0$

## Scatter Plots and Lines of Fit

### Then

You wrote linear equations given a point and the slope. (Lesson 4-3)

### Now

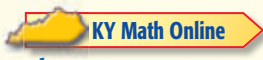
- Investigate relationships between quantities by using points on scatter plots.
- Use lines of fit to make and evaluate predictions.



**HS-DAP-S-DR4** Students will display a scatterplot and describe its shape for bivariate data.  
**HS-DAP-S-CDS5** Students will apply line-of-best fit equations for a set of two-variable data to make predictions. Also addresses HS-DAP-S-DR2, HS-DAP-S-DR5, and HS-AT-S-EI2.

### New Vocabulary

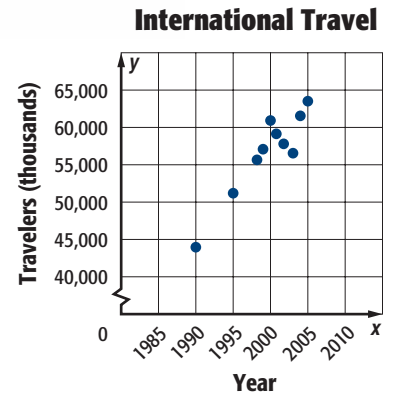
- bivariate data
- scatter plot
- line of fit
- linear interpolation



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  - Self-Check Quiz
  - Homework Help
  - Math in Motion

### Why?

The graph shows the number of people from the United States who travel to other countries. The points do not all lie on the same line; however, you may be able to draw a line that is close to all of the points. That line would show a linear relationship between the year  $x$  and the number of travelers each year  $y$ . Generally, international travel has increased.



Source: Statistical Abstract of the United States

**Investigate Relationships Using Scatter Plots** Data with two variables are called **bivariate data**. A **scatter plot** is a graph in which two sets of data are plotted as ordered pairs in a coordinate plane. Scatter plots are used to investigate a relationship between two quantities.

Concept Summary

Scatter Plots

For Your  
**FOLDABLE**

**Positive Correlation**

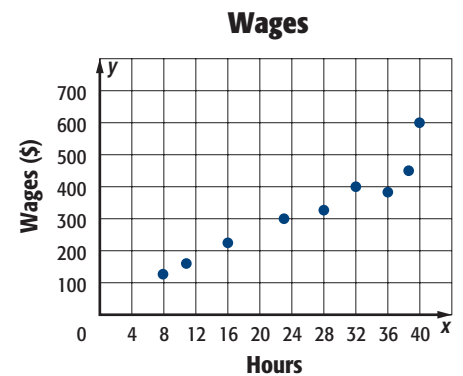
**Negative Correlation**

**No Correlation**

### Real-World EXAMPLE 1 Evaluate a Correlation

**WAGES** Determine whether the graph shows a *positive* correlation, a *negative* correlation, or *no* correlation. If there is a positive or negative correlation, describe its meaning in the situation.

The graph shows a positive correlation. As the number of hours worked increases, the wages increase.



### Check Your Progress

- Refer to the graph on international travel. Determine whether the graph shows a *positive* correlation, a *negative* correlation, or *no* correlation. If there is a positive or negative correlation, describe its meaning.



### StudyTip

#### Correlations

positive: as  $x$  increases,  
 $y$  increases  
negative: as  $x$  increases,  
 $y$  decreases  
no correlation: no  
relationship between  
 $x$  and  $y$

**Use Lines of Fit** Scatter plots can show whether there is a trend in a set of data. When the data points all lie close to a line, a **line of fit** can model the trend.

### Key Concept

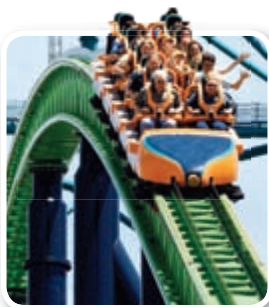
#### Using a Linear Function to Model Data

For Your

FOLDABLE

- Step 1** Make a scatter plot. Determine whether any relationship exists in the data.
- Step 2** Draw a line that seems to pass close to most of the data points.
- Step 3** Use two points on the line of fit to write an equation for the line.
- Step 4** Use the line of fit to make predictions.

▶ **Math in Motion, Interactive Lab** [glencoe.com](http://glencoe.com)



### Real-World Link

The Kingda Ka roller coaster at Six Flags Great Adventure in Jackson, New Jersey, has broken three records: tallest roller coaster at 456 feet, fastest at 128 miles per hour, and largest vertical drop of 418 feet.

Source: Ultimate Roller Coaster

### Real-World EXAMPLE 2 Write a Line of Fit

**ROLLER COASTERS** The table shows the largest vertical drops of nine roller coasters in the United States and the number of years after 1988 that they were opened. Identify the independent and the dependent variables. Is there a relationship in the data? If so, predict the vertical drop in a roller coaster built 25 years after 1988.

Years Since 1988	1	3	5	8	12	12	12	13	15
Vertical Drop (ft)	151	155	225	230	306	300	255	255	400

Source: Ultimate Roller Coaster

- Step 1** Make a scatter plot.

The independent variable is the year, and the dependent variable is the vertical drop. As the number of years increases, the vertical drop of roller coasters increases. There is a positive correlation between the two variables.

- Step 2** Draw a line of fit.

No one line will pass through all of the data points. Draw a line that passes close to the points. A line of fit is shown.

- Step 3** Write the slope-intercept form of an equation for the line of fit.

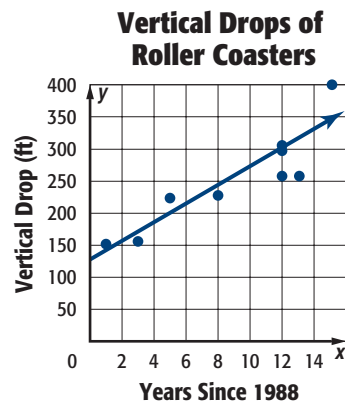
The line of fit passes close to  $(2, 150)$  and the data point  $(12, 300)$ .

Find the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} & (x_1, y_1) &= (2, 150), \\ & & (x_2, y_2) &= (12, 300) \\ &= \frac{300 - 150}{12 - 2} \\ &= \frac{150}{10} \text{ or } 15 \end{aligned}$$

Use  $m = 15$  and either the point-slope form or the slope-intercept form to write the equation of the line of fit.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 150 &= 15(x - 2) \\ y - 150 &= 15x - 30 \\ y &= 15x + 120 \end{aligned}$$



A slope of 15 means that the vertical drops increased an average of 15 feet per year. To find the vertical drop of a roller coaster built 25 years after 1988, substitute 25 for  $x$  in the equation. The vertical drop is  $15(25) + 120$  or 495 feet.

## ✓ Check Your Progress

2. **MUSIC** The table shows the dollar value in millions for the sales of CDs for the year. Make a scatter plot and determine what relationship exists, if any, in the data.

Year	2000	2001	2002	2003	2004	2005
Sales	13,215	12,909	12,044	11,233	11,447	10,520

▶ Personal Tutor [glencoe.com](http://glencoe.com)

## ReadingMath

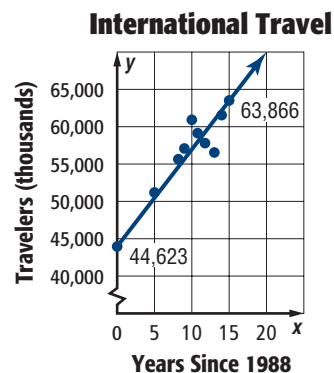
**Interpolation and Extrapolation** The Latin prefix *inter-* means between, and the Latin prefix *extra-* means beyond.

In Lesson 4-2, you learned that linear extrapolation is used to predict values *outside* the range of the data. You can also use a linear equation to predict values *inside* the range of the data. This is called **linear interpolation**.

## Real-World EXAMPLE 3 Use Interpolation or Extrapolation

**TRAVEL** Use the scatter plot to determine the number of United States travelers to international countries in 1996.

- Step 1** Draw a line of fit. The line should be as close to as many points as possible.
- Step 2** Write the slope-intercept form of the equation. The line of fit passes through  $(0, 44,623)$  and  $(15, 63,866)$ .



Find the slope.

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{63,866 - 44,623}{15 - 0} \\
 &= \frac{19,243}{15}
 \end{aligned}$$

**Slope Formula**

$$\begin{aligned}
 (x_1, y_1) &= (0, 44,623), \\
 (x_2, y_2) &= (15, 63,866)
 \end{aligned}$$

**Simplify.**

Use  $m = \frac{19,243}{15}$  and either the point-slope form or the slope-intercept form to write the equation of the line of fit.

$$y - y_1 = m(x - x_1)$$

$$y - 44,623 = \frac{19,243}{15}(x - 0)$$

$$y - 44,623 = \frac{19,243}{15}x$$

$$y = \frac{19,243}{15}x + 44,623$$

- Step 3** Evaluate the function for  $x = 1996 - 1990$  or 6.

$$y = \frac{19,243}{15}x + 44,623$$

**Equation of best-fit line**

$$= \frac{19,243}{15}(6) + 44,623$$

**$x = 6$**

$$= 7697\frac{1}{5} + 44,623 \text{ or } 52,320\frac{1}{5}$$

**Add.**

In 1996, there were approximately 52,320 thousand people who traveled from the United States to international countries.

## ✓ Check Your Progress

3. **MUSIC** Use the equation for the line of fit in Check Your Progress 2 to estimate CD sales in 2015.

▶ Personal Tutor [glencoe.com](http://glencoe.com)

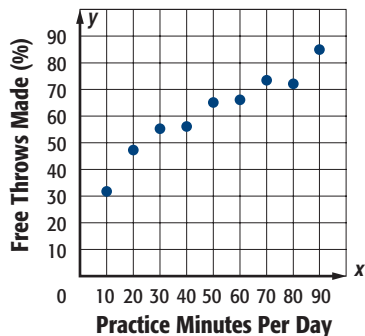
## Check Your Understanding

### Example 1 p. 245

Determine whether each graph shows a *positive* correlation, a *negative* correlation, or *no* correlation. If there is a positive or negative correlation, describe its meaning in the situation.

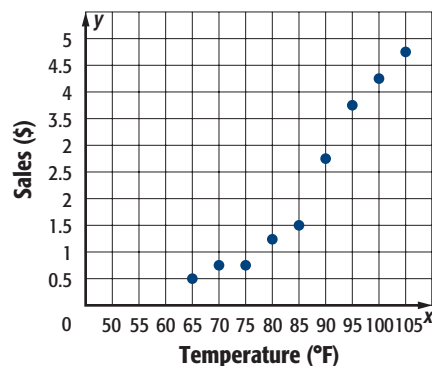
1.

#### Free Throws



2.

#### Lemonade Sales



### Example 2 p. 246

3. **MARRIAGE** The table shows the median age of females when they were first married.

- Make a scatter plot and determine what relationship exists, if any, in the data. Identify the independent and the dependant variables.
- Draw a line of fit for the scatter plot.
- Write an equation in slope-intercept form for the line of fit.
- Predict what the median age of females when they are first married will be in 2016.
- Do you think the equation can give a reasonable estimate for the median age of females when they are first married for the year 2056? Explain.

Year	Age
1996	24.8
1997	25.0
1998	25.0
1999	25.1
2000	25.1
2001	25.1
2002	25.3
2003	25.3
2005	25.5
2006	25.9

Source: U.S. Bureau of Census

### Example 3 p. 247

## Practice and Problem Solving

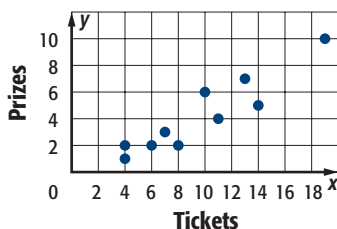
= **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

### Example 1 p. 245

Determine whether each graph shows a *positive* correlation, a *negative* correlation, or *no* correlation. If there is a positive or negative correlation, describe its meaning in the situation.

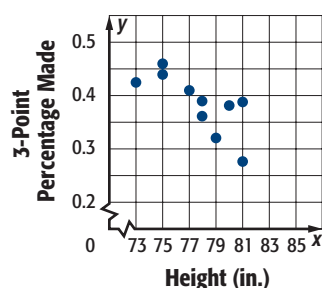
4.

