Linear Inequalities

Then

In Chapter 2, you solved equations.

Now

In Chapter 5, you will:

- Solve one-step and multi-step inequalities.
- Solve compound inequalities and inequalities involving absolute value.
- Graph inequalities in two variables.

KY Program of Studies

HS-AT-S-EI3 Students will solve one-variable inequalities using manipulatives, symbols, procedures and graphing, including graphing the solution set on a number line.
HS-AT-S-EI4 Students will solve linear inequalities in one variable including those involving the absolute value of a linear function.

Why?

PETS In the United States, about 75 million dogs are kept as pets. Approximately 16% of these were adopted from animal shelters. About 14% of dog owners own more than 3 dogs.

Math in Motion, Animation glencoe.com

Get Ready for Chapter 5

Diagnose Readiness You have two options for checking Prerequisite Skills.

*Quick*Check

Evaluate each expression for the given values. (Lesson 1-2)

1. $3x + y$ if $x = -4$ and $y = 2$

2. $-2m + 3k$ if $m = -8$ and $k = 3$

3. **CARS** The expression $\frac{m \text{ mi}}{g \text{ gal}}$ represents the gas mileage of a car. Find the gas mileage of a car that goes 295 miles on 12 gallons of gasoline. Round to the nearest tenth.

Solve each equation. (Lesson 2-2)

4. $x - 4 = 9$

5. $x + 8 = -3$

6. $4x = -16$

7. $\frac{x}{3} = 7$

8. $2x + 1 = 9$

9. $4x - 5 = 15$

10. $9x + 2 = 3x - 10$

11. $3(x - 2) = -2(x + 13)$

12. **SAVINGS** Claudia opened a savings account with $325. She saves $100 per month. Write an equation to determine how much money d, she has put in her savings account after m months. (Lesson 2-1)

Solve each equation. (Lesson 2-5)

13. $|x + 11| = 18$

14. $|3x - 2| = 16$

15. **SURVEYS** In a survey, 32% of the people surveyed chose pizza as their favorite food. The results were reported to within 2% accuracy. What is the maximum and minimum percent of people who chose pizza? (Lesson 2-5)

*Quick*Review

EXAMPLE 1

Evaluate $-3x^2 + 4x - 6$ if $x = -2$.

$-3x^2 + 4x - 6$	**Original expression**
$= -3(-2)^2 + 4(-2) - 6$	**Replace x with -2.**
$= -3(4) + 4(-2) - 6$	**Evaluate exponent.**
$= -12 + (-8) - 6$	**Multiply.**
$= -26$	**Add and subtract.**

EXAMPLE 2

Solve $-2(x - 4) = 7x - 19$.

$-2(x - 4) = 7x - 19$	**Original equation**
$-2x + 8 = 7x - 19$	**Distributive Property**
$-2x + 8 + 2x = 7x - 19 + 2x$	**Add 2x.**
$8 = 9x - 19$	**Simplify.**
$8 + 19 = 9x - 19 + 19$	**Add 19.**
$27 = 9x$	**Simplify.**
$3 = x$	**Divide by 3.**

EXAMPLE 3

Solve $|x - 4| = 9$.

If $|x - 4| = 9$, then $x - 4 = 9$ or $x - 4 = -9$.

$x - 4 = 9$	or	$x - 4 = -9$
$x - 4 + 4 = 9 + 4$		$x - 4 + 4 = -9 + 4$
$x = 13$		$x = -5$

So, the solution set is $\{-5, 13\}$.

Get Started on Chapter 5

You will learn several new concepts, skills, and vocabulary terms as you study Chapter 5. To get ready, identify important terms and organize your resources. You may wish to refer to **Chapter 0** to review prerequisite skills.

FOLDABLES® Study Organizer

Linear Inequalities Make this Foldable to help you organize your Chapter 5 notes about linear inequalities. Begin with a sheet of 11″ by 17″ paper.
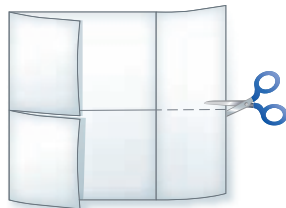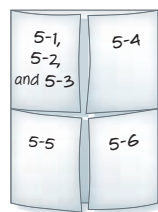
1. **Fold** each side so the edges meet in the center.

2. **Fold** in half.

3. **Unfold** and cut from each end until you reach the vertical line.
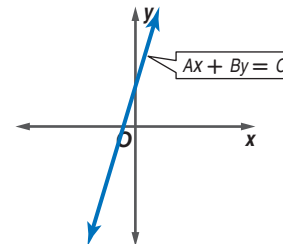
4. **Label** the front of each flap.

5-1, 5-2, and 5-3	5-4
5-5	5-6

- Study the chapter online
- Explore **Math in Motion**
- Get extra help from your own **Personal Tutor**
- Use **Extra Examples** for additional help
- Take a **Self-Check Quiz**
- **Review Vocabulary** in fun ways

New Vocabulary

English		Español
set-builder notation	• p. 284 •	notación de construcción de conjuntos
compound inequality	• p. 304 •	desigualdad compuesta
intersection	• p. 304 •	intersección
union	• p. 305 •	unión
half-plane	• p. 315 •	semiplano
boundary	• p. 315 •	frontera
open half-plane	• p. 315 •	semiplano abierto
closed half-plane	• p. 315 •	semiplano cerrada

Review Vocabulary

equivalent equations • p. 83 • ecuaciones equivalentes
equations that have the same solution

linear equation • p. 153 • ecuacion lineal
an equation in the form $Ax + By = C$, with a graph consisting of points on a straight line

$$Ax + By = C$$

solution set • p. 31 • conjunto solución
the set of elements from the replacement set that makes an open sentence true

Multilingual eGlossary glencoe.com

5-1

Solving Inequalities by Addition and Subtraction

Now

- Solve linear inequalities by using addition.
- Solve linear inequalities by using subtraction.

New Vocabulary

set-builder notation

KY Math Online

glencoe.com

- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help

Why?

The data in the table show that the recommended daily allowance of Calories for girls 11–14 years old is less than that of girls between 15–18 years old.

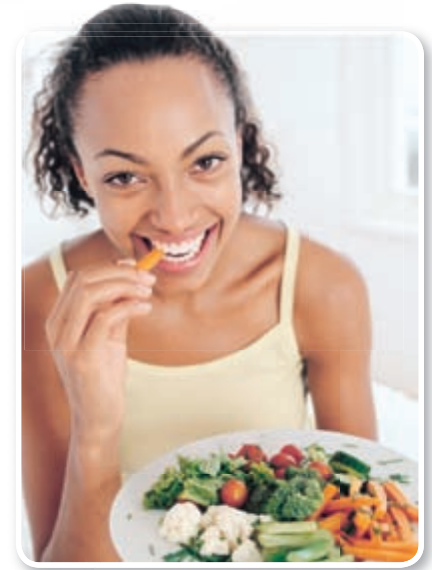| Calories	
Girls 11–14 Years	**Girls 15–18**
1845	2110

Source: *Vital Health Zone*

$$1845 < 2110$$

If a 13-year-old girl and a 16-year-old girl each eat 150 more Calories in a day than is suggested, the 16-year-old will still eat more Calories.

$$1845 + 150 \underline{\ ?\ } 2110 + 150$$
$$1995 < 2260$$

Solve Inequalities by Addition This example illustrates the Addition Property of Inequalities.

Key Concept — Addition Property of Inequalities

For Your FOLDABLE

Words   If any number is added to each side of a true inequality, the resulting inequality is also true.

Symbols   For all numbers a, b, and c, the following are true.
1. If $a > b$, then $a + c > b + c$.
2. If $a < b$, then $a + c < b + c$.

This property is also true for $\geq$ and $\leq$.

EXAMPLE 1   Solve by Adding

Solve $x - 12 \geq 8$. Check your solution.

$x - 12 \geq 8$	Original inequality
$x - 12 + 12 \geq 8 + 12$	Add 12 to each side.
$x \geq 20$	Simplify.

The solution is the set {all numbers greater than or equal to 20}.

CHECK   To check, substitute three different values into the original inequality: 20, a number less than 20, and a number greater than 20.

✔ Check Your Progress

Solve each inequality. Check your solution.

1A. $22 > m - 8$ **1B.** $d - 14 \geq -19$

▷ **Personal Tutor glencoe.com**
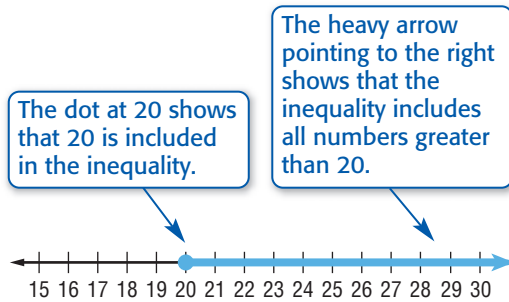
A more concise way of writing a solution set is to use **set-builder notation**. In set-builder notation, the solution set in Example 1 is $\{x \mid x \geq 20\}$.

This solution can be graphed on a number line. Be sure to check if the endpoint of the graph of an inequality should be a circle or a dot. If the endpoint is not included in the equality, use a circle, otherwise use a dot.

The dot at 20 shows that 20 is included in the inequality.

The heavy arrow pointing to the right shows that the inequality includes all numbers greater than 20.

15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Solve Inequalities by Subtraction Subtraction can also be used to solve inequalities.

Key Concept **Subtraction Property of Inequalities** **For Your FOLDABLE**

Words If any number is subtracted from each side of a true inequality, the resulting inequality is also true.

Symbols For all numbers a, b, and c, the following are true.

1. If $a > b$, then $a - c > b - c$.
2. If $a < b$, then $a - c < b - c$.

This property is also true for $\geq$ and $\leq$.

KCCT EXAMPLE 2 MA-HS-5.3.1

Solve $m + 19 > 56$.

A $\{m \mid m < 41\}$ B $\{m \mid m < 37\}$ C $\{m \mid m > 37\}$ D $\{m \mid m > 41\}$

Read the Test Item

You need to find the solution to the inequality.

Solve the Test Item

Step 1 Solve the inequality.

$$m + 19 > 56 \qquad \text{Original inequality}$$
$$m + 19 - 19 > 56 - 19 \qquad \text{Subtract 19 from each side.}$$
$$m > 37 \qquad \text{Simplify.}$$

Step 2 Write in set-builder notation.

$\{m \mid m > 37\}$ The answer is C.

✓ Check Your Progress

2. Solve the inequality $p + 8 \leq 18$.

F $\{p \mid p \geq 10\}$ G $\{p \mid p \leq 10\}$ H $\{p \mid p \leq 26\}$ J $\{p \mid p \geq 126\}$

Personal Tutor glencoe.com

Terms that are constants are not the only terms that can be subtracted. Terms with variables can also be subtracted from each side to solve inequalities.

EXAMPLE 3   Variables on Each Side

Solve $3a + 6 \leq 4a$. Then graph the solution on a number line.

$$3a + 6 \leq 4a \qquad \text{Original inequality}$$
$$3a - 3a + 6 \leq 4a - 3a \qquad \text{Subtract } 3a \text{ from each side.}$$
$$6 \leq a \qquad \text{Simplify.}$$

Since $6 \leq a$ is the same as $a \geq 6$, the solution set is $\{a \mid a \geq 6\}$.

✓ **Check Your Progress**

Solve each inequality. Then graph the solution on a number line.

3A. $9n - 1 < 10n$ **3B.** $5h \leq 12 + 4h$

Verbal problems containing phrases like *greater than* or *less than* can be solved by using inequalities. The chart shows some other phrases that indicate inequalities.

Concept Summary   **Phrases for Inequalities**   For Your **FOLDABLE**

<	>	≤	≥
less than fewer than	greater than more than	at most, no more than, less than or equal to	at least, no less than, greater than or equal to

🌐 **Real-World EXAMPLE 4   Use an Inequality to Solve a Problem**

PETS Felipe needs for the temperature of his leopard gecko's basking spot to be at least 82°F. Currently the basking spot is 62.5°F. How much warmer does the basking spot need to be for Felipe's gecko?

Words	The current temperature needs to be at least 82°F.

Variable Let $t =$ the number of degrees that the temperature needs to rise.

Inequality $62.5 + t$ $\geq$ 82

$$62.5 + t \geq 82 \qquad \text{Original inequality}$$
$$62.5 + t - 62.5 \geq 82 - 62.5 \qquad \text{Subtract 62.5 from each side.}$$
$$t \geq 19.5 \qquad \text{Simplify.}$$

Felipe needs to raise the temperature of the basking spot 19.5°F or more.

✓ **Check Your Progress**

4. SHOPPING Sanjay has $65 to spend at the mall. He bought a T-shirt for $18 and a belt for $14. If Sanjay still wants to buy a pair of jeans, how much can he spend?

✔ Check Your Understanding

Solve each inequality. Then graph the solution on a number line.

1. $x - 3 > 7$
2. $5 \geq 7 + y$
3. $g + 6 < 2$
4. $11 \leq p + 4$
5. $10 > n - 1$
6. $k + 24 > -5$
7. $8r + 6 < 9r$
8. $8n \geq 7n - 3$

Define a variable, write an inequality, and solve each problem. Check your solution.

9. A number increased by 4 is at least 10.

10. Three more than a number is less than twice the number.

11. **AMUSEMENT** A thrill ride swings passengers back and forth, swinging them a little higher each time up to 137 feet in the air. Suppose the height of the swing after 30 seconds of operation is 45 feet. How much higher will the ride swing?

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Solve each inequality. Then graph the solution on a number line.

12. $m - 4 < 3$
13. $p - 6 \geq 3$
14. $r - 8 \leq 7$
15. $t - 3 > -8$
16. $b + 2 \geq 4$
17. $13 > 18 + r$
18. $5 + c \leq 1$
19. $-23 \geq q - 30$
20. $11 + m \geq 15$
21. $h - 26 < 4$
22. $8 \leq r - 14$
23. $-7 > 20 + c$
24. $2a \leq -4 + a$
25. $z + 4 \geq 2z$
26. $w - 5 \leq 2w$
27. $3y + 6 \leq 2y$
28. $6x + 5 \geq 7x$
29. $-9 + 2a < 3a$

Define a variable, write an inequality, and solve each problem. Check your solution.

30. The sum of a number and -4 is at least 8.

31. A number decreased by 8 is less than 21.

32. Twice a number is more than the sum of that number and 9.

33. The sum of twice a number and 5 is at most 3 less than the number.

Define a variable, write an inequality, and solve each problem. Then interpret your solution.

34. **SAVINGS** Keisha is babysitting at $8 per hour to earn money for a car. So far she has saved $1300. The car that Keisha wants to buy costs at least $5440. How much money does Keisha still need to earn to buy the car?

35. **TECHNOLOGY** A recent survey found that more than 21 million people between the ages of 12 and 17 use the Internet. Of those online teens, about 16 million said they use the Internet at school. How many teens that are online do not use the Internet at school?

36. **MUSIC** A DJ added 20 more songs to his MP3 player, making the total number of songs more than 61. How many songs were originally on the player?

37. TEMPERATURE The water temperature in a swimming pool increased 4°F this morning. The temperature is now less than 81°F. What was the water temperature this morning?

38. BASKETBALL A player's goal was to score at least 150 points this season. So far, she has scored 123 points. If there is one game left in the season, how many points must the basketball player score to reach her goal?

39 SPAS Samantha received a $75 gift card for a local day spa for her birthday. She plans to get a haircut and a manicure today. How much money will be left on her gift card after her visit?

Service	Cost ($)
haircut	at least 32
manicure	at least 26

40. VOLUNTEER Kono wants to volunteer for a local charity. He knows that he can only volunteer up to 25 hours per week. If he has volunteered for the times recorded at the right, how much more time can Kono volunteer this week?

Center	Time (h)
Shelter	3 h 15 min
Kitchen	2 h 20 min

Solve each inequality. Check your solution, and then graph it on a number line.

41. $c + (-1.4) \geq 2.3$

42. $9.1g + 4.5 < 10.1g$

43. $k + \dfrac{3}{4} > \dfrac{1}{3}$

44. $\dfrac{3}{2}p - \dfrac{2}{3} \leq \dfrac{4}{9} + \dfrac{1}{2}p$

45. MULTIPLE REPRESENTATIONS In this problem, you will explore multiplication and division in inequalities.

a. GEOMETRIC Suppose a balance has 12 pounds on the left side and 18 pounds on the right side. Draw a picture to represent this situation.

b. NUMERICAL Write an inequality to represent the situation.

c. TABULAR Create a table showing the result of doubling, tripling, or quadrupling the weight on each side of the balance. Create a second table showing the result of reducing the weight on each side of the balance by $\dfrac{1}{2}$, $\dfrac{1}{3}$, or $\dfrac{1}{4}$. Include a column in each table for the inequality representing each situation.

d. VERBAL Describe the effect multiplying or dividing each side of an inequality by the same positive value has on the inequality.

If $m + 7 \geq 24$, then complete each inequality.

46. $m \geq \underline{\ ?\ }$

47. $m + \underline{\ ?\ } \geq 27$

48. $m - 5 \geq \underline{\ ?\ }$

49. $m - \underline{\ ?\ } \geq 14$

50. $m - 19 \geq \underline{\ ?\ }$

51. $m + \underline{\ ?\ } \geq 43$

H.O.T. Problems Use **H**igher-**O**rder **T**hinking Skills

52. REASONING Compare and contrast the graphs of $a < 4$ and $a \leq 4$.

53. CHALLENGE Suppose $b > d + \dfrac{1}{3}$, $c + 1 < a - 4$, and $d + \dfrac{5}{8} > a + 2$. Order $a, b, c,$ and d from least to greatest.

54. OPEN ENDED Write three linear inequalities that are equivalent to $y < -3$.

55. WRITING IN MATH Summarize the process of solving and graphing linear inequalities.

56. WRITING IN MATH Explain why $x - 2 > 5$ has the same solution set as $x > 7$.

57. Which equation represents the relationship between x and y shown in the table?

x	y
1	1
2	9
3	17
4	25
5	33
6	41

A $y = 7x - 8$

B $y = 7x + 8$

C $y = 8x - 7$

D $y = 8x + 7$

58. What is the solution set of the inequality $7 + x < 5$?

F $x < 2$ **H** $x < -2$

G $x > 2$ **J** $x > -2$

59. Francisco has $3 more than $\frac{1}{4}$ the number of dollars that Kayla has. Which expression represents how much money Francisco has?

A $3\left(\frac{1}{4}k\right)$ **C** $3 - \frac{1}{4}k$

B $\frac{1}{4}k + 3$ **D** $\frac{1}{4} + 3k$

60. GRIDDED RESPONSE The mean score for 10 students on the chemistry final exam was 178. However, the teacher had made a mistake and recorded one student's score as ten points less than the actual score. What should the mean score be?

Spiral Review

Graph each function. (Lesson 4-7)

61. $f(x) = |3x + 2|$

62. $f(x) = \begin{cases} x - 2 \text{ if } x > 1 \\ x + 3 \text{ if } x \le -1 \end{cases}$

63. $f(x) + x = 1$

64. $f(x) = \left|\frac{1}{4}x - 1\right|$

Write the slope-intercept form of an equation for the line that passes through the given point and is perpendicular to the graph of each equation. (Lesson 4-6)

65. $(-2, 0), y = x - 6$

66. $(-3, 1), y = -3x + 7$

67. $(1, -3), y = \frac{1}{2}x + 4$

68. $(-2, 7), 2x - 5y = 3$

69. TRAVEL On an island cruise in Hawaii, each passenger is given a lei. A crew member hands out 3 red, 3 blue, and 3 green leis in that order. If this pattern is repeated, what color lei will the 50th person receive? (Lesson 3-6)

Find the nth term of each arithmetic sequence described. (Lesson 3-5)

70. $a_1 = 52, d = 12, n = 102$

71. $-9, -7, -5, -3, \dots$ for $n = 18$

72. $0.5, 1, 1.5, 2, \dots$ for $n = 50$

73. JOBS Suppose your pay varies directly as the number of hours you work. Your pay for 7.5 hours is $52.50. Write a direct variation equation relating your pay to the hours worked and find your pay if you work 30 hours. (Lesson 3-4)

Skills Review

Solve each equation. (Lesson 2-2)

74. $8y = 56$ **75.** $4p = -120$ **76.** $-3a = -21$ **77.** $2c = \frac{1}{5}$

78. $\frac{r}{2} = 21$ **79.** $-\frac{3}{4}g = -12$ **80.** $\frac{2}{5}w = -4$ **81.** $-6x = \frac{2}{3}$

KY Program of Studies

HS-AT-S-EI3 **Students will solve one-variable** equations and **inequalities using manipulatives**, symbols, procedures and graphing, including graphing the solution set on a number line. *Also addresses HS-AT-S-VEO1, HS-AT-S-EI1, and HS-AT-S-EI4.*
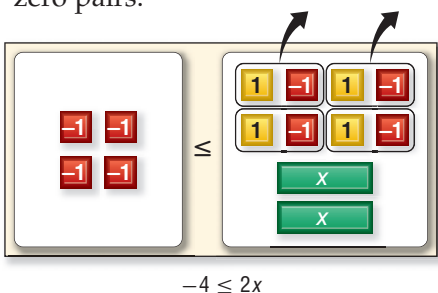
You can use algebra tiles to solve inequalities.

---

ACTIVITY — Solve Inequalities

Solve $-2x \leq 4$.

Step 1 Use a self-adhesive note to cover the equals sign on the equation mat. Then write a $\leq$ symbol on the note. Model the inequality.

$-2x \leq 4$

Step 2 Since you do not want to solve for a negative x-tile, eliminate the negative x-tiles by adding 2 positive x-tiles to each side. Remove the zero pairs.

$-2x + 2x \leq 4 + 2x$

Step 3 Add 4 negative 1-tiles to each side to isolate the x-tiles. Remove the zero pairs.

$-4 \leq 2x$
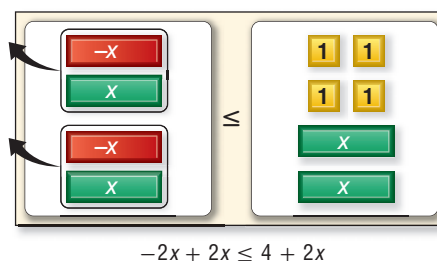
Step 4 Separate the tiles into 2 groups.

$-2 \leq x$ or $x \geq -2$
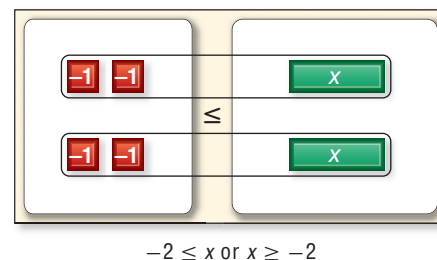
Model and Analyze

Use algebra tiles to solve each inequality.

1. $-3x < 9$ **2.** $-4x > -4$ **3.** $-5x \geq 15$ **4.** $-6x \leq -12$

5. In Exercises 1–4, is the coefficient of x in each inequality positive or negative?

6. Compare the inequality symbols and locations of the variable in Exercises 1–4 with those in their solutions. What do you find?

7. Model the solution for $3x \leq 12$. How is this different from solving $-3x \leq 12$?

8. Write a rule for solving inequalities involving multiplication and division. (*Hint:* Remember that dividing by a number is the same as multiplying by its reciprocal.)

Solving Inequalities by Multiplication and Division

Why?