#### Game Tickets at the Fair

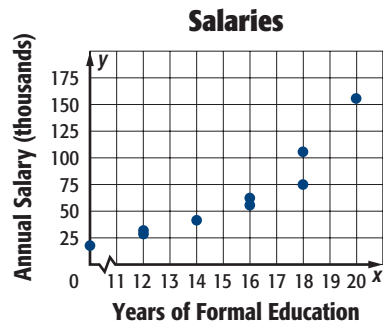


5

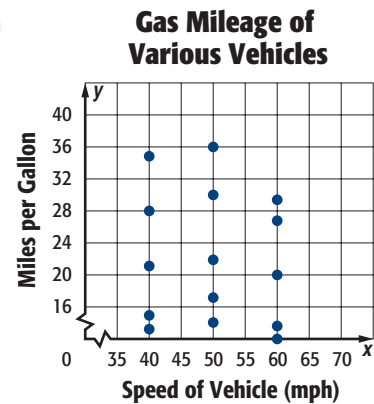
#### NBA 3-Point Percentage



6.



7



**Examples 2 and 3**  
pp. 246-247

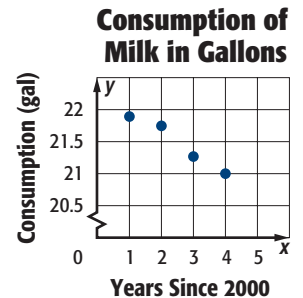


**Real-World Career**

**Camera Operator**

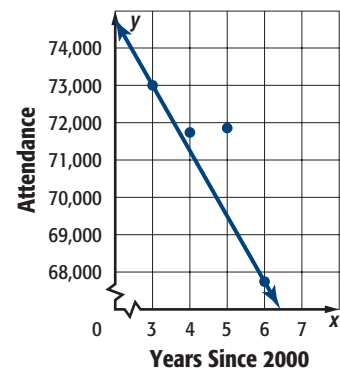
A camera operator is responsible for physically operating a camera and framing a scene. This job requires on-the-job training, while the educational requirements range from a high school diploma to a college degree.

8. **MILK** Refer to the scatter plot of gallons of milk consumption per person for selected years.
- Use the points (2, 21.75) and (4, 21) to write the slope-intercept form of an equation for the line of fit.
  - Predict the milk consumption in 2015.
  - Predict in what year milk consumption will be 10 gallons.
  - Is it reasonable to use the equation to estimate the consumption of milk for any year? Explain.



9. **FOOTBALL** Use the scatter plot that shows the average attendance at Buffalo Bills regular season football games.
- Use the points (3, 73,000) and (6, 67,650) to write the slope-intercept form of an equation for the line of fit shown in the scatter plot.
  - Predict the average attendance at a game in 2012.
  - Can you use the equation to make a decision about the average attendance in any given year in the future? Explain.

**Buffalo Bills Attendance**



10. **WEIGHT** The Body Mass Index (BMI) is a measure of body fat using height and weight. The heights and weights of twelve men with normal BMI are given in the table at the right.
- Make a scatter plot comparing the height in inches to the weight in pounds.
  - Draw a line of fit for the data.
  - Write the slope-intercept form of an equation for the line of fit.
  - Predict the normal weight for a man who is 84 inches tall.
  - A man's weight is 188 pounds. Use the equation of the line of fit to predict the height of the man.

Height (in.)	Weight (lb)
62	115
63	124
65	120
67	134
67	140
68	138
68	144
68	152
69	147
72	155
73	168
73	166





### Real-World Link

Old Faithful is located in Yellowstone National Park. The Washburn Expedition of 1870 named the geyser because of its size and frequency.

Source: The Geyser Observation and Study Association

- 11 GEYSERS** The time to the next eruption of Old Faithful can be predicted by using the duration of the current eruption.

<b>Duration (min)</b>	1.5	2	2.5	3	3.5	4	4.5	5
<b>Interval (min)</b>	48	55	70	72	74	82	93	100

- Identify the independent and the dependant variables. Make a scatter plot and determine what relationship, if any, exists in the data. Draw a line of fit for the scatter plot.
  - Let  $x$  represent the duration of the previous interval. Let  $y$  represent the length of the interval between eruptions. Write the slope-intercept form of the equation for the line of fit. Predict the interval that will occur after an eruption that lasts 7.5 minutes.
  - Make a critical judgment about using the equation to predict the duration of the next eruption. Would the equation be a useful model?
- 12. COLLECT DATA** Use a tape measure to measure both the foot size and the height in inches of ten individuals.
- Record your data in a table.
  - Make a scatter plot and draw a line of fit for the data.
  - Write an equation for the line of fit.
  - Make a conjecture about the relationship between foot size and height.

## H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills

- 13. OPEN ENDED** Describe a real-life situation that can be modeled using a scatter plot. Decide whether there is a *positive*, *negative*, or *no* correlation. Explain what this correlation means.
- 14. WHICH ONE DOESN'T BELONG?** Analyze the following situations and determine which one does not belong.

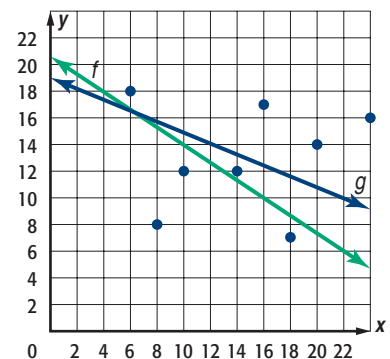
hours worked and amount of money earned

height of an athlete and favorite color

seedlings that grow an average of 2 centimeters each week

number of photos stored on a camera and capacity of camera

- 15. CHALLENGE** Determine which line of fit is better for the scatter plot at the right. Explain your reasoning.
- 16. REASONING** What can make a scatter plot and line of fit more useful for accurate predictions? Does an accurate line of fit always predict what will happen in the future? Explain.
- 17. WRITING IN MATH** Make a scatter plot that shows the height of a person and age. Explain how you could use the scatter plot to predict the age of a person given his or her height. How can the information from a scatter plot be used to identify trends and make decisions?





18. Which equation best describes the relationship between the values of  $x$  and  $y$  shown in the table?

- A  $y = x - 5$
- B  $y = 2x - 5$
- C  $y = 3x - 7$
- D  $y = 4x - 7$

$x$	$y$
-1	-7
0	-5
2	-1
4	3

19. **STATISTICS** Mr. Hernandez collected data on the heights and average stride lengths of a random sample of high school students. He then graphed the data on a scatter plot. What kind of correlation did he most likely see?

- F positive
- G constant
- H negative
- J no

20. **GEOMETRY** Mrs. Aguilar's rectangular bedroom measures 13 feet by 11 feet. She wants to purchase carpet for the bedroom that costs \$2.95 per square foot, including tax. How much will it cost to carpet her bedroom?

- A \$70.80
- B \$141.60
- C \$145.95
- D \$421.85

21. **SHORT RESPONSE** Nikia bought a one-month membership to a Fitness Center for \$35. Each time she goes, she rents a locker for \$0.25. If she spent a total of \$40.50 to use the Fitness Center last month, how many days did she go?

**Spiral Review**

Determine whether the graphs of each pair of equations are *parallel*, *perpendicular*, or *neither*. (Lesson 4-4)

22.  $y = -2x + 11$   
 $y + 2x = 23$

23.  $3y = 2x + 14$   
 $2x + 3y = 2$

24.  $y = -5x$   
 $y = 5x - 18$

25.  $y = 3x + 2$   
 $y = -\frac{1}{3}x - 2$

Write each equation in standard form. (Lesson 4-3)

26.  $y - 13 = 4(x - 2)$

27.  $y - 5 = -2(x + 2)$

28.  $y + 3 = -5(x + 1)$

29.  $y + 7 = \frac{1}{2}(x + 2)$

30.  $y - 1 = \frac{5}{6}(x - 4)$

31.  $y - 2 = -\frac{2}{5}(x - 8)$

Graph each equation. (Lesson 4-1)

32.  $y = 2x + 3$

33.  $4x + y = -1$

34.  $3x + 4y = 7$

Find the slope of the line that passes through each pair of points. (Lesson 3-3)

35. (3, 4), (10, 8)

36. (-4, 7), (3, 5)

37. (3, 7), (-2, 4)

38. (-3, 2), (-3, 4)

39. (-2, -6), (-1, 10)

40. (1, -5), (-3, -5)

41. **DRIVING** Latisha drove 248 miles in 4 hours. At that rate, how long will it take her to drive an additional 93 miles? (Lesson 2-6)

**Skills Review**

Express each relation as a graph. Then determine the domain and range. (Lesson 1-6)

42.  $\{(4, 5), (5, 4), (-2, -2), (4, -5), (-5, 4)\}$

43.  $\{(7, 6), (3, 4), (4, 5), (-2, 6), (-3, 2)\}$

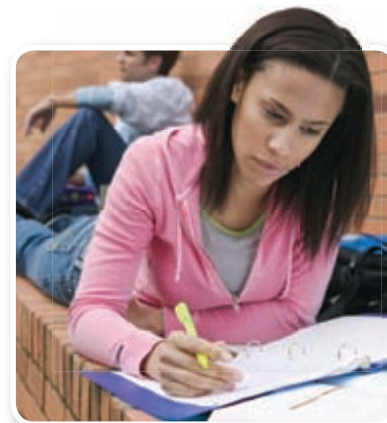


You may be considering attending a college or technical school in the future. What factors cause tuition to rise—increased building costs, higher employee salaries, or the amount of bottled water drank?

Let's see how bottled water and college tuition are related. The table shows the average college tuition and fees for public colleges and the per person consumption of bottled water per year for 2001 through 2005.

Year	2001	2002	2003	2004	2005
Water Consumed (gallons)	18.8	20.9	22.4	24.0	26.1
Tuition (\$)	3725	4081	4694	5132	5491

Source: *The New York Times Almanac*



### ACTIVITY

Follow the steps to learn about correlation and causation.

- Step 1** Graph the ordered pairs (gallons, tuition) to create a scatter plot. For example, one ordered pair is (18.8, 2562). Describe the graph.
- Step 2** The graph shows a correlation. Is the correlation *positive* or *negative*? Explain.
- Step 3** Do you think drinking more bottled water *causes* college tuition costs to rise? Explain.
- Step 4** **Causation** occurs when a change in one variable produces a change in another variable. Correlation can be observed between many variables, but causation can only be determined from data collected from a controlled experiment. Describe an experiment that could illustrate causation.

### Exercises

For each exercise, determine whether each situation illustrates *correlation* or *causation*. Explain your reasoning, including other factors that might be involved.

1. A survey showed that sleeping with the light on was positively correlated to nearsightedness.
2. A controlled experiment showed a positive correlation between the number of cigarettes smoked and the probability of developing lung cancer.
3. A random sample of students found that owning a cell phone had a negative correlation with riding the bus to school.
4. A controlled experiment showed a positive correlation between the number of hours using headphones when listening to music and the level of hearing loss.
5. DeQuan read in the newspaper that shark attacks are positively correlated with monthly ice cream sales.

## Regression and Median-Fit Lines

### Then

You used lines of fit and scatter plots to evaluate trends and make predictions. (Lesson 4-5)

### Now

- Write equations of best-fit lines using linear regression.
- Write equations of median-fit lines.

#### KY Program of Studies

**HS-DAP-S-CDS4** Students will determine regression coefficients, regression equations and correlation coefficients for bivariate data using technological tools.

**HS-DAP-S-CDS5** Students will apply line-of-best fit equations for a set of two-variable data to make predictions. Also addresses HS-DAP-S-CDS6, HS-AT-S-PRF14, and HS-AT-S-EI2.

### New Vocabulary

- best-fit line
- linear regression
- correlation coefficient
- median-fit line

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### Why?

Spider-Man comics have been around since 1962. Since then Spider-Man has made appearances in magazines, books, and movies.

The table shows the number of comic books and appearances that he has made. We can estimate how many appearances he will make in 2012.

Spider-Man Comics and Appearances

Year	Number
1962	1
1972	39
1982	110
1992	164
2002	278
2012	?

**Equations of Best-Fit Lines** You have learned how to find and write equations for lines of fit. Many calculators use an algorithm that finds a more precise line of fit called the **best-fit line**. This algorithm is called **linear regression**.

Your calculator may also compute a number called the **correlation coefficient**. This number will tell you if your correlation is positive or negative and how closely the equation is modeling the data. The closer the correlation coefficient is to 1 or  $-1$ , the more closely the equation models the data.

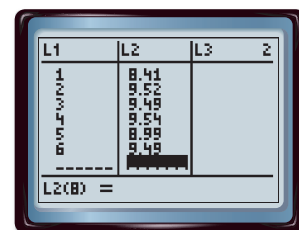
### Real-World EXAMPLE 1 Write an Equation for a Best-Fit Line

**MOVIES** The table shows the amount of money made by movies in the United States. Use a graphing calculator to write an equation for the best-fit line for that data.

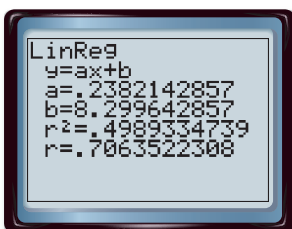
Year	2000	2001	2002	2003	2004	2005	2006
Income (\$ billion)	7.66	8.41	9.52	9.49	9.54	8.99	9.49

Before you begin, make sure that your Diagnostic setting is on. You can find this under the CATALOG menu. Press D and then scroll down and click DiagnosticOn. Then press **ENTER**.

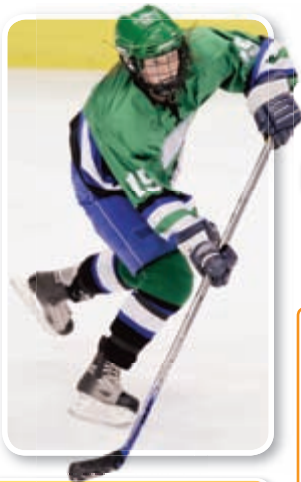
**Step 1** Enter the data by pressing **STAT** and selecting the Edit option. Let the year 2000 be represented by 0. Enter the years since 2000 into List 1 (L1). These will represent the  $x$ -values. Enter the income (\$ billion) into List 2 (L2). These will represent the  $y$ -values.



**Step 2** Perform the regression by pressing **STAT** and selecting the **CALC** option. Scroll down to **LinReg (ax+b)** and press **ENTER**.



- ← slope
- ←  $y$ -intercept
- ← correlation coefficient



### Real-World Link

In 1994, Minnesota became the first state to sanction girls' ice hockey as a high school varsity sport.

Source: ESPN SportsZone

**Step 3** Write the equation of the regression line by rounding the  $a$  and  $b$  values on the screen. The form that we chose for the regression was  $ax + b$ , so the equation is  $y = 0.24x + 8.30$ . The correlation coefficient is about 0.7064, which means that the equation models the data fairly well.

### Check Your Progress

Write an equation of the best-fit line for the data in each table. Name the correlation coefficient. Round to the nearest ten-thousandth.

**1A. HOCKEY** The table shows the number of goals of leading scorers for the Mustang Girls Hockey Team.

Mustang Girls Hockey Leading Scorers								
Year	2003	2004	2005	2006	2007	2008	2009	2010
Goals	30	23	41	35	31	43	33	45

**1B. HOCKEY** The table gives the number of goals scored by the team each season.

Mustang Girls Hockey Team Goals								
Year	2003	2004	2005	2006	2007	2008	2009	2010
Goals	63	44	55	63	81	85	93	84

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We can use points on the best-fit line to estimate values that are not in the data. Recall that when we estimate values that are between known values, this is called *linear interpolation*. When we estimate a number outside of the range of the data, it is called *linear extrapolation*.

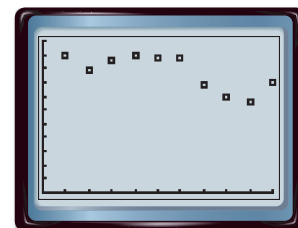
### Real-World EXAMPLE 2 Use Interpolation and Extrapolation

**PAINTBALL** The table shows the points received by the top ten paintball teams at a tournament. How many points did the 20th-ranked team receive?

Top Ten Teams										
Rank	1	2	3	4	5	6	7	8	9	10
Score	100	89	96	99	97	98	78	70	64	80

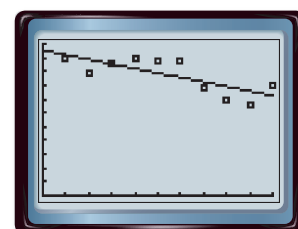
Write an equation of the best-fit line for the data. Then extrapolate to find the missing value.

**Step 1** Enter the data from the table into the lists as you did before. Let rank be the  $x$ -values and the score be the  $y$ -values. Then graph the scatter plot.



[0, 10] scl: 1 by [0, 110] scl: 10

**Step 2** Perform the linear regression using the data in the lists. Find the equation of the best-fit line. The equation of the best-fit line is  $y = -3.31x + 105.3$ .



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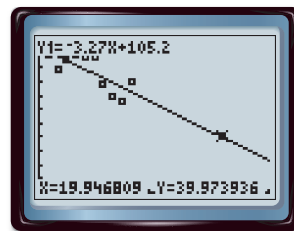
### StudyTip

#### Alternative Method

Instead of Step 3 in Example 2, you could substitute 3 for  $x$  in the equation of the best-fit line. The result would be 46.75 or about 47.

**Step 3** Graph the best-fit line. Then use the **Trace** feature and the arrow keys until you find a point where  $x = 20$ . When  $x = 20$ ,  $y \approx 39$ .

It is estimated that the 20th ranked team received 39 points.



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### Check Your Progress

**ONLINE GAMES** Use linear interpolation to find the percent of Americans that play online games for the following ages.

Percent of Americans Who Play Online Games					
Age	15	20	30	40	50
Percent	81	54	37	29	25

Source: Pew Internet & American Life Survey

**2A.** 35 year olds

**2B.** 18 year olds

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**Equations of Median-Fit Lines** A second type of best-fit line that can be found using a graphing calculator is a **median-fit line**. The equation of a median-fit line is calculated using the medians of the coordinates of the data points.



### Real-World Link

Paintball is more popular with 12- to 17-year-olds than any other age group. In 2002, 3,649,000 teenagers participated in paintball while 2,195,000 18- to 24-year-olds participated.

Source: *Statistical Abstract of the United States*

### EXAMPLE 3 Use a Median-Fit Line

**PAINTBALL** Find and graph the equation of a median-fit line for the data on the paintball tournament in Example 2. Then predict the score of the 15th ranked team.

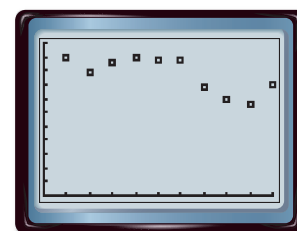
**Step 1** The data should be in the lists. Graph the scatter plot.

**Step 2** To find the median-fit equation, press the **STAT** key and select the **CALC** option. Scroll down to the **Med-Med** option and press **ENTER**. The value of  $a$  is the slope, and the value of  $b$  is the  $y$ -intercept.

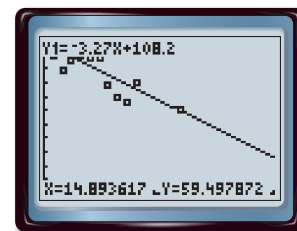
The equation for the median-fit line is  $y = -3.71x + 108.26$ .

**Step 3** Copy the equation to the  $Y=$  list and graph. Use the trace option until you reach  $x = 15$ .

The 15th place team scored about 60 points.



[0, 10] scl: 1 by [0, 110] scl: 10



[0, 10] scl: 1 by [0, 110] scl: 10

Notice that the equations for the regression line and the median-fit line are very similar.

### Check Your Progress

**3.** Use the data from Check Your Progress 2 and a median-fit line to estimate the numbers of 18- and 35-year-olds who play online games. Compare these values with the answers from the regression line.

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## Check Your Understanding

### Example 1 p. 253

1. **POTTERY** A local university is keeping track of the number of art students who use the pottery studio each day. Write an equation of the regression line for the data. Then find the correlation coefficient.

Students Throwing Pottery							
Day	1	2	3	4	5	6	7
Students	10	15	18	15	13	19	20

### Example 2 p. 254

2. **COMPUTERS** The table below shows the percent of Americans with a broadband connection at home in a recent year sorted by age. Use linear extrapolation and a regression equation to estimate the percentage of 15-year-olds with broadband at home.

Percentage of Americans with Broadband At Home						
Age	25	30	35	40	45	50
Percent	40	42	36	35	36	32

### Example 3 p. 255

3. **VACATION** The Smiths want to rent a house on the lake that sleeps eight people. The cost of the house per night is based on how close it is to the water.

Rental Properties							
Distance from Lake (mi)	0.0 (houseboat)	0.3	0.5	1.0	1.25	1.5	2.0
Price/Night (\$)	785	325	250	200	150	140	100

- a. Find and graph an equation for the median-fit line.  
b. What would you estimate is the cost of a rental 1.75 miles from the lake?

## Practice and Problem Solving

 = **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

### Examples 1 and 2 pp. 253–254

Write an equation of the regression line for the data in each table. Then find the correlation coefficient.

4. **SKYSCRAPERS** The table gives the rank and the number of stories of the ten tallest buildings in the world.

Tallest Buildings and Stories										
Rank	1	2	3	4	5	6	7	8	9	10
Stories	101	88	110	88	88	80	69	102	78	70

5. **MUSIC** The table gives the number of annual violin auditions held by a youth symphony each year since 2000.

Youth Symphony Violin Auditions							
Year	2000	2001	2002	2003	2004	2005	2006
Auditions	22	19	25	37	32	35	42

6. **RETAIL** The table gives the sales of jeans at a clothing chain since 2004.

Jeans Sales By Year						
Year	2004	2005	2006	2007	2008	2009
Sales (Millions of Dollars)	6.84	7.6	10.9	15.4	17.6	21.2

**Example 3**  
p. 255

- 7 MARATHON** The Boston Marathon has been run each year since 1897. The number of entrants every five years since 1975 is shown.

Year	1975	1980	1985	1990	1995	2000	2005
Entrants	2395	5417	5594	9412	9416	17,813	20,453

Source: Boston Athletic Association

- Find an equation for the median-fit line.
  - According to the equation, how many entrants were there in 2003?
- 8. CAMPING** A lakeside campground kept a record of the number of campsites rented over the week that includes July 4 for several years.

Campsites Rented July 4th Week						
Year	2002	2003	2004	2005	2006	2007
Sites Rented	34	45	42	53	58	47

- Find an equation for the regression line.
  - Predict the number of campsites that will be rented in 2010.
  - Predict the number of campsites that will be rented in 2020.
- 9. ICE CREAM** An ice cream company kept a count of the tubs of cookie dough ice cream delivered to each of their stores in a particular area.

Store Delivery of Cookie Dough Ice Cream					
Store Size (ft <sup>2</sup> )	2100	2225	3135	3569	4587
Tubs (hundreds)	110	102	215	312	265

- Find an equation for the median-fit line.
  - Graph the points and the median-fit line.
  - How many tubs would be delivered to a 1500-square-foot store? a 5000-square-foot store?
- 10. COLLEGE TESTING** The ACT is a college entrance exam that evaluates students' readiness to perform college-level work. The table below shows the number of participants who took the test in the given years.

Years	1990	1995	2000	2001	2002	2003	2004	2005	2006
Participants (thousands)	817	945	1065	1070	1116	1175	1171	1186	1206

Source: ACT

- Find an equation for the regression line.
  - According to the equation, how many participants were there in 1998?
  - How many students would you predict will participate in 2011?
- 11. RETAIL** The prices of the eight top-selling brands of jeans at Jeanie's Jeans are given in the table below.

Sales Rank	1	2	3	4	5	6	7	8
Price (\$)	43	44	50	61	64	135	108	78

- Find the equation for the regression line.
- According to the equation, what would be the price of a pair of the 12th best-selling brands of jeans?
- Is this a reasonable prediction? Explain.



**Real-World Link**

The average American eats 20 quarts of ice cream per year.

Source: Real Simple Magazine





**Math History Link**

**Florence Nightingale David (1909–1993)**

A renowned statistician born in Ivington, England, Florence Nightingale David received the first Elizabeth L. Scott Award for her “efforts in opening the door to women in statistics; ... for research contributions to ... statistical methods ...; and her spirit as a lecturer and as a role model.”

12. **STATE FAIRS** Opening day attendance at the North Carolina State Fair for 2000 was 53,331, for 2001 it was 47,940, for 2002 it was 54,036, for 2003 it was 61,364, for 2004 it was 61,289, and for 2005 it was 52,201.
- Construct a table of values for the given data.
  - Graph the points, and find a regression line.
  - Predict the attendance on opening day in 2012.