Terrell received a gift card for $20 of music downloads. If each download costs $0.89, the number of downloads he can purchase can be represented by the inequality $0.89d \leq 20$.
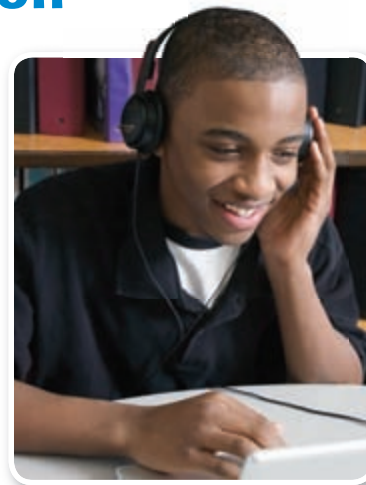
Solve Inequalities by Multiplication If you multiply each side of an inequality by a positive number, then the inequality remains true.

$4 > 2$	**Original inequality**
$4(3) \overset{?}{=} 2(3)$	**Multiply each side by 3.**
$12 > 6$	**Simplify.**

Notice that the direction of the inequality remains the same.

If you multiply each side of an inequality by a negative number, the inequality symbol changes direction.

$7 < 9$	**Original inequality**
$7(-2) \overset{?}{=} 9(-2)$	**Multiply each side by −2.**
$-14 > -18$	**Simplify.**

These examples demonstrate the **Multiplication Property of Inequalities**.

Key Concept

For Your FOLDABLE

Multiplication Property of Inequalities

Words	Symbols	Examples
If both sides of an inequality that is true are multiplied by a positive number, the resulting inequality is also true.	For any real numbers a and b, and c is a positive real number, if $a > b$, $ac > bc$. And, if $a < b$, $ac < bc$.	$6 > 3.5$ $6(2) > 3.5(2)$ $12 > 7$ or $2.1 < 5$ $2.1(0.5) < 5(0.5)$ $1.05 < 2.5$
If both sides of an inequality that is true are multiplied by a negative number, the direction of the inequality sign is reversed to make the resulting inequality also true.	For any real numbers a and b, and c is a negative real number, if $a > b$, $ac < bc$. And, if $a < b$, $ac > bc$.	$7 > 4.5$ $7(-3) < 4.5(-3)$ $-21 < -13.5$ or $3.1 < 5.2$ $3.1(-4) > 5.2(-4)$ $-12.4 > -20.8$

This property also holds for inequalities involving ≤ and ≥.
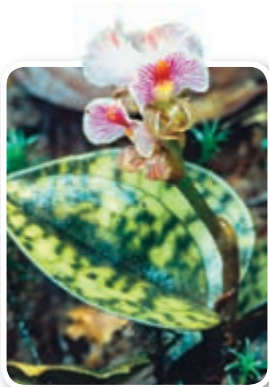
Real-World EXAMPLE 1   Write and Solve an Inequality

SURVEYS Of the students surveyed at Madison High School, fewer than eighty-four said they have never purchased an item online. This is about one eighth of those surveyed. How many students were surveyed?

Understand You know the number of students who have never purchased an item online and the portion this is of the number of students surveyed.

Plan Let n = the number of students surveyed. Write an open sentence that represents this situation.

Words	One eighth	times	the number of students surveyed	is less than	84.
Inequality	$\frac{1}{8}$	$\cdot$	n	$<$	84.

Solve Solve for n.

$\frac{1}{8}n < 84$ **Original inequality**

$(8)\frac{1}{8}n < (8)84$ **Multiply each side by 8.**

$n < 672$ **Simplify.**

Check To check this answer, substitute a number less than 672 into the original inequality. If $n = 80$, then $\frac{1}{8}(80)$ or $10 < 84$, so the solution checks.

The solution set is $\{n \mid n < 672\}$. This means that there are fewer than 672 students who were surveyed at Madison High School.

☑ **Check Your Progress**

1. **BIOLOGY** Mount Kinabalue in Malaysia has the greatest concentration of wild orchids on Earth. It contains more than 750 species, which is approximately one fourth of all orchid species in Malaysia. How many orchid species are there in Malaysia?

▷ **Personal Tutor glencoe.com**

You can also use multiplicative inverses with the Multiplication Property of Inequalities to solve an inequality.

EXAMPLE 2   Solve by Multiplying

Solve $-\frac{3}{7}r < 21$. Check your solution.

$-\frac{3}{7}r < 21$ **Original inequality**

$\left(-\frac{7}{3}\right)\left(-\frac{3}{7}r\right) > 21\left(-\frac{7}{3}\right)$ **Multiply each side by $-\frac{7}{3}$. Reverse the inequality symbol.**

$r > -49$ **Simplify. Check by substituting values.**

☑ **Check Your Progress**

Solve each inequality. Check your solution.

2A. $-\frac{n}{6} \leq 8$ **2B.** $-\frac{4}{3}p > -10$

2C. $\frac{1}{5}m \geq -3$ **2D.** $\frac{3}{8}t < 5$

▷ **Personal Tutor glencoe.com**

Solve Inequalities by Division If you divide each side of an inequality by a positive number, then the inequality remains true.

$$-10 < -5 \qquad \text{Original inequality}$$

$$\frac{-10}{5} \; ? \; \frac{-5}{5} \qquad \text{Divide each side by } -5.$$

$$-2 < -1 \qquad \text{Simplify.}$$

Notice that the direction of the inequality remains the same. If you divide each side of an inequality by a negative number, the inequality symbol changes direction.

$$15 < 18 \qquad \text{Original inequality}$$

$$\frac{15}{-3} \; ? \; \frac{18}{-3} \qquad \text{Divide each side by } -3.$$

$$-5 > -6 \qquad \text{Simplify.}$$

These examples demonstrate the **Division Property of Inequalities.**

Key Concept

For Your FOLDABLE

Division Property of Inequalities

Words	Symbols	Examples	
If both sides of an inequality that is true are divided by a positive number, the resulting inequality is also true.	For any real numbers a and b, and c is a positive real number, if $a > b$, $\frac{a}{c} > \frac{b}{c}$. And, if $a < b$, $\frac{a}{c} < \frac{b}{c}$.	$4.5 > 2.1$ $\frac{4.5}{3} > \frac{2.1}{3}$ $1.5 > 0.7$	$1.5 < 5$ or $\frac{1.5}{0.5} < \frac{5}{0.5}$ $3 < 10$
If both sides of an inequality that is true are divided by a negative number, the direction of the inequality sign is reversed to make the resulting inequality also true.	For any real numbers a and b, and c is a negative real number, if $a > b$, $\frac{a}{c} < \frac{b}{c}$. And, if $a < b$, $\frac{a}{c} > \frac{b}{c}$.	$6 > 2.4$ $\frac{6}{-6} < \frac{2.4}{-6}$ $-1 < -0.4$	$-1.8 < 3.6$ or $\frac{-1.8}{-9} < \frac{3.6}{-9}$ $0.2 > -0.4$

This property also holds for inequalities involving $\leq$ and $\geq$.

EXAMPLE 3 Divide to Solve an Inequality

Solve each inequality. Check your solution.

a. $60t > 8$

$$60t > 8 \qquad \text{Original inequality}$$

$$\frac{60t}{60} > \frac{8}{60} \qquad \text{Divide each side by 60.}$$

$$t > \frac{2}{15} \qquad \text{Simplify.}$$

b. $-7d \leq 147$

$$-7d \leq 147 \qquad \text{Original inequality}$$

$$\frac{-7d}{-7} \geq \frac{147}{-7} \qquad \text{Divide each side by } -7.$$

$$d \geq -21 \qquad \text{Simplify.}$$

✓ Check Your Progress

3A. $8p < 58$ **3B.** $-42 > 6r$ **3C.** $-12h > 15$ **3D.** $-\frac{1}{2}n < 6$

Personal Tutor glencoe.com

Example 1
p. 291

1. **FUNDRAISING** The Jefferson Band Boosters raised more than $5500 from sales of their band DVD. It sold for $15. How many DVDs did they sell? Define a variable, and write an inequality to represent this situation. Solve the inequality and interpret your solution.

Examples 2 and 3
pp. 291–292

Solve each inequality. Check your solution.

2. $30 > \frac{1}{2}n$ 3. $-\frac{3}{4}r \le -6$ 4. $-\frac{c}{6} \ge 7$ 5. $\frac{h}{2} < -5$

6. $9t > 108$ 7. $-84 < 7v$ 8. $-28 \le -6x$ 9. $40 \ge -5z$

Practice and Problem Solving

= **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Example 1
p. 291

Define a variable, write an inequality, and solve each problem. Then interpret your solution.

10. **CELL PHONE PLAN** Mario has a prepaid cell phone. He can purchase a certain plan that offers up to $50 at $0.13 per minute. How many minutes can Mario talk on this plan?

11. **SAVINGS** Rodrigo is saving money for his vacation over spring break. He needs at least $560 to pay for his expenses, and he is saving $25 from each of his weekly paychecks. How long will it be before he can pay for his trip?

Examples 2 and 3
pp. 291–292

Solve each inequality. Check your solution.

12. $\frac{1}{4}m \le -17$ 13 $\frac{1}{2}a < 20$ 14. $-11 > -\frac{c}{11}$

15. $-2 \ge -\frac{d}{34}$ 16. $-10 \le \frac{x}{-2}$ 17. $-72 < \frac{f}{-6}$

18. $\frac{2}{3}h > 14$ 19. $-\frac{3}{4}j \ge 12$ 20. $-\frac{1}{6}n \le -18$

21. $6p \le 96$ 22. $4r < 64$ 23. $32 > -2y$

24. $-26 < 26t$ 25. $-6v > -72$ 26. $-33 \ge -3z$

27. $4b \le -3$ 28. $-2d < 5$ 29. $-7f > 5$

30. **CHEERLEADING** To remain on the cheerleading squad, Lakita must attend at least 15 of the study table sessions offered. This is $\frac{3}{5}$ of the number of sessions offered. What is the least amount of study table sessions?

31. **BRACELETS** Homemade bracelets are on sale for $4.75 each. How many bracelets can Caitlin buy for herself and her friends if she wants to spend no more than $22?

32. **CHARITY** The National Honor Society at Pleasantville High School wants to raise at least $500 for a local charity by getting donations for their annual walk-a-thon. Each student earns $0.50 for every quarter of a mile walked. How many miles will the students need to walk?

33. **MUSEUM** The American history classes at a high school are planning a trip to a local museum. The admission to the museum is $8 per person. Determine how many people can go to the museum if the classes have $260.

34. **GASOLINE** Jan has $24 to spend on gasoline. If gasoline costs $3.15 per gallon, how many gallons of gasoline, to the nearest tenth, can Jan buy for her car?
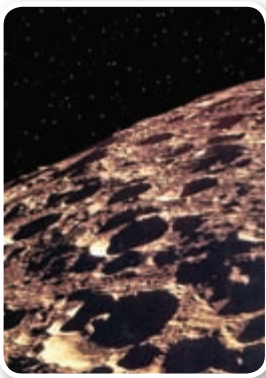
Math History Link

Thomas Harriot
(1560–1621)

Harriot was a prolific astronomer. He was the first to map the moon's surface and to see sunspots. Harriot is best known for his work in algebra.
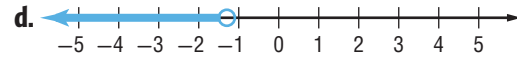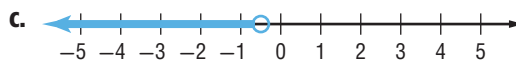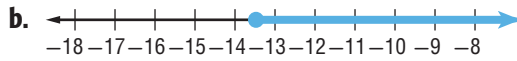
Match each inequality to the graph of its solution.

35. $-\frac{2}{3}h \leq 9$ **36.** $25j \geq 8$ **37.** $3.6p < -4.5$ **38.** $2.3 < -5t$

a.

```
 -5 -4 -3 -2 -1  0  1  2  3  4  5
```

b.

```
-18 -17 -16 -15 -14 -13 -12 -11 -10 -9 -8
```

c.

```
 -5 -4 -3 -2 -1  0  1  2  3  4  5
```

d.

```
 -5 -4 -3 -2 -1  0  1  2  3  4  5
```

39. **CANDY** Fewer than 42 employees at a factory stated that they preferred fudge over fruit candy. This is about two thirds of the employees. How many employees are there?

40. **TRAVEL** A certain travel agency employs more than 275 people at all of its branches. Approximately three fifths of all the people are employed by the west branch. How many people work at the west branch?

41. **MULTIPLE REPRESENTATIONS** In this problem, you will discover a relationship between the height and volume of a certain pyramid. The equation for the volume of a pyramid is $\frac{1}{3}$ the area of the base times the height.

    a. **GEOMETRIC** Draw a pyramid with a square base b cm long and a height of h cm.

    b. **NUMERICAL** Suppose the pyramid has a volume of 72 cm^3. Write an equation to find the height.

    c. **TABULAR** Create a table showing the value of h when $b = 1, 3, 6, 9,$ and 12.

    d. **ANALYTIC** Write and solve an inequality for the maximum lengths of b and h for this pyramid.

    e. **NUMERICAL** Write an inequality for the possible lengths of b such that $b < h$. Write an inequality for the possible lengths of h such that $b > h$.

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

42. **FIND THE ERROR** Taro and Jamie are solving $6d \geq -84$. Is either of them correct? Explain your reasoning.

| Taro	Jamie
$6d \geq -84$	$6d \geq -84$
$\frac{6d}{6} \geq \frac{-84}{6}$	$\frac{6d}{6} \leq \frac{-84}{6}$
$d \geq -14$	$d \leq -14$

43. **CHALLENGE** Solve $-96c < 12d$ for c using two methods. Show your work.

44. **CHALLENGE** Determine whether the inequalities $x^2 > 1$ and $x > 1$ are equivalent. Explain your reasoning.

45. **REASONING** Explain whether the statement *If $a > b$, then $\frac{1}{a} > \frac{1}{b}$ is sometimes, always,* or *never* true.

46. **OPEN ENDED** Create a real-world situation to represent the inequality $-\frac{5}{8} \geq x$.

47. **WRITING IN MATH** Explain the circumstances under which the inequality symbol changes directions. Use examples to support your explanation.
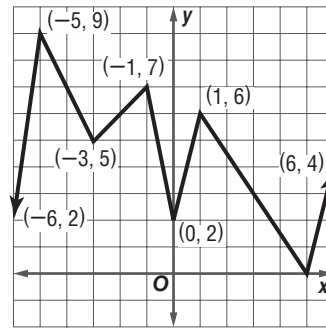
48. Juan's long-distance phone company charges 9¢ for each minute. Which inequality can be used to find how long he can talk to a friend if he does not want to spend more than $2.50 on the call?

A $0.09 \geq 2.50m$

B $0.09 \leq 2.50m$

C $0.09m \geq 2.50$

D $0.09m \leq 2.50$

49. SHORT RESPONSE Find the value of x.
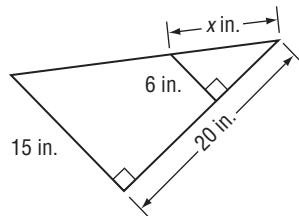
50. What is the greatest rate of decrease of this function?

F -5 **H** -2

G -3 **J** 1

51. What is the value of x if $4x - 3 = -2x$?

A -2 **C** $\frac{1}{2}$

B $-\frac{1}{2}$ **D** 2

Spiral Review

Solve each inequality. Check your solution, and then graph it on a number line. (Lesson 5-1)

52. $-8 + 4a < 6a$

53. $2y + 11 \geq -24y$

54. $7 - 2b > 12b$

Determine the domain and range for each function. (Lesson 4-7)

55. $f(x) = |2x - 5|$

56. $h(x) = \{x - 1\}$

57. $g(x) = \begin{cases} -3x + 4 \text{ if } x > 2 \\ x - 1 \text{ if } x \leq 2 \end{cases}$

58. HOME DECOR Pam is having blinds installed at her home. The cost c of installation for any number of blinds b can be described by $c = 25 + 6.5b$. Graph the equation and determine how much it would cost if Pam has 8 blinds installed. (Lesson 3-1)

59. RESCUE A boater radioed for a helicopter to pick up a sick crew member. At the time of the message, the boat is 660 kilometers from the helicopter and heading toward it at a speed of 30 kilometers per hour. If the helicopter is flying at 300 kilometers per hour, how long will it take to reach the boat? (Lesson 2-8)

Solve each open sentence. (Lesson 2-5)

60. $|x + 3| = 10$

61. $|2x - 8| = 6$

62. $|3x + 1| = -2$

Skills Review

Solve each equation. (Lessons 2-3 and 2-4)

63. $4y + 11 = 19$

64. $2x - 7 = 9 + 4x$

65. $\frac{1}{4} + 2x = 4x - 8$

66. $\frac{1}{3}(6w - 3) = 3w + 12$

67. $\frac{7r + 5}{2} = 13$

68. $\frac{1}{2}a = \frac{a - 3}{4}$

Solving Multi-Step Inequalities

Why?

Salespeople often rely on commissions for part or all of their earnings. For instance, a salesperson may make a base monthly salary and earn a commission on each of her sales. To find the number of sales she needs to make to pay her monthly bills, you can use a multi-step inequality.
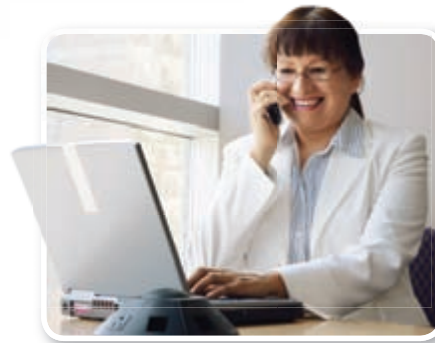
Solve Multi-Step Inequalities Multi–step inequalities can be solved by undoing the operations in the same way you would solve a multi-step equation.

⊕ Real-World EXAMPLE 1   Solve a Multi-Step Inequality

SALES **Write and solve an inequality to find the sales Mrs. Jones needs if she earns a base monthly salary of $2000 plus a 10% commission on her sales. Her goal is to make at least $4000 per month. What sales does she need to meet her goal?**

base salary + (commission $\times$ sales) $\geq$ income needed

$2000 + 0.10x \geq 4000$ **Substitution**

$0.10x \geq 2000$ **Subtract 2000 from each side.**

$x \geq 20,000$ **Divide each side by 0.10.**

She must make at least $20,000 in sales to meet her monthly goal.

✓ Check Your Progress

1. **MONEY** The Print Shop advertises a special to print 400 flyers for less than the competition. The price includes a $3.50 set-up fee. If the competition charges $35.50, what does the Print Shop charge for each flyer?

▷ **Personal Tutor glencoe.com**

When multiplying or dividing by a negative number, the direction of the inequality symbol changes. This holds true for multi-step inequalities.

EXAMPLE 2   Inequality Involving a Negative Coefficient

Solve $-11y - 13 > 42$.

$-11y - 13 > 42$ **Original inequality**

$-11y > 55$ **Add 13 to each side and simplify.**

$\dfrac{-11y}{-11} < \dfrac{55}{-11}$ **Divide each side by -11, and reverse the inequality.**

$y < -5$ **Simplify.**

✓ Check Your Progress   Solve each inequality.

2A. $23 \geq 10 - 2w$ **2B.** $43 > -4y + 11$

▷ **Personal Tutor glencoe.com**

You can translate sentences into multi-step inequalities and then solve them using the Properties of Inequalities.

EXAMPLE 3 Write and Solve an Inequality

Define a variable, write an inequality, and solve the problem.

Five minus 6 times a number is more than four times a number plus 45.

Five	**minus**	**six times a number**	**is more**	**four times a number**	**plus**	**forty-five.**
5	−	$6n$	$>$	$4n$	+	45

$5 - 10n > 45$ **Subtract 4n from each side and simplify.**

$-10n > 40$ **Subtract 5 from each side and simplify.**

$\dfrac{-10n}{-10} < \dfrac{40}{-10}$ **Divide each side by −10, and reverse the inequality.**

$n < -4$ **Simplify.**

The solution set is $\{n \mid n < -4\}$.

✓ Check Your Progress

3. *Two more than half of a number is greater than twenty-seven.*

▷ **Personal Tutor glencoe.com**

Solve Inequalities Involving the Distributive Property When solving inequalities that contain grouping symbols, use the Distributive Property to remove the grouping symbols first. Then use the order of operations to simplify the resulting inequality.

EXAMPLE 4 Distributive Property

Solve $4(3t - 5) + 7 \geq 8t + 3$.

$4(3t - 5) + 7 \geq 8t + 3$ **Original inequality**

$12t - 20 + 7 \geq 8t + 3$ **Distributive Property**

$12t - 13 \geq 8t + 3$ **Combine like terms.**

$4t - 13 \geq 3$ **Subtract 8t from each side and simplify.**

$4t \geq 16$ **Add 13 to each side.**

$\dfrac{4t}{4} \geq \dfrac{16}{4}$ **Divide each side by 4.**

$t \geq 4$ **Simplify.**

The solution set is $\{t \mid t \geq 4\}$.

✓ Check Your Progress

Solve each inequality. Check your solution.

4A. $6(5z - 3) \leq 36z$ **4B.** $2(h + 6) > -3(8 - h)$

▷ **Personal Tutor glencoe.com**

If solving an inequality results in a statement that is always true, the solution set is the set of all real numbers. This solution set is written as $\{x \mid x \text{ is a real number.}\}$. If solving an inequality results in a statement that is never true, the solution set is the empty set, which is written as the symbol $\varnothing$. The empty set has no members.

EXAMPLE 5 **Empty Set and All Reals**

Solve each inequality. Check your solution.

a. $9t - 5(t - 5) \leq 4(t - 3)$

$9t - 5(t - 5) \leq 4(t - 3)$	**Original inequality**
$9t - 5t + 25 \leq 4t - 12$	**Distributive Property**
$4t + 25 \leq 4t - 12$	**Combine like terms.**
$4t + 25 - 4t \leq 4t - 12 - 4t$	**Subtract 4t from each side.**
$25 \leq -12$	**Simplify.**

Since the inequality results in a false statement, the solution is the empty set, ∅.

b. $3(4m + 6) \leq 42 + 6(2m - 4)$

$3(4m + 6) \leq 42 + 6(2m - 4)$	**Original inequality**
$12m + 18 \leq 42 + 12m - 24$	**Distributive Property**
$12m + 18 \leq 12m + 18$	**Combine like terms.**
$12m + 18 - 12m \leq 12m + 18 - 12m$	**Subtract 12m from each side.**
$18 \leq 18$	**Simplify.**

All values of x make the inequality true. All real numbers are the solution.

✓ **Check Your Progress**

Solve each inequality. Check your solution.

5A. $18 - 3(8c + 4) \geq -6(4c - 1)$ **5B.** $46 \leq 8m - 4(2m + 5)$

▶ **Personal Tutor** glencoe.com

✓ **Check Your Understanding**

Example 1
p. 296

Write a verbal expression for each algebraic expression.

1. CANOEING A canoe was advertised as having an "800-pound capacity," meaning that it can hold at most 800 pounds. If four people plan to use the canoe with 60 pounds of supplies, write and solve an inequality to find the allowable average weight per person.

2. SHOPPING Rita is ordering a movie for $11.95 and a few CDs. She has $50 to spend. Shipping and sales tax will be about $10. If each CD cost $9.99, write and solve an inequality to find the greatest number of CDs that she can buy.

Example 2
p. 296

Solve each inequality. Check your solution.

③ $6h - 10 \geq 32$ **4.** $-3 \leq \frac{2}{3}r + 9$

5. $-3x + 7 > 43$ **6.** $4m - 17 < 6m + 25$

Example 3
p. 297

Define a variable, write an inequality, and solve each problem. Then check your solution.