13. **MUSIC** The table shows the amount of money raised each year from concessions at the “Battle of the Bands” competition. Find the equation of the regression line and give the correlation coefficient to the nearest ten-thousandth place.

Battle of the Bands Concessions									
Year	1998	1999	2000	2001	2002	2003	2004	2005	2006
Amount Raised (\$)	1236	1560	1423	1740	2230	2563	3215	4517	4219

14. **FIREFIGHTERS** The table shows statistics on firefighters by age according to the U.S. Fire Administration.
- Find an equation for the median-fit line.
  - Graph the points and the median-fit line.
  - Does the median-fit line give you an accurate picture of the number of firefighters? Explain.

Age	Number of Firefighters
18	40,919
25	245,516
35	330,516
45	296,665
55	167,087
65	54,559

15. **ATHLETICS** The table shows the total number of teens who participated in high school athletics in various years.

Year Since 1970	1	10	20	30	35
Athletes (\$)	3,960,932	5,356,913	5,298,671	6,705,223	7,159,904

- Find an equation for the regression line.
  - According to the equation, how many participants were there in 1988?
16. **ART** An art auction house sold several paintings this season. A count was kept on the number of paintings that sold by the year in which they were painted.

Paintings Sorted by Year of Execution						
Year Painted	1950	1955	1960	1965	1970	1975
Paintings Sold	8	5	25	21	9	22

- Find the equation for the linear regression line.
  - How many paintings were sold that were painted in 1961?
  - Is the linear regression equation an accurate model of the data? Explain why or why not.
17. **SCHOOL** The table shows the average cost of a public 2-year technical school between 2000 and 2006.

Average cost of Public 2-year Institution						
Year	2000–2001	2001–2002	2002–2003	2003–2004	2004–2005	2005–2006
Cost (\$)	4839	5137	5601	6020	6375	6492

Source: U.S. Department of Education

- Find the equation of the median-fit line.
- Graph the points and the median-fit line.
- What would you estimate the cost to be in 2020–2021?

18. **SPACE EXPLORATION** As of 2006, the names of the space shuttles launched each year since 1993 are given below.

Space Shuttle Launches	
Year	Launches
1993	STS-61, STS-58, STS-51, STS-57, STS-55, STS-56, STS-54
1994	STS-66, STS-68, STS-64, STS-65, STS-59, STS-62, STS-60
1995	STS-74, STS-73, STS-69, STS-70, STS-71, STS-67, STS-63
1996	STS-80, STS-79, STS-78, STS-77, STS-76, STS-75, TSTS-72
1997	STS-87, STS-86, STS-85, STS-94, STS-84, STS-83, STS-82, STS-81
1998	STS-88, STS-95, STS-91, STS-90, STS-89
1999	STS-103, STS-93, STS-96
2000	STS-97, STS-92, STS-106, STS-101, STS-99
2001	STS-108, STS-105, STS-104, STS-100, STS-102, STS-98
2002	STS-113, STS-112, STS-111, STS-110, STS-109
2003	STS-107
2004	
2005	STS-114
2006	STS-116, STS-115, STS-121

- Construct a table of values for the data provided that could be used for graphing and linear regression.
- Find an equation for the regression line.
- Predict the number of launches there will be in 2015. Is your prediction reasonable? What influences might cause the actual number of flights to be different from your prediction? Explain.

### H.O.T. Problems

Use **H**igher-**O**rders **T**hinking Skills



#### Real-World Link

The World Superpipe Championship is held in Park City, Utah. This park is home to one of the largest half-pipes in the world, with walls 22 feet high.

Source: Park City Mountain Resort

19. **CHALLENGE** Below are the results of the World Superpipe Championships in 2008.

Men	Score	Rank	Women	Score
Shaun White	93.00	1	Torah Bright	96.67
Mason Aguirre	90.33	2	Kelly Clark	93.00
Janne Korpi	85.33	3	Soko Yamaoka	85.00
Luke Mitrani	85.00	4	Ellery Hollingsworth	79.33
Keir Dillion	81.33	5	Sophie Rodriguez	71.00

Find an equation of the regression line for each, and graph them on the same coordinate plane. Compare and contrast the men's and women's graphs.

- REASONING** For a class project, the scores that 10 randomly selected students earned on the first 8 tests of the school year are given. Explain how to find a line of best-fit for the data. Determine whether this line of best fit could be used to predict the scores of other students. Explain your reasoning.
- OPEN ENDED** For 10 different people, measure their heights and the lengths of their heads from chin to top. Use these data to generate a linear regression equation and a median-fit equation. Make a prediction using both of the equations.
- WRITING IN MATH** Using the data at the beginning of the lesson, describe the steps you would take to determine the number of appearances Spiderman will make in 2012.

**23. GEOMETRY** Sam is going to put a border around a poster.  $x$  represents the poster's width, and  $y$  represents the poster's length. Which equation represents how much border Sam will use if he doubles both the length and the width of the poster?

- A  $4xy$                       C  $4(x + y)$   
 B  $(x + y)^4$                 D  $16(x + y)$

**24. SHORT RESPONSE** Tatiana wants to run 5 miles at an average pace of 9 minutes per mile. After 4 miles, her average pace is 9 minutes 10 seconds. In how many minutes must she complete the final mile to reach her goal?

**25.** What is the slope of the line that passes through  $(1, 3)$  and  $(-3, 1)$ ?

- F  $-2$                               H  $\frac{1}{2}$   
 G  $-\frac{1}{2}$                             J  $2$

**26.** What is an equation of the line that passes through  $(0, 1)$  and has a slope of 3?

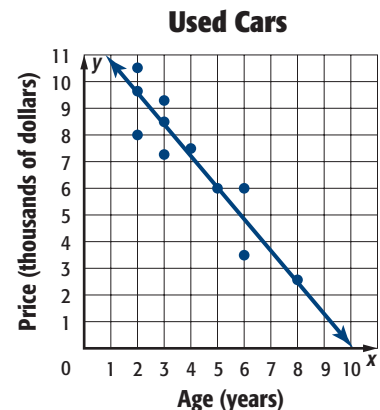
- A  $y = 3x - 1$   
 B  $y = 3x - 2$   
 C  $y = 3x + 4$   
 D  $y = 3x + 1$

### Spiral Review

**27. USED CARS** Gianna wants to buy a specific make and model of a used car. She researched prices from dealers and private sellers in her area and made the scatter plot and line of fit shown. (Lesson 4-5)

- Describe the relationship in the data.
- Use the line of fit to predict the price of a car that is 7 years old.
- Is it reasonable to use this line of fit to predict the price of a 10-year old car? Explain.

**28. GEOMETRY** A quadrilateral has sides with equations  $y = -2x$ ,  $2x + y = 6$ ,  $y = \frac{1}{2}x + 6$ , and  $x - 2y = 9$ . Is the figure a rectangle? Explain your reasoning. (Lesson 4-4)



Write each equation in standard form. (Lesson 4-3)

29.  $y - 2 = 3(x - 1)$                                       30.  $y - 5 = 6(x + 1)$   
 31.  $y + 2 = -2(x - 5)$                                     32.  $y + 3 = \frac{1}{2}(x + 4)$   
 33.  $y - 1 = \frac{2}{3}(x + 9)$                                     34.  $y + 3 = -\frac{1}{4}(x + 2)$

Find the slope of the line that passes through each pair of points. (Lesson 3-3)

35.  $(3, 4), (10, 8)$                       36.  $(-4, 7), (3, 5)$                       37.  $(3, 7), (-2, 4)$                       38.  $(-3, 2), (-3, 4)$

### Skills Review

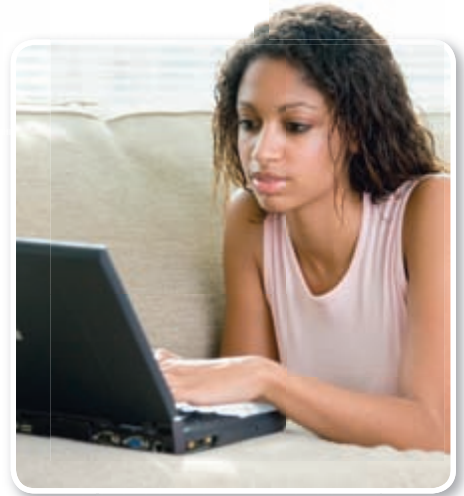
If  $f(x) = x^2 - x + 1$ , find each value. (Lesson 1-9)

39.  $f(-1)$                               40.  $f(5) - 3$                               41.  $f(a)$                                       42.  $f(b + 2)$

Graph each equation. (Lesson 3-1)

43.  $y = x + 2$                               44.  $x + 5y = 4$                               45.  $2x - 3y = 6$                               46.  $5x + 2y = 6$

## Special Functions



### Then

You wrote equations of lines that pass through a particular point, either parallel or perpendicular to a given line.

(Lesson 4-4)

### Now

- Identify and graph step functions.
- Identify and graph absolute value and piecewise-defined functions.



**HS-AT-S-PRF13** Students will graph linear, absolute value, quadratic and exponential functions and identify their key characteristics.  
**HS-AT-S-PRF14** Students will recognize and solve problems that can be modeled using linear, absolute value, quadratic or exponential functions. Also addresses HS-AT-S-VE012.

### New Vocabulary

- step function
- piecewise-linear function
- greatest integer function
- absolute value function
- piecewise-defined function



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- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

### Why?

Kim is ordering books from a Web site. The online store charges for shipping based on the dollar amount of the order. If the order is less than \$10, shipping costs \$3. If the order is more than \$10 but, less than \$20, it will cost \$5 to ship it.

**Step Functions** The graph of a **step function** is a series of line segments. Because each part of a step function is linear, this type of function is called a **piecewise-linear function**.

One example of a step function is the **greatest integer function**, written as  $f(x) = \llbracket x \rrbracket$ , where  $f(x)$  is the greatest integer not greater than  $x$ . For example,  $\llbracket 6.8 \rrbracket = 6$  because 6 is the greatest integer that is not greater than 6.8.



### Key Concept

### Greatest Integer Function

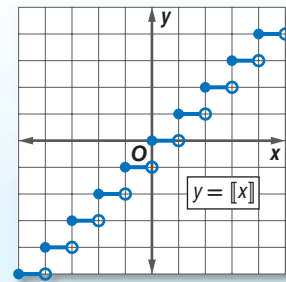
For Your FOLDABLE

**Parent function:**  $f(x) = \llbracket x \rrbracket$

**Type of graph:** disjointed line segments

**Domain:** all real numbers

**Range:** all integers

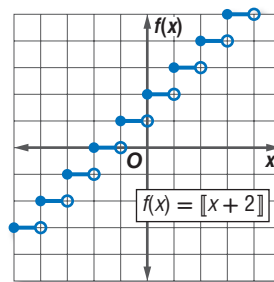


### EXAMPLE 1 Greatest Integer Function

Graph  $f(x) = \llbracket x + 2 \rrbracket$ . State the domain and range.

First, make a table of values. Select a few values between integers. On the graph, dots represent points that are included. Circles represent points not included.

$x$	$x + 2$	$\llbracket x + 2 \rrbracket$
0	2	2
0.25	2.25	2
0.5	2.5	2
1	3	3
1.25	3.25	3
1.5	3.5	3
2	4	4
2.25	4.25	4



Note that this is the graph of  $f(x) = \llbracket x \rrbracket$  shifted 2 units to the left.

Because the dots and circles overlap, the domain is all real numbers. The range is all integers.

### Check Your Progress

- Graph  $g(x) = 2\llbracket x \rrbracket$ . State the domain and range.

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Step functions can be used to represent many real-world situations involving money.

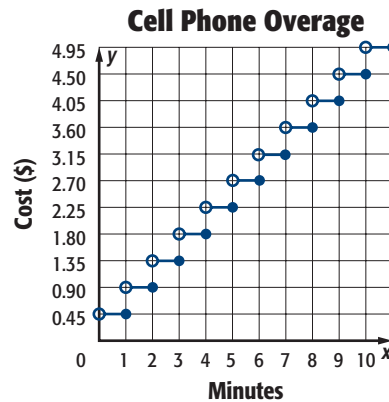


### Real-World EXAMPLE 2 Step Function

**CELL PHONE PLANS** Cell phone companies charge by the minute, not by the second. If you exceed the minutes in your plan, you have to pay overage fees. A cell phone company charges \$0.45 per minute or any fraction thereof for exceeding the number of minutes allotted on each plan. Draw a graph that represents this situation.