7. Four times a number minus six is greater than eight plus two times a number.

8. Negative three times a number plus four is less than five times a number plus eight.

Examples 4 and 5
pp. 297–298

Solve each inequality. Check your solution.

9. $-6 \leq 3(5v - 2)$ **10.** $-5(g + 4) > 3(g - 4)$ **11.** $3 - 8x \geq 9 + 2(1 - 4x)$

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1 and 2
p. 296

Solve each inequality. Check your solution.

12. $5b - 1 \geq -11$

13 $21 > 15 + 2a$

14. $-9 \geq \frac{2}{5}m + 7$

15. $\frac{w}{8} - 13 > -6$

16. $-a + 6 \leq 5$

17. $37 < 7 - 10w$

18. $8 - \frac{z}{3} \geq 11$

19. $-\frac{5}{4}p + 6 < 12$

20. $3b - 6 \geq 15 + 24b$

21. $15h + 30 < 10h - 45$

Example 3
p. 297

Define a variable, write an inequality, and solve each problem. Check your solution.

22. Three fourths of a number decreased by nine is at least forty-two.

23. Two thirds of a number added to six is at least twenty-two.

24. Seven tenths of a number plus 14 is less than forty-nine.

25. Eight times a number minus twenty-seven is no more than the negative of that number plus eighteen.

26. Ten is no more than 4 times the sum of twice a number and three.

27. Three times the sum of a number and seven is greater than five times the number less thirteen.

28. The sum of nine times a number and fifteen is less than or equal to the sum of twenty-four and ten times the number.

Examples 4 and 5
pp. 297–298

Solve each inequality. Check your solution.

29. $-3(7n + 3) < 6n$

30. $21 \geq 3(a - 7) + 9$

31. $2y + 4 > 2(3 + y)$

32. $3(2 - b) < 10 - 3(b - 6)$

33. $7 + t \leq 2(t + 3) + 2$

34. $8a + 2(1 - 5a) \leq 20$

Define a variable, write an inequality, and solve each problem. Then interpret your solution.

35. **CARS** A car salesperson is paid a base salary of $35,000 a year plus 8% of sales. What are the sales needed to have an annual income greater than $65,000?

36. **ANIMALS** Keith's dog weighs 90 pounds. The veterinarian told him that a healthy weight for his dog would be less than 75 pounds. If Keith's dog can lose an average of 1.25 pounds per week on a certain diet, how long will it take the dog to reach a healthy weight?

37. Solve $6(m - 3) > 5(2m + 4)$. Show each step and justify your work.

38. Solve $8(a - 2) \leq 10(a + 2)$. Show each step and justify your work.

39. **MUSICAL** A high school drama club is performing a musical in which the proceeds benefit a local charity. Tickets are being sold for $5 each. They also received donations of $565. They want to raise at least $1500 for the local charity.

   a. Write an inequality that describes this situation. Then solve the inequality.

   b. Graph the solution.

40. **ICE CREAM** Benito has $6 to spend at the ice cream stand. A sundae costs $3.25 plus $0.65 per topping. Write and solve an inequality to find how many toppings he can order.

⊕ Real-World Career

Veterinarian
Veterinarians take care of sick and injured animals. Vets can work anywhere from a zoo to a research facility to owning their own practice. Vets need to earn a bachelor's degree, attend vet college for 4 years, and take a test to get licensed.

41 **SCIENCE** The normal body temperature of a camel is 97.7°F in the morning. If it has had no water by noon, its body temperature can be greater than 104°F.

    a. Write an inequality that represents a camel's body temperature at noon.

    b. If C represents degrees Celsius, then $F = \frac{9}{5}C + 32$. Write and solve an inequality to find the camel's body temperature at noon in Celsius.

42. **NUMBER THEORY** Find all sets of three consecutive positive even integers with a sum no greater than 36.

43. **NUMBER THEORY** Find all sets of four consecutive positive odd integers whose sum is less than 42.

Solve each inequality. Check your solution.

44. $2(x - 4) \leq 2 + 3(x - 6)$ **45.** $\frac{2x - 4}{6} \geq -5x + 2$

46. $5.6z + 1.5 < 2.5z - 4.7$ **47.** $0.7(2m - 5) \geq 21.7$

GRAPHING CALCULATOR **Use a graphing calculator to solve each inequality.**

48. $3x + 7 > 4x + 9$ **49.** $13x - 11 \leq 7x + 37$ **50.** $2(x - 3) < 3(2x + 2)$

51. $\frac{1}{2}x - 9 < 2x$ **52.** $2x - \frac{2}{3} \geq x - 22$ **53.** $\frac{1}{3}(4x + 3) \geq \frac{2}{3}x + 2$

54. **MULTIPLE REPRESENTATIONS** In this problem, you will solve compound inequalities. A number x is greater than 4, and the same number is less than 9.

    a. **NUMERICAL** Write two separate inequalities for the statement.

    b. **GRAPHICAL** Graph the solution set for the first inequality in red. Graph the solution set for the second inequality in blue. Highlight the portion of the graph in which the red and blue overlap.

    c. **TABULAR** Make a table using ten points from your number line, including points from each region. Use one column for each inequality and a third column titled "Both are True." Complete the table by writing true or false.

    d. **VERBAL** Describe the relationship between the colored regions of the graph and the chart.

    e. **LOGICAL** Make a prediction of what the graph of $4 < x < 9$ looks like.

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

55. **REASONING** Explain how you could solve $-3p + 7 \geq -2$ without multiplying or dividing each side by a negative number.

56. **CHALLENGE** If $ax + b < ax + c$ has infinitely many solutions, what will be the solution of $ax + b > ax + c$? Explain how you know.

57. **OPEN ENDED** Write two different multi-step inequalities that have the same graph.

58. **WHICH ONE DOESN'T BELONG?** Identify the inequality that does not belong with the other three. Explain.

$4y + 9 > -3$	$3y - 4 > 5$	$-2y + 1 < -5$	$-5y + 2 < -13$

59. **WRITING IN MATH** Explain when the solution set of an inequality will be the empty set or the set of all real numbers. Provide an example of each to support your explanation.
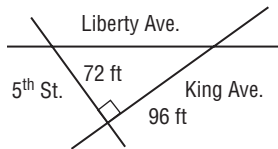
60. What is the solution set of the inequality $4t + 2 < 8t - (6t - 10)$?

   A $\{t < -6.5\}$ **C** $\{t < 4\}$

   B $\{t > -6.5\}$ **D** $\{t > 4\}$

61. GEOMETRY The section of Liberty Ave. between the intersection of 5th St. and King Ave. is temporarily closed. Traffic is being detoured right on 5th St. for 72 feet, left on King Ave. for 96 feet and then back on Liberty Ave. How long is the closed section of Liberty Ave.?

   F 100 ft

   G 120 ft

   H 144 ft

   J 180 ft

62. SHORT RESPONSE Rhiannon is paid $52 for working 4 hours. At this rate, how many hours of work will it take her to earn $845?

63. GEOMETRY Classify the triangle.
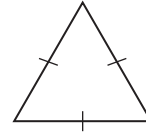
   A acute

   B isosceles

   C obtuse

   D equilateral

Spiral Review

Solve each inequality. Check your solution. (Lesson 5-2)

64. $\dfrac{y}{2} \leq -5$ **65.** $12b > -48$ **66.** $-\dfrac{2}{3}t \leq -30$

Solve each inequality. Check your solution, and graph it on a number line. (Lesson 5-1)

67. $6 - h > -8$ **68.** $p - 9 < 2$ **69.** $3 \geq 4 - m$

Solve each equation by graphing. Verify your answer algebraically. (Lesson 3-2)

70. $2x - 7 = 4x + 9$ **71.** $5 + 3x = 7x - 11$ **72.** $2(x - 3) = 5x + 12$

73. THEME PARKS In 2006, 119.8 million people visited the top 20 theme parks in North America. That represents an increase of about 1.5% in the number of visitors from 2005. About how many people visited theme parks in North America in 2005? (Lesson 2-7)

If $f(x) = 4x - 3$ and $g(x) = 2x^2 + 5$, find each value. (Lesson 1-7)

74. $f(-2)$ **75.** $g(2) - 5$ **76.** $f(c + 3)$

77. COSMETOLOGY A hair stylist gave 12 haircuts. She earned $29.95 for each haircut and received an average tip of $4 for each. Write and evaluate an expression to determine the total amount that she earned. (Lesson 1-4)

Skills Review

Graph each set of numbers on a number line.

78. $\{-4, -2, 2, 4\}$ **79.** $\{-3, 0, 1, 5\}$

80. {integers less than 3}   **81.** {integers greater than or equal to -2}

82. {integers between -3 and 4}   **83.** {integers less than -1}

Solve each inequality. Then graph it on a number line. (Lesson 5-1)

1. $x - 8 > 4$

2. $m + 2 \geq 6$

3. $p - 4 < -7$

4. $12 \leq t - 9$

5. **CONCERTS** Lupe's allowance for the month is $60. She wants to go to a concert for which a ticket costs $45. (Lesson 5-1)

   a. Write and solve an inequality that shows how much money she can spend that month after buying a concert ticket.

   b. She spends $9.99 on music downloads and $2 on lunch in the cafeteria. Write and solve an inequality that shows how much she can spend after these purchases and the concert ticket.

Define a variable, write an inequality, and solve each problem. Check your solution. (Lesson 5-2)

6. The sum of a number and −2 is no more than 6.

7. A number decreased by 4 is more than −1.

8. Twice a number increased by 3 is less than the number decreased by 4.

9. **MULTIPLE CHOICE** Jane is saving money to buy a new cell phone that costs no more than $90. So far, she has saved $52. How much more money does Jane need to save? (Lesson 5-2)

   A $38

   B more than $38

   C no more than $38

   D at least $38

Solve each inequality. Check your solution. (Lesson 5-2)

10. $\frac{1}{3}y \geq 5$

11. $4 < \frac{c}{5}$

12. $-8x > 24$

13. $2m \leq -10$

14. $\frac{x}{2} < \frac{5}{8}$

15. $-9a \geq -45$

16. $\frac{w}{6} > -3$

17. $\frac{k}{7} < -2$

18. **ANIMALS** The world's heaviest flying bird is the great bustard. A male bustard can be up to 4 feet long and weight up to 40 pounds. (Lesson 5-2)

    a. Write inequalities to describe the ranges of lengths and weights of male bustards.

    b. Male bustards are usually about four times as heavy as females. Write and solve an inequality that describes the range of weights of female bustards.

19. **GARDENING** Bill is building a fence around his garden to keep deer out. The garden is in the shape of a square, and Bill has 60 feet of fencing. Find the maximum length of a side of the garden. (Lesson 5-3)

x ft
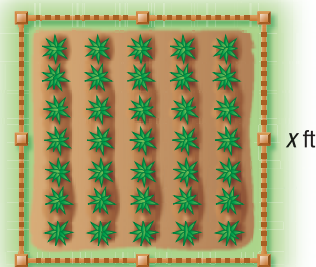
Solve each inequality. Check your solution. (Lesson 5-3)

20. $4a - 2 > 14$

21. $2x + 11 \leq 5x - 10$

22. $-p + 4 < -9$

23. $\frac{d}{4} + 1 \geq -3$

24. $-2(4b + 1) < -3b + 8$

Define a variable, write an inequality, and solve each problem. Check your solution. (Lesson 5-3)

25. Three times a number increased by 8 is no more than the number decreased by 4.

26. Two thirds of a number plus 5 is greater than 17.

27. **MULTIPLE CHOICE** Kyle has $15 to spend on bowling. Shoe rental costs $2, and each game bowled costs $3. How many games can Kyle bowl without spending more than $15? (Lesson 5-3)

    F 2

    G 3

    H 4

    J 5

A compound statement is made up of two simple statements connected by the word *and* or *or*. Before you can determine whether a compound statement is true or false, you must understand what the words *and* and *or* mean.
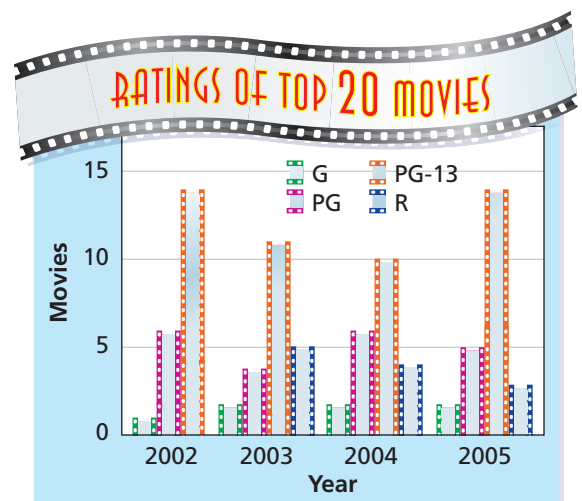
A spider has eight legs, *and* a dog has five legs.