The total cost for the extra minutes will be a multiple of \$0.45, and the graph will be a step function. If the time is greater than 0 but less than or equal to 1 minute, the charge will be \$0.45. If the time is greater than 2 but is less than or equal to 3 minutes, you will be charged for 3 minutes or \$1.35.

$x$	$f(x)$
$0 < x \leq 1$	0.45
$1 < x \leq 2$	0.90
$2 < x \leq 3$	1.35
$3 < x \leq 4$	1.80
$4 < x \leq 5$	2.25
$5 < x \leq 6$	2.70
$6 < x \leq 7$	3.15



### Real-World Link

North Americans are the most likely to have cell phones with 93.2% of the population owning phones.

Source: IT Facts

### Check Your Progress

- PARKING GARAGES** A parking garage charges \$4 for the first hour and \$1 for each additional hour. Draw a graph that represents this situation.

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### Review Vocabulary

**absolute value** the distance a number is from zero on a number line; written  $|n|$  (Lesson 2-5)

**Absolute Value Functions** Another type of piecewise-linear function is the **absolute value function**. Recall that the absolute value of a number is always nonnegative. So in the absolute value parent function, written as  $f(x) = |x|$ , all of the values of the range are nonnegative.

### Key Concept

### Absolute Value Function

For Your FOLDABLE

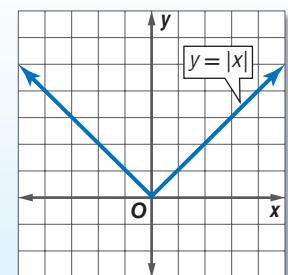
**Parent function:**  $f(x) = |x|$ , defined as

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$$

**Type of graph:** V-shaped

**Domain:** all real numbers

**Range:** all nonnegative real numbers



The absolute value function is called a **piecewise-defined function** because it is written using two or more expressions.

### StudyTip

#### Absolute Value of 0

Since the only number with an absolute value of 0 is 0, the absolute value bars may be removed in the second step of Example 3.

### EXAMPLE 3 Absolute Value Function

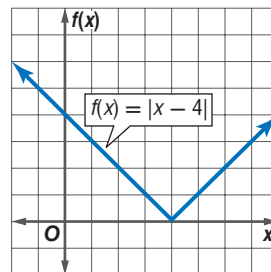
Graph  $f(x) = |x - 4|$ . State the domain and range.

Since  $f(x)$  cannot be negative, the minimum point of the graph is where  $f(x) = 0$ .

$$\begin{aligned} f(x) &= |x - 4| && \text{Original function} \\ 0 &= x - 4 && \text{Replace } f(x) \text{ with } 0. \\ 4 &= x && \text{Add 4 to each side.} \end{aligned}$$

Next make a table of values. Include values for  $x > 4$  and  $x < 4$ .

$f(x) =  x - 4 $	
$x$	$f(x)$
-2	6
0	4
2	2
4	0
5	1
6	2
7	3
8	4



The domain is all real numbers. The range is all real numbers greater than or equal to 0. Note that this is the graph of  $f(x) = |x|$  shifted 4 units to the right.

### Check Your Progress

3. Graph  $f(x) = |2x + 1|$ . State the domain and range.

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Not all piecewise-defined functions are absolute value functions. Step functions are also piecewise-defined functions. In fact, all piecewise-linear functions are piecewise-defined.

### StudyTip

#### Piecewise Functions

To graph a piecewise-defined function, graph each "piece" separately. There should be a dot or line that contains each member of the domain.

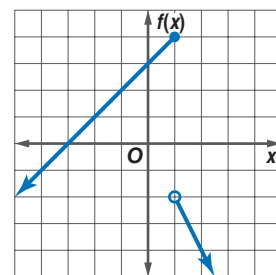
### EXAMPLE 4 Piecewise-Defined Function

Graph  $f(x) = \begin{cases} -2x & \text{if } x > 1 \\ x + 3 & \text{if } x \leq 1 \end{cases}$ . State the domain and range.

Graph the first expression. Create a table of values for when  $x > 1$ ,  $f(x) = -2x$  and draw the graph. Since  $x$  is not equal to 1, place a circle at  $(1, -2)$ .

Next, graph the second expression. Create a table of values for when  $x > 1$ ,  $f(x) = x + 3$  and draw the graph. Since  $x$  is equal to 1, place a dot at  $(1, 4)$ .

The domain is all real numbers. The range is  $\{y \mid y \leq 4\}$ .

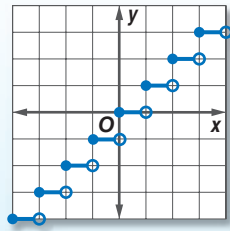


### Check Your Progress

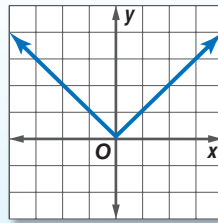
4. Graph  $f(x) = \begin{cases} 2x + 1 & \text{if } x > 0 \\ 3 & \text{if } x \leq 0 \end{cases}$ . State the domain and range.

[glencoe.com](http://glencoe.com)

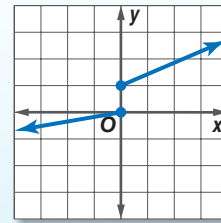
Step Function



Absolute Value Function



Piecewise-Defined Function



Check Your Understanding

**Example 1**  
p. 261

Graph each function. State the domain and range.

1.  $f(x) = \frac{1}{2} \lceil x \rceil$

2.  $g(x) = -\lfloor x \rfloor$

3.  $h(x) = \lceil 2x \rceil$

**Example 2**  
p. 262

4. **SHIPPING** Elan is ordering a gift for his dad from an online retailer. The table shows the shipping rates. Graph the step function to display the shipping rates.

Order Total (\$)	Shipping Cost (\$)
0–15	3.99
15.01–30	5.99
30.01–50	6.99
50.01–75	7.99
75.01–100	8.99
Over \$100	9.99

**Examples 3 and 4**  
p. 263

Graph each function. State the domain and range.

5.  $f(x) = |x - 3|$

6.  $g(x) = |2x + 4|$

7.  $f(x) = \begin{cases} 2x - 1 & \text{if } x > -1 \\ -x & \text{if } x \leq -1 \end{cases}$

8.  $g(x) = \begin{cases} -3x - 2 & \text{if } x > -2 \\ -x + 1 & \text{if } x \leq -2 \end{cases}$

Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.  
**Extra Practice** begins on page 815.

**Example 1**  
p. 261

Graph each function. State the domain and range.

9.  $f(x) = 3 \lceil x \rceil$

10.  $f(x) = \lfloor -x \rfloor$

11.  $g(x) = -2 \lceil x \rceil$

12.  $g(x) = \lfloor x \rfloor + 3$

13.  $h(x) = \lceil x \rceil - 1$

14.  $h(x) = \frac{1}{2} \lceil x \rceil + 1$

**Example 2**  
p. 262

15. **CAB FARES** Lauren wants to take a taxi from a hotel to a friend's house. The rate is \$3 plus \$1.50 per mile. Every fraction of a mile is rounded up to the next mile.

- a. Draw a graph to represent the cost of using a taxi cab.
- b. What is the cost if the trip is 8.5 miles long?

16. **POSTAGE RATES** The United States Postal Service increases the rate of postage periodically. The table shows the cost to mail a letter from 1988 through 2007. Draw a step graph to represent the data.

Year	1988	1991	1995	1999	2001	2002	2006	2007
Cost (\$)	0.25	0.29	0.32	0.33	0.34	0.37	0.39	0.41

### Examples 3 and 4

p. 263

Graph each function. State the domain and range.

17.  $f(x) = |2x - 1|$

19.  $g(x) = |-3x - 5|$

21.  $f(x) = \left| \frac{1}{2}x - 2 \right|$

23.  $g(x) = |x + 2| + 3$

25.  $f(x) = \begin{cases} \frac{1}{2}x - 1 & \text{if } x > 3 \\ -2x + 3 & \text{if } x \leq 3 \end{cases}$

27.  $f(x) = \begin{cases} 2x + 3 & \text{if } x \geq -3 \\ -\frac{1}{3}x + 1 & \text{if } x < -3 \end{cases}$

29.  $f(x) = \begin{cases} 3x + 2 & \text{if } x > -1 \\ -\frac{1}{2}x - 3 & \text{if } x \leq -1 \end{cases}$

18.  $f(x) = |x + 5|$

20.  $g(x) = |-x - 3|$

22.  $f(x) = \left| \frac{1}{3}x + 2 \right|$

24.  $g(x) = |2x - 3| + 1$

26.  $f(x) = \begin{cases} 2x - 5 & \text{if } x > 1 \\ 4x - 3 & \text{if } x \leq 1 \end{cases}$

28.  $f(x) = \begin{cases} 3x + 4 & \text{if } x \geq 1 \\ x + 3 & \text{if } x < 1 \end{cases}$

30.  $f(x) = \begin{cases} 2x + 1 & \text{if } x < -2 \\ -3x - 1 & \text{if } x \geq -2 \end{cases}$

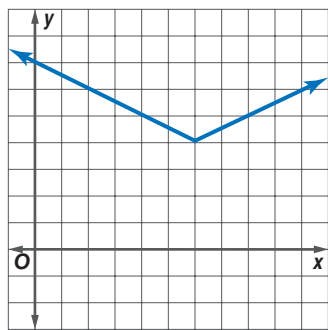
#### StudyTip

##### Nonlinear Functions

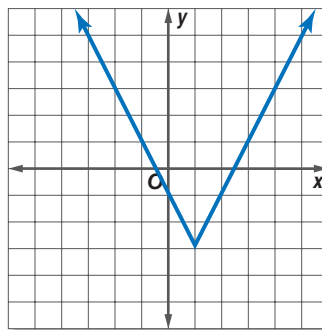
The greatest integer function, absolute value function, and piecewise defined functions are examples of nonlinear functions.

Determine the domain and range of each function.

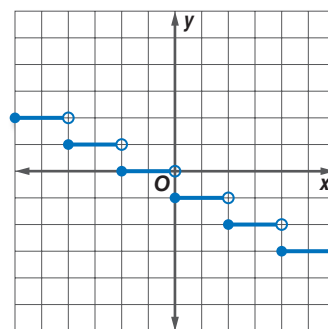
31.



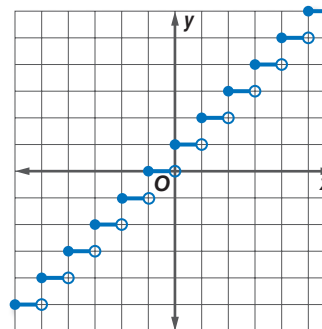
32.



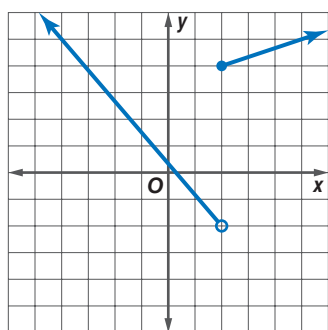
33.



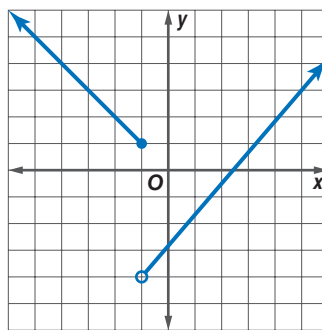
34.



35.



36.



37. **BOATING** According to Boat Minnesota, the maximum number of people that can safely ride in a boat is determined by the boat's length and width. The table shows some safety guidelines for a boat's length that is 6 feet wide. Graph this relation.

Length of Boat (ft)	18–19	20–22	23–24
Number of People	7	8	9



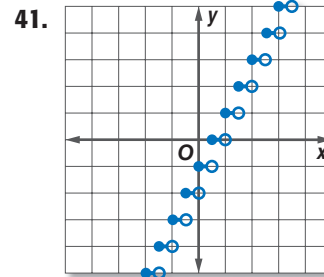
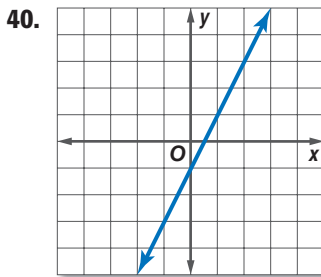
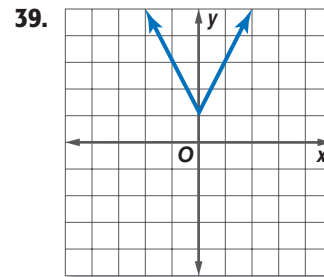
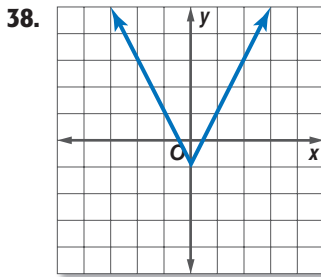
For Exercises 38–41, match each graph to one of the following equations.

**A**  
 $y = 2x - 1$

**B**  
 $y = \lceil 2x \rceil - 1$

**C**  
 $y = |2x| - 1$

**D**  
 $y = \begin{cases} 2x + 1 & \text{if } x > 0 \\ -2x + 1 & \text{if } x \leq 0 \end{cases}$



**Real-World Link**

Leasing a car differs from buying a car. When leasing a car, you may pay a lower monthly cost; however, you do not own the car at the end of the lease term. Most leasing agreements limit the number of miles you can drive the car before incurring additional costs.

Source: Federal Reserve

42. **CAR LEASE** Marcus is leasing his car. As part of his leasing agreement, he will be charged \$0.20 per mile for each mile over 12,000. Any fraction of a mile is rounded up to the next mile. Make a step graph to represent the cost of going over the mileage on the leasing agreement.
43. **BASEBALL** A baseball team is ordering T-shirts with the team logo on the front and the players' names on the back. A graphic design store charges \$10 to set up the artwork plus \$10 per shirt, \$4 each for the team logo, and \$2 to print the last name for an order of 12 shirts or less. For orders of 11–20 shirts, a 5% discount is given. For orders of more than 20 shirts, a 10% discount is given.
- Organize the information into a table. Include a column showing the total order price for each size order.
  - Write an equation for the piecewise relation representing the total price for an order of  $x$  shirts.
  - Graph the piecewise relation.
44. Consider the function  $f(x) = |2x + 3|$ .
- Make a table of values where  $x$  is all integers between  $-5$  and  $5$ .
  - Plot the points on a coordinate grid.
  - Graph the function.
45. Consider the function  $f(x) = |2x| + 3$ .
- Make a table of values where  $x$  is all integers between  $-5$  and  $5$ .
  - Plot the points on a coordinate grid.
  - Graph the function.
  - Describe how this graph is different from the graph in Exercise 44.



### Real-World Link

In addition to traditional dance classes like ballet and tap, more studios are offering classes in hip-hop, jazz/hip-hip, and cardio dance.

Source: Ariel Dance Studio

46. **DANCE CLASSES** A local dance studio must have at least 5 students enrolled in a class, or else the class will be canceled. Once 10 students are enrolled, a second class is started. Draw a graph that describes this situation.
47. **THEATERS** A certain theater will not have a show unless it has sold 50 tickets for that show. Once the capacity of 250 seats are sold, the theater begins selling tickets for the next show. Draw a graph that describes this situation.

Graph each function.

48.  $f(x) = \frac{1}{2}|x| + 2$       49.  $g(x) = \frac{1}{3}|x| + 4$       50.  $h(x) = -2|x - 3| + 2$   
 51.  $f(x) = -4|x + 2| - 3$       52.  $g(x) = -\frac{2}{3}|x + 6| - 1$       53.  $h(x) = -\frac{3}{4}|x - 8| + 1$

54. **MULTIPLE REPRESENTATIONS** In this problem, you will explore piecewise-defined functions.

a. **TABULAR** Copy and complete the table of values for  $f(x) = \lfloor |x| \rfloor$  and  $g(x) = \lceil |x| \rceil$ .

$x$	$\lfloor  x  \rfloor$	$f(x) = \lfloor  x  \rfloor$	$ x $	$g(x) = \lceil  x  \rceil$
-3	-3	3	3	3
-2.5				
-2				
0				
0.5				
1				
1.5				

- b. **GRAPHICAL** Graph each function on a coordinate plane.  
 c. **ANALYTICAL** Compare and contrast the graphs of  $f(x)$  and  $g(x)$ .

### H.O.T. Problems

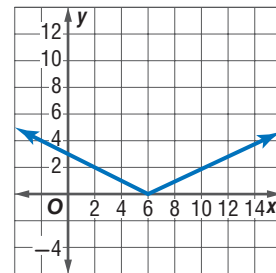
Use Higher-Order Thinking Skills

55. **REASONING** Determine whether the piecewise relation below represents a function. Explain why or why not.

$$y = \begin{cases} -2x + 4 & \text{if } x \geq 2 \\ -\frac{1}{2}x - 1 & \text{if } x \leq 4 \end{cases}$$

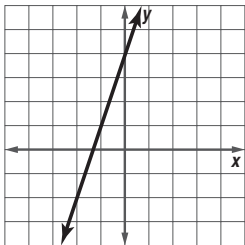
**CHALLENGE** Refer to the graph.

56. Write an absolute value function that represents the graph.  
 57. Write a piecewise function to represent the graph.  
 58. What are the domain and range of the graph?  
 59. **WRITING IN MATH** Refer to the information on cell phone plans in Example 2. Explain why the graph of this description is called a *step graph*.



60. **CHALLENGE** A bicyclist travels up and down a hill. The hill has a cross section that can be modeled by the equation  $y = -\frac{4}{3}|x - 300| + 400$  where  $x$  and  $y$  are measured in feet.
- If  $0 \leq x \leq 600$ , find the slope for the uphill portion of the trip and then the downhill portion of the trip.
  - Graph this function.
  - What are the domain and range of the graph?

61. Which equation represents a line that is perpendicular to the graph and passes through the point at  $(2, 0)$ ?

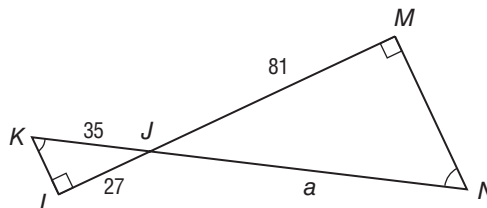


- A  $y = 3x - 6$
- B  $y = -3x + 6$
- C  $y = -\frac{1}{3}x + \frac{2}{3}$
- D  $y = \frac{1}{3}x - \frac{2}{3}$

62. A giant tortoise travels at a rate of 0.17 mile per hour. Which equation models the time  $t$  it would take the giant tortoise to travel 0.8 mile?

- F  $t = \frac{0.8}{0.17}$
- G  $t = (0.17)(0.8)$
- H  $t = \frac{0.17}{0.8}$
- J  $0.8 = \frac{0.17}{t}$

63. **GEOMETRY** If  $\triangle JKL$  is similar to  $\triangle JNM$  what is the value  $a$ ?



- A 62.5
- B 105
- C 125
- D 155.5

64. **GRIDDED RESPONSE** What is the difference in the value of  $2.1(x + 3.2)$ , when  $x = 5$  and when  $x = 3$ ?

**Spiral Review**

Write an equation of the regression line for the data in each table. (Lesson 4-6)

65.

$x$	1	3	5	7	9
$y$	3	8	15	18	21

66.

$x$	3	5	7	9	11
$y$	7.2	23.5	41.2	56.4	73.1

67.

$x$	1	2	3	4	5
$y$	21	33	39	54	64

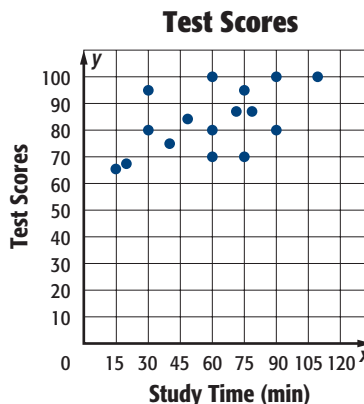
68.

$x$	2	4	6	8	10
$y$	1.4	2.4	2.9	3.3	4.2

69. **TESTS** Determine whether the graph at the right shows a *positive* correlation, a *negative* correlation, or *no* correlation. If there is a correlation, describe its meaning in the situation. (Lesson 4-5)

Suppose  $y$  varies directly as  $x$ . (Lesson 3-4)

- 70. If  $y = 2.5$  when  $x = 0.5$ , find  $y$  when  $x = 20$ .
- 71. If  $y = -6.6$  when  $x = 9.9$ , find  $y$  when  $x = 6.6$ .
- 72. If  $y = 2.6$  when  $x = 0.25$ , find  $y$  when  $x = 1.125$ .
- 73. If  $y = 6$  when  $x = 0.6$ , find  $x$  when  $y = 12$ .



**Skills Review**

Solve each equation. (Lesson 2-2)

- 74.  $104 = k - 67$
- 75.  $-4 + x = -7$
- 76.  $\frac{m}{7} = -11$
- 77.  $\frac{2}{3}p = 14$
- 78.  $-82 = n + 18$
- 79.  $\frac{9}{t} = -27$

# Graphing Technology Lab

## Piecewise-Linear Functions



HS-AT-S-PRF13 Students will graph linear, absolute value, quadratic and exponential functions and identify their key characteristics. Also addresses HS-AT-S-PRF14.

You can use a graphing calculator to graph and analyze various piecewise functions, including greatest integer functions and absolute value functions.

### ACTIVITY 1 Greatest Integer Functions

Graph  $f(x) = \llbracket x \rrbracket$  in the standard viewing window.

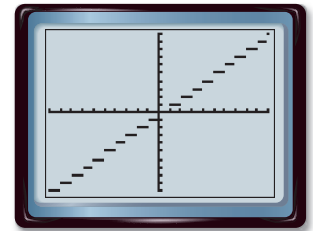
The calculator may need to be changed to dot mode for the function to graph correctly. Press **MODE** then use the arrow and **ENTER** keys to select **DOT**.

Enter the equation in the **Y=** list. Then graph the equation.

**KEYSTROKES:** **Y=** **MATH** **▶** 5 **X,T,θ,n** **)** **Zoom** 6

**1A.** How does the graph of  $f(x) = \llbracket x \rrbracket$  compare to the graph of  $f(x) = x$ ?

**1B.** What are the domain and range of the function  $f(x) = \llbracket x \rrbracket$ ? Explain.



$[-10, 10]$  scl: 1 by  $[-10, 10]$  scl: 1

The graphs of piecewise functions are affected by changes in parameters.

### ACTIVITY 2 Absolute Value Functions

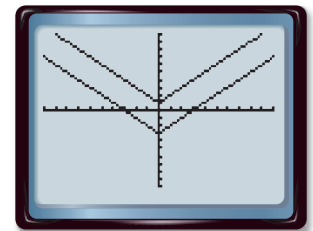
Graph  $y = |x| - 3$  and  $y = |x| + 1$  in the standard viewing window.

Enter the equations in the **Y=** list. Then graph.

**KEYSTROKES:** **Y=** **MATH** **▶** 1 **X,T,θ,n** **)** **-** 3 **ENTER** **MATH** **▶** 1  
**X,T,θ,n** **)** **+** 1 **Zoom** 6

**2A.** Compare and contrast the graphs to the graph of  $y = |x|$ .

**2B.** How does the value of  $c$  affect the graph of  $y = |x| + c$ ?



$[210, 10]$  scl: 1 by  $[210, 10]$  scl: 1

### Analyze The Results

1. A parking garage charges \$4 for every hour or fraction of an hour. Is this situation modeled by a *linear* function or a *step* function? Explain your reasoning.
2. A maintenance technician is testing an elevator system. The technician starts the elevator at the fifth floor of a building. It is sent to the ground floor, then back to the fifth floor. Assume the elevator travels at a constant rate. Should the height of the elevator be modeled by a step function or an absolute value function? Explain your reasoning.
3. **MAKE A CONJECTURE** Explain why the greatest integer function is sometimes called the *floor function*.
4. Graph  $y = -|x|$  in the standard viewing window. How is this graph related to the graph of  $y = |x|$ ?
5. **MAKE A CONJECTURE** Describe the transformation of the parent graph to  $y = |x + c|$ . Use a graphing calculator with different values of  $c$  to test your conjecture.

## Chapter Summary

### Key Concepts

#### Slope-Intercept Form (Lessons 4-1 and 4-2)

- The slope-intercept form of a linear equation is  $y = mx + b$ , where  $m$  is the slope and  $b$  is the  $y$ -intercept.
- If you are given two points through which a line passes, use them to find the slope first.