For a compound statement connected by the word *and* to be true, both simple statements must be true.

A spider has eight legs. → true   A dog has five legs. → false

Since one of the statements is false, the compound statement is false.

A compound statement connected by the word *or* may be *exclusive* or *inclusive.* For example, the statement "With your lunch, you may have milk *or* juice," is exclusive. In everyday language, *or* means one or the other, but not both. However, in mathematics, *or* is inclusive. It means one or the other or both.

A spider has eight legs, *or* a dog has five legs.

For a compound statement connected by the word *or* to be true, at least one of the simple statements must be true. Since it is true that a spider has eight legs, the compound statement is true.

Exercises

Determine whether each compound statement is *true* or *false*. Explain your answer.

1. Most top 20 movies in 2004 were rated PG-13, *or* most top 20 movies in 2002 were rated G.

2. In 2005 more top 20 movies were rated PG than were rated G, *and* more were rated PG than rated PG-13.

3. For the years shown most top 20 movies are rated PG-13, *and* no top 20 movies in 2002 were rated R.

4. No top 20 movies in 2005 were rated G, *or* most top 20 movies in 2005 were *not* rated PG.

5. $11 < 5$ or $9 < 7$

6. $-2 > 0$ and $3 < 7$

7. $5 > 0$ and $-3 < 0$

8. $-2 > -3$ or $0 = 0$

9. $8 \neq 8$ or $-2 > -5$

10. $5 > 10$ and $4 > -2$

Source: National Association of Theater Owners
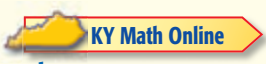
Solving Compound Inequalities

KY Program of Studies

HS-AT-S-VEO1 Students will write expressions, equations, **inequalities** and relations **in equivalent forms.**
HS-AT-S-EI1 Students will write equivalent forms of equations, **inequalities** and systems of equations and inequalities **and solve them with fluency** - mentally or with paper and pencil in simple cases and using technology in all cases. *Also addresses HS-AT-S-EI3 and HS-AT-S-EI4.*

## New Vocabulary
compound inequality
intersection
union

KY Math Online

glencoe.com
- Extra Examples
- Personal Tutor
- Self-Check Quiz
- Homework Help
- Math in Motion

Why?

The Mind Eraser roller coaster at Six Flags in Baltimore, Maryland, is an inverted steel track roller coaster. To ride this coaster, you must be at least 52 inches tall, and your height cannot exceed 72 inches. If h represents the height of a rider, we can write two inequalities to represent this.

at least 52 inches   cannot exceed 72 inches
$$h \geq 52 \qquad\qquad h \leq 72$$

The inequalities $h \geq 52$ and $h \leq 72$ can be combined and written without using *and* as $52 \leq h \leq 72$.

Inequalities Containing *and* When considered together, two inequalities such as $h \geq 52$ and $h \leq 72$ form a **compound inequality**. A compound inequality containing *and* is only true if both inequalities are true. Its graph is where the two inequalities overlap. This is called the **intersection**. The solution must be a solution of *both* inequalities or the compound inequality has no solutions.

The intersection can be found by graphing each inequality and then determining where the graphs intersect.

$$x \geq 3$$

$$x < 7$$

$$x \geq 3 \text{ and } x < 7$$
$$3 \leq x < 7$$
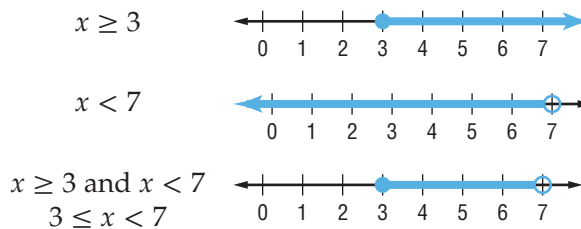
The statement $3 \leq x < 7$ can be read as *x is greater than or equal to 3 and less than 7* or *x is between 3 and 7 including 3.*
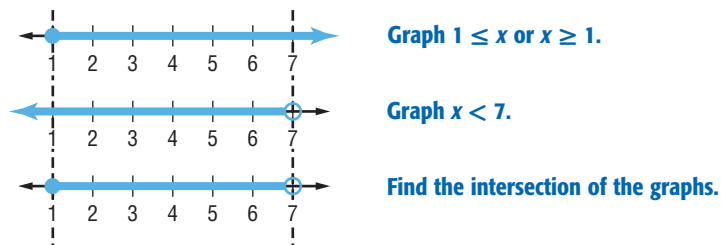
EXAMPLE 1   Solve and Graph an Intersection

Solve $-2 \leq x - 3 < 4$. Then graph the solution set.

First, express $-2 \leq x - 3 < 4$ using *and*. Then solve each inequality.

$-2 \leq x - 3$	**and**	$x - 3 < 4$	**Write the inequalities.**
$-2 + 3 \leq x - 3 + 3$		$x - 3 + 3 < 4 + 3$	**Add 3 to each side.**
$1 \leq x$		$x < 7$	**Simplify.**

The solution set is $\{x \mid 1 \leq x < 7\}$. Now graph the solution set.

Graph $1 \leq x$ or $x \geq 1$.

Graph $x < 7$.

Find the intersection of the graphs.

✔ **Check Your Progress**

Solve each compound inequality. Then graph the solution set.

1A. $y - 3 \geq -11$ and $y - 3 \leq -8$ **1B.** $6 \leq r + 7 < 10$

▷ **Personal Tutor** glencoe.com

Inequalities Containing *or* Another type of compound inequality contains the word *or*. A compound inequality containing *or* is true if at least one of the inequalities is true. Its graph is the **union** of the graphs of two inequalities. So, its solution is a solution of *either* inequality, not necessarily both.

$x > 2$

$x \leq -1$

$x > 2$ or $x \leq -1$
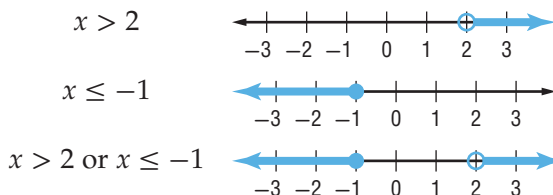
When solving problems involving inequalities, *within* is meant to be inclusive, so use $\geq$ or $\leq$. *Between* is meant to be exclusive, so use $<$ or $>$.

🌐 **Real-World EXAMPLE 2**   **Write and Graph a Compound Inequality**

SOUND The human ear can only detect sounds between the frequencies 20 Hertz and 20,000 Hertz. Write and graph a compound inequality that describes the frequency of sounds human cannot hear.
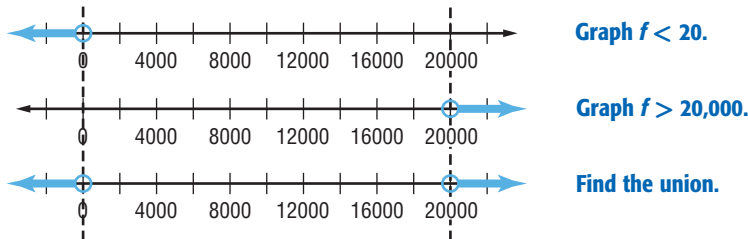
The problem states that human can hear the frequencies between 20 Hz and 20,000 Hz. We are asked to find the frequencies humans cannot hear.

Words	The frequency	is at most	20 Hertz	or	The frequency	is at least	20,000 Hertz.

Variable   Let *f* be the frequency.

Inequality	f	$<$	20	or	f	$>$	20,000

Now, graph the solution set.

Graph $f < 20$.

Graph $f > 20,000$.

Find the union.

Notice that the graphs do not intersect. Humans cannot hear sounds at a frequency less than 20 Hertz or greater than 20,000 Hertz. The compound inequality is $\{f \mid f < 20 \text{ or } f > 20,000\}$.
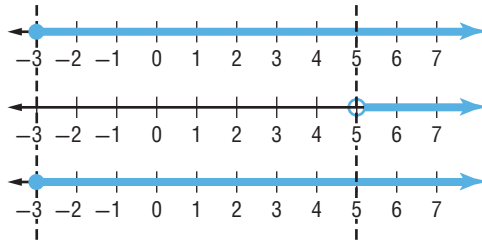
✔ **Check Your Progress**

2. MANUFACTURING A company is manufacturing an action figure that must be at least 11.2 centimeters and at most 11.4 centimeters tall. Write and graph a compound inequality that describes how tall the action figure can be.

▷ **Personal Tutor** glencoe.com

EXAMPLE 3   Solve and Graph a Union

Solve $-2m + 7 \leq 13$ or $5m + 12 > 37$. Then graph the solution set.

$-2m + 7 \leq 13$	**or**	$5m + 12 > 37$
$-2m + 7 - 7 \leq 13 - 7$	**Subtract.**	$5m + 12 - 12 > 37 - 12$
$-2m \leq 6$	**Simplify.**	$5m > 25$
$\dfrac{-2m}{-2} \leq \dfrac{6}{-2}$	**Divide.**	$\dfrac{5m}{5} > \dfrac{25}{5}$
$m \geq -3$	**Simplify.**	$m > 5$

StudyTip

Intersections and Unions The graphs of compound inequalities containing *and* will be an intersection. The graphs of compound inequalities containing *or* will be a union.

Graph $m \geq -3$.

Graph $m > 5$.

Find the union.

Notice that the graph of $m \geq -3$ contains every point in the graph of $m > 5$. So, the union is the graph of $m \geq -3$. The solution set is $\{m \mid m \geq -3\}$.

✔ **Check Your Progress**

Solve each compound inequality. Then graph the solution set.

3A. $a + 1 < 4$ or $a - 1 \geq 3$ **3B.** $x \leq 9$ or $2 + 4x < 10$

▶ **Personal Tutor glencoe.com**

✔ # Check Your Understanding

Examples 1 and 3
pp. 304, 306

Solve each compound inequality. Then graph the solution set.

1. $4 \leq p - 8$ and $p - 14 \leq 2$ **2.** $r + 6 < -8$ or $r - 3 > -10$

3. $4a + 7 \geq 31$ or $a > 5$ **4.** $2 \leq g + 4 < 7$

Example 2
p. 305

5. BIKES The recommended air pressure for the tires of a mountain bike is at least 35 pounds per square inch (psi), but no more than 80 pounds per square inch. If a bike's tires have 24 pounds per square inch, what is the recommended range of air that should be put into the tires?

Practice and Problem Solving

● = **Step-by-Step Solutions** begin on page R12.
Extra Practice begins on page 815.

Examples 1 and 3
pp. 304, 306

Solve each compound inequality. Then graph the solution set.