#### Point-Slope Form (Lesson 4-3)

- The linear equation  $y - y_1 = m(x - x_1)$  is written in point-slope form, where  $(x_1, y_1)$  is a given point on a non-vertical line and  $m$  is the slope of the line.

#### Parallel and Perpendicular Lines (Lesson 4-4)

- Parallel lines have the same slope.
- Lines that intersect at right angles are called perpendicular lines. The slopes of perpendicular lines are opposite reciprocals.

#### Scatter Plots and Lines of Fit (Lesson 4-5)

- Data with two variables are called bivariate data.
- A scatter plot is a graph in which two sets of data are plotted as ordered pairs in a coordinate plane.

#### Regression and Median-Fit Lines (Lesson 4-6)

- A graphing calculator can be used to find regression lines and median-fit lines.

#### Special Functions (Lesson 4-7)

- The greatest integer function is written as  $f(x) = \llbracket x \rrbracket$ , where  $f(x)$  is the greatest integer not greater than  $x$ .
- The absolute value function is written as  $f(x) = |x|$ , where  $f(x)$  is the distance between  $x$  and 0 on a number line.

## FOLDABLES® Study Organizer

Be sure the Key Concepts are noted in your Foldable.



## Key Vocabulary

<b>absolute value function</b> (p. 262)	<b>median-fit line</b> (p. 255)
<b>best-fit line</b> (p. 253)	<b>parallel lines</b> (p. 237)
<b>bivariate data</b> (p. 245)	<b>perpendicular lines</b> (p. 238)
<b>correlation coefficient</b> (p. 253)	<b>piecewise-defined function</b> (p. 262)
<b>greatest integer function</b> (p. 261)	<b>piecewise-linear function</b> (p. 261)
<b>linear extrapolation</b> (p. 224)	<b>point-slope form</b> (p. 231)
<b>linear interpolation</b> (p. 247)	<b>scatter plot</b> (p. 245)
<b>linear regression</b> (p. 253)	<b>slope-intercept form</b> (p. 214)
<b>line of fit</b> (p. 246)	<b>step function</b> (p. 261)

## Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. The  $y$ -intercept is the point where the graph crosses the  $y$ -axis.
2. The process of using a linear equation to make predictions about values that are beyond the range of the data is called linear regression.
3. A graph in which two sets of data are plotted as ordered pairs in a coordinate plane is called a step function.
4. The correlation coefficient describes whether the correlation between the variables is positive or negative and how closely the equation is modeling the data.
5. Lines in the same plane that do not intersect are called parallel lines.
6. Lines that intersect at acute angles are called perpendicular lines.
7. A function that is defined differently for different parts of its domain is called a piecewise-defined function.
8. The range of the greatest integer function is the set of all real numbers.
9. A piecewise-linear function is also called a step function.

## Lesson-by-Lesson Review

### 4-1 Graphing Equations in Slope-Intercept Form (pp. 214–221)

HS-G-S-CG2,  
HS-AT-S-EI11

Write an equation of a line in slope-intercept form with the given slope and  $y$ -intercept. Then graph the equation.

10. slope: 3,  $y$ -intercept: 5
11. slope:  $-2$ ,  $y$ -intercept:  $-9$
12. slope:  $\frac{2}{3}$ ,  $y$ -intercept: 3
13. slope:  $-\frac{5}{8}$ ,  $y$ -intercept:  $-2$

Graph each equation.

14.  $y = 4x - 2$
15.  $y = -3x + 5$
16.  $y = \frac{1}{2}x + 1$
17.  $3x + 4y = 8$

18. **SKI RENTAL** A local ski slope charges \$5 per hour to rent ski equipment. The cost of a lift ticket is \$15. Write an equation in slope-intercept form for the total cost of skiing for  $h$  hours.

### EXAMPLE 1

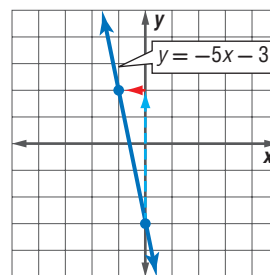
Write an equation of a line in slope-intercept form with slope  $-5$  and  $y$ -intercept  $-3$ . Then graph the equation.

$$y = mx + b \quad \text{Slope-intercept form}$$

$$y = -5x + (-3) \quad m = -5 \text{ and } b = -3$$

$$y = -5x - 3 \quad \text{Simplify.}$$

To graph the equation, plot the  $y$ -intercept  $(0, -3)$ . Then move up 5 units and left 1 unit. Plot the point. Draw a line through the two points.



### 4-2 Writing Equations in Slope-Intercept Form (pp. 224–230)

HS-AT-S-EI11,  
HS-AT-S-EI17

Write an equation of the line that passes through the given point and has the given slope.

19.  $(1, 2)$ , slope 3
20.  $(2, -6)$ , slope  $-4$
21.  $(-3, -1)$ , slope  $\frac{2}{5}$
22.  $(5, -2)$ , slope  $-\frac{1}{3}$

Write an equation of the line that passes through the given points.

23.  $(2, -1)$ ,  $(5, 2)$
24.  $(-4, 3)$ ,  $(1, 13)$
25.  $(3, 5)$ ,  $(5, 6)$
26.  $(2, 4)$ ,  $(7, 2)$
27. **CAMP** In 2000, a camp had 450 campers. Five years later, the number of campers rose to 750. Write a linear equation that represents the number of campers that attend camp.

### EXAMPLE 2

Write an equation of the line that passes through  $(3, 2)$  with a slope of 5.

**Step 1** Find the  $y$ -intercept.

$$y = mx + b \quad \text{Slope-intercept form}$$

$$2 = 5(3) + b \quad m = 5, y = 2, \text{ and } x = 3$$

$$2 = 15 + b \quad \text{Simplify.}$$

$$-13 = b \quad \text{Subtract 15 from each side.}$$

**Step 2** Write the equation in slope-intercept form.

$$y = mx + b \quad \text{Slope-intercept form}$$

$$y = 5x - 13 \quad m = 5 \text{ and } b = -13$$

**4-3** Point-Slope Form (pp. 231–236)
 HS-AT-S-EI11,  
HS-AT-S-EI17

Write an equation in point-slope form for the line that passes through the given point with the slope provided.

28. (6, 3), slope 5  
 29. (–2, 1), slope –3  
 30. (–4, 2), slope 0

Write each equation in standard form.

31.  $y - 3 = 5(x - 2)$   
 32.  $y - 7 = -3(x + 1)$   
 33.  $y + 4 = \frac{1}{2}(x - 3)$   
 34.  $y - 9 = -\frac{4}{5}(x + 2)$

Write each equation in slope-intercept form.

35.  $y - 2 = 3(x - 5)$   
 36.  $y - 12 = -2(x - 3)$   
 37.  $y + 3 = 5(x + 1)$   
 38.  $y - 4 = \frac{1}{2}(x + 2)$

**EXAMPLE 3**

Write an equation in point-slope form for the line that passes through (3, 4) with a slope of –2.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

$$y - 4 = -2(x - 3) \quad \text{Replace } m \text{ with } -2 \text{ and } (x_1, y_1) \text{ with } (3, 4).$$

**EXAMPLE 4**

Write  $y + 6 = -4(x - 3)$  in standard form.

$$y + 6 = -4(x - 3) \quad \text{Original equation}$$

$$y + 6 = -4x + 12 \quad \text{Simplify.}$$

$$4x + y + 6 = 12 \quad \text{Add } 4x \text{ to each side.}$$

$$4x + y = 6 \quad \text{Subtract } 6 \text{ from each side.}$$

**4-4** Parallel and Perpendicular Lines (pp. 237–243)
 HS-G-S-CG1,  
HS-AT-S-EI11

Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of each equation.

39. (2, 5),  $y = x - 3$   
 40. (0, 3),  $y = 3x + 5$   
 41. (–4, 1),  $y = -2x - 6$   
 42. (–5, –2),  $y = -\frac{1}{2}x + 4$

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the given equation.

43. (2, 4),  $y = 3x + 1$   
 44. (1, 3),  $y = -2x - 4$   
 45. (–5, 2),  $y = \frac{1}{3}x + 4$   
 46. (3, 0),  $y = -\frac{1}{2}x$

**EXAMPLE 5**

Write an equation in slope-intercept form for the line that passes through (–2, 4) and is parallel to the graph of  $y = 6x - 3$ .

The slope of the line with equation  $y = 6x - 3$  is 6. The line parallel to  $y = 6x - 3$  has the same slope, 6.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

$$y - 4 = 6(x - (-2)) \quad \text{Substitute.}$$

$$y - 4 = 6(x + 2) \quad \text{Simplify.}$$

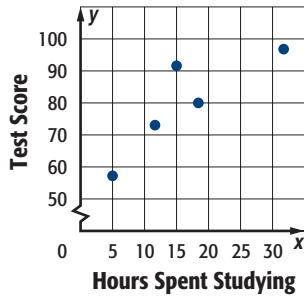
$$y - 4 = 6x + 12 \quad \text{Distributive Property}$$

$$y = 6x + 16 \quad \text{Add } 4 \text{ to each side.}$$

**4-5 Scatter Plots and Lines of Fit** (pp. 245–252)

HS-DAP-S-DR4  
HS-DAP-S-CDS5

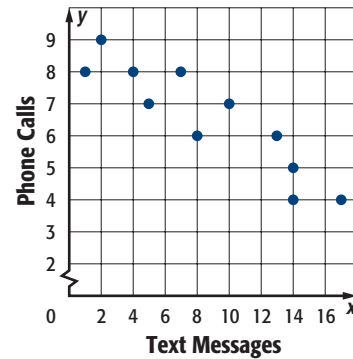
47. Determine whether the graph shows a *positive* correlation, a *negative* correlation, or *no* correlation. If there is a positive or negative correlation, describe its meaning in the situation.



48. **ATTENDANCE** A scatter plot of data compares the number of years since a business has opened and its annual number of sales. It contains the ordered pairs (2, 650) and (5, 1280). Write an equation in slope-intercept form for the line of fit for this situation.

**EXAMPLE 6**

The scatter plot displays the number of text messages and the number of phone calls made daily. Write an equation for the line of fit.



First, find the slope using (2, 9) and (17, 4).

$$m = \frac{4 - 9}{17 - 2} = \frac{-5}{15} \text{ or } -\frac{1}{3} \quad \text{Substitute and simplify.}$$

Then find the *y*-intercept.

$$9 = -\frac{1}{3}(2) + b \quad \text{Substitute.}$$

$$9\frac{2}{3} = b \quad \text{Add } \frac{2}{3} \text{ to each side.}$$

Write the equation.  $y = -\frac{1}{3}x + 9\frac{2}{3}$

**4-6 Regression and Median-Fit Lines** (pp. 253–260)

HS-DAP-S-CDS4  
HS-DAP-S-CDS5

49. **OUTERWEAR SALE** The table shows the number of sales made at a outerwear store during a sale. Write an equation of the regression line. Then estimate the daily sales on day 10 of the sale.

Days Since Sale Began	1	2	3	4	5	6	7
Daily Sales (\$)	15	21	32	30	40	38	51

50. **MOVIES** The table shows ticket sales during the first week after a movie opens. Write an equation of the regression line. Then estimate the daily ticket sales on the 15th day after a movie opens.

Days Since Movie Opened	1	2	3	4	5	6	7
Daily Ticket Sales (\$)	85	92	89	78	65	68	55

**EXAMPLE 7**

**ATTENDANCE** The table shows the annual attendance at an amusement park. Write an equation of the regression line for the data.

Year (since 2000)	0	1	2	3	4	5	6
Attendance (thousands)	75	80	72	68	65	60	53

**Step 1** Enter the data by pressing  $\boxed{2\text{nd}} \text{ [STAT]}$  and selecting the **Edit** option.

**Step 2** Perform the regression by pressing  $\boxed{2\text{nd}} \text{ [STAT]}$  and selecting the **CALC** option. Scroll down to **LinReg** ( $ax + b$ ) and press  $\boxed{\text{ENTER}}$ .

**Step 3** Write the equation of the regression line by rounding the *a*- and *b*-values on the screen.  
 $y = -4.04x + 79.68$



## 4-7 Special Functions (pp. 261–268)

HS-AT-S-PRF13  
HS-AT-S-PRF14

Graph each function. State the domain and range.

51.  $f(x) = \lceil x \rceil$

52.  $f(x) = \lfloor 2x \rfloor$

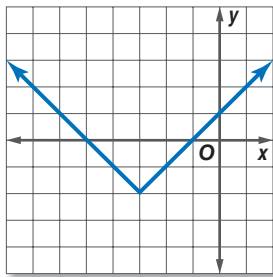
53.  $f(x) = |x|$

54.  $f(x) = |2x - 2|$

55.  $f(x) = \begin{cases} x - 2 & \text{if } x < 1 \\ 3x & \text{if } x \geq 1 \end{cases}$

56.  $f(x) = \begin{cases} 2x - 3 & \text{if } x \leq 2 \\ x + 1 & \text{if } x > 2 \end{cases}$

57. Determine the domain and range of the function graphed below.



## EXAMPLE 8

Graph  $f(x) = |x + 3|$ . State the domain and range.

Since  $f(x)$  cannot be negative, the minimum point of the graph is where  $f(x) = 0$ .

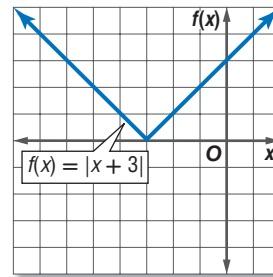
$$f(x) = |x + 3| \quad \text{Original function}$$

$$0 = x + 3 \quad \text{Replace } f(x) \text{ with } 0.$$

$$-3 = x \quad \text{Subtract 3 from each side.}$$

Next, make a table of values. Include values for  $x > -3$  and  $x < -3$ .

$x$	-5	-4	-3	-2	-1
$f(x)$	2	1	0	1	2



The domain is all real numbers, and the range is  $\{y \mid y \geq 0\}$ .



- Graph the equation  $y = 2x - 3$ .
- MULTIPLE CHOICE** A popular pizza parlor charges \$12 for a large cheese pizza plus \$1.50 for each additional topping. Write an equation in slope-intercept form for the total cost  $C$  of a pizza with  $t$  toppings.
  - $C = 12t + 1.50$
  - $C = 13.50t$
  - $C = 12 + 1.50t$
  - $C = 1.50t - 12$

Write an equation of a line in slope-intercept form that passes through the given point and has the given slope.

- $(-4, 2)$ ; slope  $-3$
- $(3, -5)$ ; slope  $\frac{2}{3}$

Write an equation of the line in slope-intercept form that passes through the given points.

- $(1, 4)$ ,  $(3, 10)$
- $(2, 5)$ ,  $(-2, 8)$
- $(0, 4)$ ,  $(-3, 0)$
- $(7, -1)$ ,  $(9, -4)$

- PAINTING** The data in the table show the size of a room in square feet and the length of time it takes to paint the room in minutes.

Room Size	100	150	200	400	500
Painting Time	160	220	270	500	680

- Use the points  $(100, 160)$  and  $(500, 680)$  to write the slope-intercept form of an equation for the line of fit.
  - Predict the amount of time required to paint a room measuring 750 square feet.
- SALARY** The table shows the relationship between years of on-the-job experience and salary for jobs in the field of teaching.

Years Experience	1	5	10	15	20
Salary (thousands of dollars)	28	31	42	49	64

- Write an equation for the best-fit line.
- Find the correlation coefficient and explain what it tells us about the relationship between years of experience and salary.

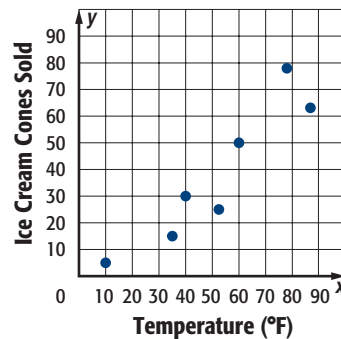
Write an equation in slope-intercept form for the line that passes through the given point and is parallel to the graph of each equation.

- $(2, -3)$ ,  $y = 4x - 9$
- $(-5, 1)$ ,  $y = -3x + 2$

Write an equation in slope-intercept form for the line that passes through the given point and is perpendicular to the graph of the equation.

- $(1, 4)$ ,  $y = -2x + 5$
- $(-3, 6)$ ,  $y = \frac{1}{4}x + 2$

- MULTIPLE CHOICE** The scatter plot shows data concerning relationship between outside temperature and daily ice cream cone sales. What type of correlation is shown in the plot?



- positive correlation
- negative correlation
- no correlation
- not enough information

Graph each function.

- $f(x) = |x - 1|$
- $f(x) = -|2x|$
- $f(x) = \lceil x \rceil$
- $f(x) = \begin{cases} 2x - 1 & \text{if } x < 2 \\ x - 3 & \text{if } x \geq 2 \end{cases}$
- The table shows the number of children from Russia adopted by U.S. citizens.

Years Since 2000	0	1	2	3	4
Number of Children	4269	4279	4939	5209	6936

- Write the slope intercept form of the equation for the line of fit.
- Predict the number of children from Russia who will be adopted in 2025.

## Short Answer Questions

Short answer questions require you to provide a solution to the problem, along with a method, explanation, and/or justification used to arrive at the solution.



### Strategies for Solving Short Answer Questions

#### Step 1

Short answer questions are typically graded using a **rubric**, or a scoring guide. The following is an example of a short answer question scoring rubric.

Scoring Rubric	
Criteria	Score
<b>Full Credit:</b> The answer is correct and a full explanation is provided that shows each step.	2
<b>Partial Credit:</b> <ul style="list-style-type: none"> <li>The answer is correct, but the explanation is incomplete.</li> <li>The answer is incorrect, but the explanation is correct.</li> </ul>	1
<b>No Credit:</b> Either an answer is not provided or the answer does not make sense.	0

#### Step 2

In solving short answer questions, remember to...

- explain your reasoning or state your approach to solving the problem.
- show all of your work or steps.
- check your answer if time permits.

#### EXAMPLE

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

The table shows production costs for building different numbers of skateboards. Determine the missing value,  $x$ , that will result in a linear model.

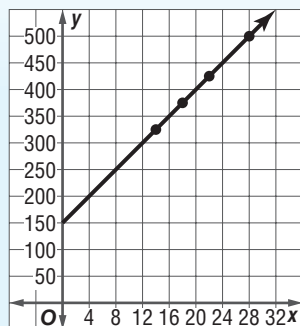
Skateboards Built	Production Costs
14	\$325
28	\$500
$x$	\$375
22	\$425

Read the problem carefully. You are given several data points and asked to find the missing value that results in a linear model.

Example of a 2-point response:

Set up a coordinate grid and plot the three given points:  
(14, 325), (28, 500), (22, 425).

Then draw a straight line through them and find the  $x$ -value that produces a  $y$ -value of 375.



So, building 18 skateboards would result in production costs of \$375. These data form a linear model.

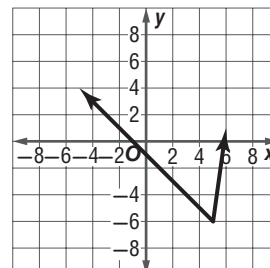
The steps, calculations, and reasoning are clearly stated. The student also arrives at the correct answer. So, this response is worth the full 2 points.

## Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

- Given points  $M(-1, 7)$ ,  $N(3, -5)$ ,  $O(6, 1)$ , and  $P(-3, -2)$ , determine two segments that are perpendicular to each other.
- Write the equation of a line that is parallel to  $4x + 2y = 8$  and has a  $y$ -intercept of 5.

- Three vertices of a quadrilateral are shown on the coordinate grid. Determine a fourth vertex that would result in a trapezoid.





### Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. A shipping company charges \$8.50 to ship packages that weigh up to 1 pound and \$7.25 for each additional pound. Which of the following piecewise-defined functions represents the cost  $C$  of shipping a package that weighs  $p$  pounds?

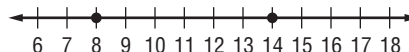
**Shipping Rates**

- ◆ Up to 1 pound: **\$8.50**
- ◆ Each additional pound: **\$7.25**



- A  $C(p) = \begin{cases} 8.50 & p \leq 1 \\ 8.50 + 7.25p & p > 1 \end{cases}$
- B  $C(p) = \begin{cases} 7.25 & p \leq 1 \\ 7.25 + 8.50p & p > 1 \end{cases}$
- C  $C(p) = \begin{cases} 8.50 & p \leq 1 \\ 8.50p + 7.25(p - 1) & p > 1 \end{cases}$
- D  $C(p) = \begin{cases} 8.50 & p \leq 1 \\ 8.50 + 7.25(p - 1) & p > 1 \end{cases}$
2. Refer to the information given in Exercise 2. How much would it cost a customer to ship a package that weighs 5 pounds 11 ounces? (Assume that partial pounds are rounded up to the nearest whole pound.)
- F \$42.35                      H \$48.20
- G \$44.75                      J \$52.00
3. Jaime bought a car in 2005 for \$28,500. By 2008, the car was worth \$23,700. Based on a linear model, what will the value of the car be in 2012?
- A \$17,300                      C. \$18,100
- B \$17,550                      D. \$18,475

4. If the graph of a line has a positive slope and a negative  $y$ -intercept, what happens to the  $x$ -intercept if the slope and the  $y$ -intercept are doubled?
- F The  $x$ -intercept becomes four times larger.
- G The  $x$ -intercept becomes twice as large.
- H The  $x$ -intercept becomes one-fourth as large.
- J The  $x$ -intercept remains the same.
5. Which absolute value equation has the graph below as its solution?



- A  $|x - 3| = 11$
- B  $|x - 4| = 12$
- C  $|x - 11| = 3$
- D  $|x - 12| = 4$
6. The table below shows the relationship between certain temperatures in degrees Fahrenheit and degrees Celsius. Which of the following linear equations correctly models this relationship?
- F  $F = \frac{8}{5}C + 35$
- G  $F = \frac{4}{5}C + 42$
- H  $F = \frac{9}{5}C + 32$
- J  $F = \frac{12}{5}C + 26$

Celsius (C)	Fahrenheit (F)
10°	50°
15°	59°
20°	68°
25°	77°
30°	86°

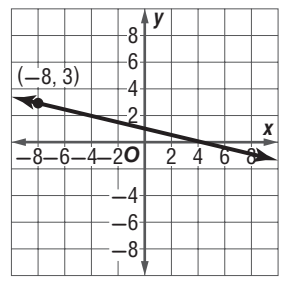
### Test-Taking Tip

**Question 3** Find the average annual depreciation between 2005 and 2008. Then extend the pattern to find the car's value in 2012.

## Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

7. What is the equation of the line graphed below?



Express your answer in point slope form using the point  $(-8, 3)$ .

8. **GRIDDED RESPONSE** The linear equation below is a best fit model for the peak depth of the Mad River when  $x$  inches of rain fall. What would you expect the peak depth of the river to be after a storm that produces  $1\frac{3}{4}$  inches of rain? Round your answer to the nearest tenth of a foot if necessary.

$$y = 2.5x + 14.8$$

9. Jacob formed an advertising company in 1992. Initially, the company only had 14 employees. In 2008, the company had grown to a total of 63 employees. Find the percent of change in the number of employees working at Jacob's company. Round to the nearest tenth of a percent if necessary.

10. The table shows the total amount of rain during a storm.

Hour	1	2	3	4
Inches	0.45	0.9	1.35	1.8

- a. Write an equation to fit the data in the table.
  - b. Describe the relationship between the hour and the amount of rain received.
11. An electrician charges a \$25 consultation fee plus \$35 per hour for labor.

a. Copy and complete the following table showing the charges for jobs that take 1, 2, 3, 4, or 5 hours.

Hours, $h$	Total Cost, $C$
1	
2	
3	
4	
5	

- b. Write an equation in slope-intercept form for the total cost of a job that takes  $h$  hours.
- c. If the electrician bills in quarter hours, how much would it cost for a job that takes 3 hours 15 minutes to complete?

## Extended Response

Record your answer on a sheet of paper. Show your work.

12. Explain how you can determine whether two lines are parallel or perpendicular.

Need Extra Help?												
If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12
Go to Lesson or Page...	4-7	4-7	4-5	3-1	2-5	4-2	4-3	4-5	2-7	3-6	4-2	4-4
	MA- HS- 5.1.2	MA- HS- 5.1.2	MA- HS- 4.1.1	MA- HS- 3.3.1	MA- HS- 5.3.1	MA- HS- 5.1.3	MA- HS- 5.3.3	MA- HS- 4.1.1	MA- HS- 1.4.1	MA- HS- 5.1.3	MA- HS- 5.1.5	MA- HS- 5.3.3