6. $f - 6 < 5$ and $f - 4 \geq 2$ **7** $n + 2 \leq -5$ and $n + 6 \geq -6$

8. $y - 1 \geq 7$ or $y + 3 < -1$ **9.** $t + 14 \geq 15$ or $t - 9 < -10$

10. $-5 < 3p + 7 \leq 22$ **11.** $-3 \leq 7c + 4 < 18$

12. $5h + -4 \geq 6$ and $7h + 11 < 32$ **13.** $22 \geq 4m - 2$ or $5 - 3m \leq -13$

14. $-4a + 13 \geq 29$ and $10 < 6a - 14$ **15.** $-y + 5 \geq 9$ or $3y + 4 < -5$
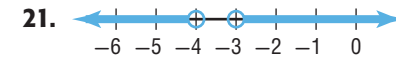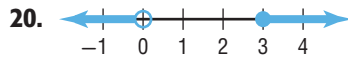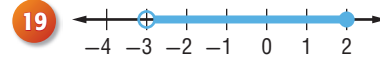
16. SPEED The posted speed limit on an interstate highway is shown. Write an inequality that represents the sign. Graph the inequality.

SPEED
LIMIT
70
MINIMUM
40

17. NUMBER THEORY Find all sets of two consecutive positive odd integers with a sum greater than or equal to 8 and no greater than 24.

Write a compound inequality for each graph.

18.

19

20.

21.

22.

23.
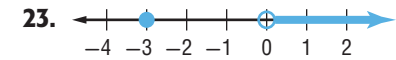
Solve each compound inequality. Then graph the solution set.

24. $3b + 2 < 5b - 6 \leq 2b + 9$

25. $-2a + 3 \geq 6a - 1 < 9a - 10$

26. $10m - 7 < 17m$ or $-6m > 36$

27. $5n - 1 < -16$ or $-3n - 1 < 8$

28. COUPON Juanita has a coupon for 10% off any digital camera at a local electronics store. She is looking at digital cameras that range in price from $100 to $250.

    a. How much are the cameras after the coupon is used?

    b. If the tax amount is 6.5%, how much should Juanita expect to spend?

Define a variable, write an inequality, and solve each problem. Then check your solution.

29. Eight less than a number is no more than 14 and no less than 5.

30. The sum of 3 times a number and 4 is between −8 and 10.

31. The product of −5 and a number is greater than 35 or less than 10.

32. One half a number is greater than 0 and less than or equal to 1.

33. SNAKES Most snakes live where the temperature ranges from 75°F to 90°F. Write an inequality to represent temperatures where snakes will *not* thrive.

34. FUNDRAISING Yumas is selling gift cards to raise money for a class trip. He can earn prizes depending on how many cards he sells. So far, he has sold 34 cards. How many more does he need to sell to earn a prize in category 4?

Cards	Prize
1–15	1
16–30	2
31–45	3
46–60	4
+61	5

35. TURTLES Atlantic sea turtle eggs that incubate below 23°C or above 33°C rarely hatch. Write an inequality for the temperatures at which the eggs should be incubated.

Real-World Link

The organization Field Trip Earth monitors the locations of Atlantic sea turtles. This data allows the scientists to track migration patterns as part of a wildlife conservation project.

Source: Field Trip Earth

36. GEOMETRY The *Triangle Inequality Theorem* states that the sum of the measures of any two sides of a triangle is greater than the measure of the third side.

    a. Write and solve three inequalities to express the relationships among the measures of the sides of the triangle shown at the right.

    b. What are four possible lengths for the third side of the triangle?

    c. Write a compound inequality for the possible values of x.
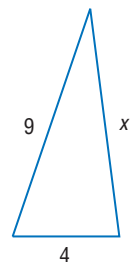
37 **HURRICANES** The Saffir-Simpson Hurricane Scale rates hurricanes on a scale from 1 to 5 based on their intensity.

Category	Wind Speed (mph)	Example (year)
1	74–95	Gaston (2004)
2	96–110	Frances (2004)
3	111–130	Ivan (2004)
4	131–155	Charley (2004)
5	> 155	Andrew (1992)

   a. Write a compound inequality for the wind speeds of a category 3 and a category 4 hurricane.

   b. What is the union of the two graphs? the intersection?

38. ⟳ **MULTIPLE REPRESENTATIONS** In this problem, you will investigate measurements. The **absolute error** of a measurement is equal to one half the unit of measure. The **relative error** of a measure is the ratio of the absolute error to the expected measure.

   a. TABULAR Copy and complete the table.

Measure	Absolute Error	Relative Error
14.3 cm	$\frac{1}{2}(0.1) = 0.05$ cm	$\dfrac{\text{absolute error}}{\text{expected measure}} = \dfrac{0.05 \text{ cm}}{14.3 \text{ cm}}$ ≈ 0.0035 or 0.4%
1.85 cm		
61.2 cm		
237 cm		

   b. NUMERICAL Determine the absolute error by taking the absolute value of 60 seconds subtracted from each student's time.

   c. ANALYTICAL If the absolute error is 6 seconds, write two possibilities for a student's estimated time.

   d. LOGICAL What estimates would have an absolute error of less than 6 seconds?

   e. GRAPHICAL Make a line plot of the responses and highlight all values such that $|60 - x| < 6$. How many guesses were within 6 seconds?

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

39. FIND THE ERROR Chloe and Jonas are solving the compound inequality $3 < 2x - 5 < 7$. Is either of them correct? Explain your reasoning.

Chloe
$3 < 2x - 5 < 7$
$3 < 2x < 12$
$\frac{3}{2} < x < 6$

Jonas
$3 < 2x - 5 < 7$
$8 < 2x < 7$
$4 < x < \frac{7}{2}$

40. REASONING Write a compound inequality for which the graph is the empty set and one for which the graph is the set of all real numbers.

41. OPEN ENDED Create an example of a compound inequality containing *or* that has infinitely many solutions.

42. CHALLENGE Solve $6x + 5 \le 2x - 3 < 3x - 8$. Rewrite the expression to make the compound inequality true.

43. WRITING IN MATH Explain how compound inequalities can be used to fulfill two separate conditions. Give an example of another compound inequality you might encounter at an amusement park. Does the example represent an intersection or a union?

44. What is the solution set of the inequality $-7 < x + 2 < 4$?

    A $-5 < x < 6$ **C** $-9 < x < 2$

    B $-5 < x < 2$ **D** $-9 < x < 6$

45. GEOMETRY What is the surface area of the rectangular solid?

    F 249.6 cm^2

    G 278.4 cm^2

    H 313.6 cm^2

    J 371.2 cm^2

8 cm, 5.8 cm, 8 cm
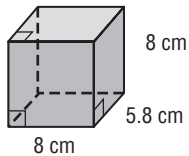
46. GRIDDED RESPONSE What is the next term in the series?

$$\frac{13}{2}, \frac{18}{5}, \frac{23}{8}, \frac{28}{11}, \frac{33}{14}$$

47. After paying a $15 membership fee, members of a video club can rent movies for $2. Nonmembers can rent movies for $4. What is the least number of movies which must be rented for it to be less expensive for members?

    A 9   **C** 7

    B 8   **D** 6

Spiral Review

48. BABYSITTING Marilyn earns $150 per month delivering newspapers plus $7 an hour babysitting. If she wants to earn at least $300 this month, how many hours will she have to babysit? (Lesson 5-3)

49. MAGAZINES Carlos is selling magazine subscriptions to earn extra money. He has earned more than $260. Each magazine subscription was sold for $12. How many subscriptions did Carlos sell? (Lesson 5-2)

50. PUNCH Raquel is mixing lemon-lime soda and a fruit juice blend that is 45% juice. If she uses 3 quarts of soda, how many quarts of fruit juice must be added to produce punch that is 30% juice? (Lesson 2-9)

Solve each proportion. If necessary, round to the nearest hundredth. (Lesson 2-6)

51. $\dfrac{14}{x} = \dfrac{20}{8}$

52. $\dfrac{0.47}{6} = \dfrac{1.41}{m}$

53. $\dfrac{16}{7} = \dfrac{9}{b}$

54. $\dfrac{2 + y}{5} = \dfrac{10}{3}$

55. $\dfrac{8}{9} = \dfrac{2r - 3}{4}$

56. $\dfrac{6 - 2y}{8} = \dfrac{2}{18}$

Determine whether a valid conclusion follows from the statement below for each given condition. If a valid conclusion does not follow, write *no valid conclusion* and explain why. (Lesson 1-8)

    If a DVD box set costs less than $70, then Ian will buy one.

57. A DVD box set costs $59.   **58.** A DVD box set costs $89.

59. Ian will not buy a DVD box set.   **60.** Ian bought 2 DVD box sets.

Evaluate each expression. Name the property used in each step. (Lesson 1-3)

61. $5 + (4 - 2^2)$

62. $\dfrac{3}{8}[8 \div (7 - 4)]$

63. $2(4 \cdot 9 - 3) + 5 \cdot \dfrac{1}{5}$

Skills Review

Solve each equation. (Lesson 2-3)

64. $4p - 2 = -6$

65. $18 = 5p + 3$

66. $9 = 1 + \dfrac{m}{7}$

67. $1.5a - 8 = 11$

68. $20 = -4c - 8$

69. $\dfrac{b + 4}{-2} = -17$

70. $\dfrac{n - 3}{8} = 20$

71. $6y - 16 = 44$

72. $130 = 11k + 9$

Inequalities Involving Absolute Value

Why?

Some companies use absolute value inequalities to control the quality of their product. To make baby carrots for snacks, long carrots are sliced into 3-inch sections and peeled. If the machine that slices the carrots is accurate to within $\frac{1}{8}$ of an inch, the length of a baby carrot ranges from $2\frac{7}{8}$ inches to $3\frac{1}{8}$ inches.

Absolute Value Inequalities (<) The inequality $|x| < 3$ means that the distance between x and 0 is less than 3.
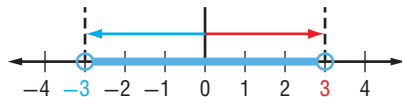
So, $x > -3$ and $x < 3$. The solution set is $\{x \mid -3 < x < 3\}$.

When solving absolute value inequalities, there are two cases to consider.

Case 1 The expression inside the absolute value symbols is positive.

Case 2 The expression inside the absolute value symbols is negative.

The solution is the intersection of the solutions of these two cases.

EXAMPLE 1 **Solve Absolute Value Inequalities (<)**

Solve each inequality. Then graph the solution set.

a. $|m + 2| < 11$

Rewrite $|m + 2| < 11$ for Case 1 *and* Case 2.

Case 1 $m + 2$ is positive.	**and**	**Case 2** $m + 2$ is negative.

$$m + 2 < 11 \qquad\qquad\qquad -(m + 2) < 11$$
$$m + 2 - 2 < 11 - 2 \qquad\qquad m + 2 > -11$$
$$m < 9 \qquad\qquad\qquad m + 2 - 2 > -11 - 2$$
$$m > -13$$

So, $m < 9$ and $m > -13$.

The solution set is $\{m \mid -13 < m < 9\}$.
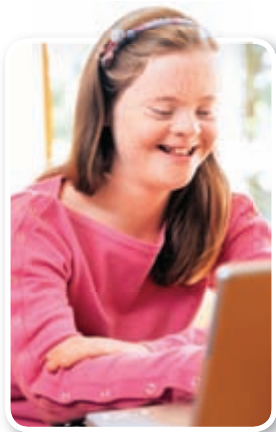

b. $|y - 1| < -2$

$|y - 1|$ cannot be negative. So it is not possible for $|y - 1|$ to be less than -2. Therefore, there is no solution, and the solution set is the empty set, $\varnothing$.

✔ **Check Your Progress**

1A. $|n - 8| \leq 2$ **1B.** $|2c - 5| < -3$

Personal Tutor glencoe.com

Real-World EXAMPLE 2   Apply Absolute Value Inequalities

INTERNET A recent survey showed that 65% of young adults watched video clips on the Internet. The margin of sampling error was within 3 percentage points. Find the range of young adults who use video sharing sites.

The difference between the actual number of viewers and the number from the survey is less than or equal to 3. Let x be the actual number of viewers. Then $|x - 65| \leq 3$.

Solve each case of the inequality.

Case 1   and   **Case 2**

$$x - 65 \leq 3 \qquad\qquad -(x - 65) \leq 3$$
$$x - 65 + 65 \leq 3 + 65 \qquad\qquad x - 65 \geq -3$$
$$x \leq 68 \qquad\qquad x \geq 62$$

The range of young adults who use video sharing sites is $\{x \,|\, 62 \leq x \leq 68\}$.

✓ Check Your Progress

2. **CHEMISTRY** The melting point of ice is 0°C. During a chemistry experiment, Jill observed ice melting within 2°C of this measurement. Write the range of temperatures that Jill observed ice melting.

Personal Tutor glencoe.com

Absolute Value Inequalities (>) The inequality $|x| > 3$ means that the distance between x and 0 is greater than 3.
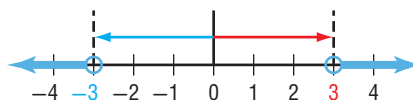
So, $x < -3$ or $x > 3$. The solution set is $\{x \,|\, x < -3 \text{ or } x > 3\}$.

As in the previous example, we must consider both cases.

Case 1 The expression inside the absolute value symbols is positive.

Case 2 The expression inside the absolute value symbols is negative.

EXAMPLE 3   Solve Absolute Value Inequalities (>)

Solve $|3n + 6| \geq 12$. **Then graph the solution set.**
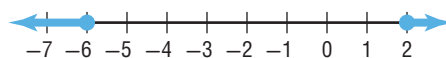
Rewrite $|3n + 6| \geq 12$ for Case 1 *or* Case 2.

Case 1   or   **Case 2**

$3n + 6$ is positive. $3n + 6$ is negative.

$$3n + 6 \geq 12 \qquad\qquad -(3n + 6) \geq 12$$
$$3n + 6 - 6 \geq 12 - 6 \qquad\qquad 3n + 6 \leq -12$$
$$3n \geq 6 \qquad\qquad 3n \leq -18$$
$$n \geq 2 \qquad\qquad n \leq -6$$

So, $n \geq 2$ or $n \leq -6$. The solution set is $\{n \,|\, n \geq 2 \text{ or } n \leq -6\}$.

✓ Check Your Progress

Solve each inequality. Then graph the solution set.

3A. $|2k + 1| > 7$ **3B.** $|r - 6| \geq -5$

Personal Tutor glencoe.com

Examples 1 and 3
pp. 310–311

Solve each inequality. Then graph the solution set.

1. $|a - 5| < 3$ **2.** $|u + 3| < 7$ **3.** $|t + 4| \leq -2$

4. $|c + 2| > -2$ **5.** $|n + 5| \geq 3$ **6.** $|p - 2| \geq 8$

Example 2
p. 311

7. STOCK MARKET Jerome bought stock in his favorite fast-food restaurant chain. The stock last sold at $70.85. However, it has fluctuated up to $0.75 in a day. Find the range of prices for which the stock could trade in a day.

Practice and Problem Solving

Examples 1 and 3
pp. 310–311

Solve each inequality. Then graph the solution set.

8. $|x + 8| < 16$ **9** $|r + 1| \leq 2$ **10.** $|2c - 1| \leq 7$

11. $|3h - 3| < 12$ **12.** $|m + 4| < -2$ **13.** $|w + 5| < -8$

14. $|r + 2| > 6$ **15.** $|k - 4| > 3$ **16.** $|2h - 3| \geq 9$

17. $|4p + 2| \geq 10$ **18.** $|5v + 3| > -9$ **19.** $|-2c - 3| > -4$

Example 2
p. 311

20. SCUBA DIVING The pressure of a scuba tank should be within 500 pounds per square inch (psi) of 2500 psi. Write the range of optimum pressures for scuba tanks.

Solve each inequality. Then graph the solution set.

21. $|4n + 3| \geq 18$ **22.** $|5t - 2| \leq 6$ **23.** $\left|\dfrac{3h + 1}{2}\right| < 8$

24. $\left|\dfrac{2p - 8}{4}\right| \geq 9$ **25.** $\left|\dfrac{7c + 3}{2}\right| \leq -5$ **26.** $\left|\dfrac{2g + 3}{2}\right| > -7$

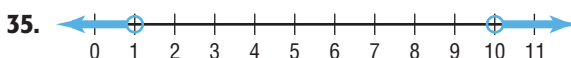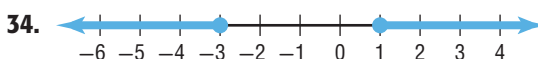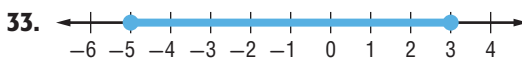**27.** $|-6r - 4| < 8$ **28.** $|-3p - 7| > 5$ **29.** $|-h + 1.5| < 3$

30. MUSIC DOWNLOADS Kareem is allowed to download $10 worth of music each month. This month he has spent within $3 of his allowance.

     a. What is the range of money he has spent on music downloads this month?

     b. Graph the range of the money that he spent.

31. CHEMISTRY Water is one of the few compounds in our homes that can change its state. To keep water from being a liquid, it must be more than 90°F from 122°F.

     a. Write the range of temperatures that water is not a liquid.

     b. Graph this range.

     c. Write the absolute value inequality that describes this situation.

Write an open sentence involving absolute value for each graph.

32.
```
  +--+--+--⊕--+--+--⊕--+--+--+-->
 -5 -4 -3 -2 -1  0  1  2  3  4  5
```

33.
```
  +--+--●--+--+--+--+--+--●--+-->
 -6 -5 -4 -3 -2 -1  0  1  2  3  4
```

34.
```
<--+--+--+--●--+--+--+--●--+--+-->
 -6 -5 -4 -3 -2 -1  0  1  2  3  4
```

35.
```
<--+--⊕--+--+--+--+--+--+--⊕--+--+-->
   0  1  2  3  4  5  6  7  8  9  10 11
```

36. ANIMALS A sheep's normal body temperature is 39°C. However, a healthy sheep may have body temperatures 1°C above or below this temperature. What is the range of body temperatures for a sheep?

37 MINIATURE GOLF Ginger's score was within 5 strokes of her average score of 52. Determine the range of scores for Ginger's game.

Express each statement using an inequality involving absolute value. Do *not* solve.

38. The pH of a swimming pool must be within 0.3 of a pH of 7.5.

39. The temperature inside a refrigerator should be within 1.5 degrees of 38°F.

40. Ramona's bowling score was within 6 points of her average score of 98.

41. The cruise control of a car set at 55 miles per hour should keep the speed within 3 miles per hour of 55.

42. MULTIPLE REPRESENTATIONS In this problem, you will investigate the graphs of absolute value inequalities on a coordinate plane.

a. GRAPHICAL Graph $f(x) = |x - 1|$.

b. TABULAR Copy and complete the table. Substitute the x and $f(x)$ values for each point into each inequality. Mark whether the resulting statement is *true* or *false*.

Point	$f(x) \geq |x - 1|$	true/false	$f(x) \leq |x - 1|$	true/false
(−4, 2)				
(−2, 2)				
(0, 2)				
(2, 2)				
(4, 2)				

c. GRAPHICAL For each point that made $f(x) \geq |x - 1|$ a true statement, plot the point on the graph in red. For each point that made $f(x) \leq |x - 1|$, plot the point on the graph in blue.

d. LOGICAL Make a conjecture of what the graphs of $f(x) \geq |x - 1|$ and $f(x) \leq |x - 1|$ look like. Complete the table with other points to verify your conjecture.

e. GRAPHICAL Use what you discovered in graphing $f(x) \geq |x - 1|$ to graph $f(x) \geq |x - 3|$.

H.O.T. Problems Use **H**igher-**O**rder **T**hinking Skills

43. FIND THE ERROR Lucita sketched a graph of her solution to $|2a - 3| > 1$. Is she correct? Explain your reasoning.

44. REASONING The graph of an absolute value inequality is *sometimes, always,* or *never* the union of two graphs. Explain.

45. CHALLENGE Demonstrate why the solution of $|t| > 0$ is not all real numbers. Explain your reasoning.

46. OPEN ENDED Write an absolute value inequality to represent a real-world situation. Interpret the solution.

47. WRITING IN MATH Explain how to determine whether an absolute value inequality uses a compound inequality with *and* or a compound inequality with *or*. Then summarize how to solve absolute value inequalities.

48. The formula for acceleration in a circle is $a = \frac{v^2}{r}$. Which of the following shows the equation solved for v?

A $v = ar$ **C** $v^2 = ar$

B $v = \sqrt{ar}$ **D** $v = \frac{\sqrt{a}}{r}$

49. An engraver charges a $3 set-up fee plus an additional $0.25 per word engraved. Which table shows the total price p for engraving w words?

F

w	p
15	$3
20	$4.25
25	$5.50
30	$7.75

H

w	p
15	$3.75
20	$5
25	$6.25
30	$8.50

G

w	p
15	$6.75
20	$7
25	$7.25
30	$7.50

J

w	p
15	$6.75
20	$8
25	$9.25
30	$10.50

50. SHORT RESPONSE The table shows the items purchased at the school store during the first day of class. What is the probability that the first item purchased will be a notebook?

Item	Number Purchased
pencil	57
pen	38
eraser	6
folder	25
notebook	18

51. Assume that n is an integer. Solve for n.

$$|2n - 3| = 5$$

A $\{-4, -1\}$
B $\{-1, 4\}$
C $\{1, 1\}$
D $\{4, 4\}$

Spiral Review

Solve each compound inequality. Then graph the solution set. (Lesson 5-4)

52. $b + 3 < 11$ and $b + 2 > -3$ **53.** $6 \leq 2t - 4 \leq 8$ **54.** $2c - 3 \geq 5$ or $3c + 7 \leq -5$

55. MONEY In a recent year, the sum of the number of $2 bills and $50 bills in circulation was 1,857,573,945. The number of $50 bills was 494,264,809 more than the number of $2 bills. How many of each type of bill was in circulation? (Lesson 5-3)

56. GEOMETRY One angle of a triangle measures 10° more than the second. The measure of the third angle is twice the sum of the measure of the first two angles. Find the measure of each angle. (Lesson 2-4)

Solve each equation. Then check your solution. (Lesson 2-2)

57. $c - 7 = 11$ **58.** $2w = 24$ **59.** $9 + p = -11$ **60.** $\frac{t}{5} = 20$

Skills Review

Graph each equation. (Lesson 3-1)

61. $y = 4x - 1$ **62.** $y - x = 3$ **63.** $2x - y = -4$ **64.** $3y + 2x = 6$

65. $4y = 4x - 16$ **66.** $2y - 2x = 8$ **67.** $-9 = -3x - y$ **68.** $-10 = 5y - 2x$

Graphing Inequalities in Two Variables

Why?

Hannah has budgeted $35 every three months for car maintenance. From this she must buy oil costing $3 and filters that cost $7 each. How much oil and how many filters can Hannah buy and stay within her budget?

Graph Linear Inequalities The graph of a linear inequality is the set of points that represent all of the possible solutions of that inequality. An equation defines a **boundary**, which divides the coordinate plane into two **half-planes**.
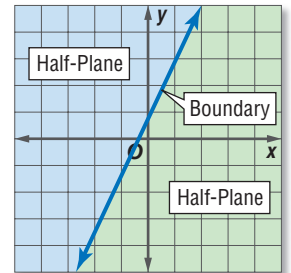
The boundary may or may not be included in the graph of an inequality. When it is included, the solution is a **closed half-plane**. When not included, the solution is an **open half-plane**.

Key Concept — Graphing Linear Inequalities

For Your FOLDABLE

Step 1 Graph the boundary. Use a solid line when the inequality contains $\leq$ or $\geq$. Use a dashed line when the inequality contains $<$ or $>$.

Step 2 Use a test point to determine which half-plane should be shaded.

Step 3 Shade the half-plane that contains the solution.

EXAMPLE 1 — Graph an Inequality ($<$ or $>$)
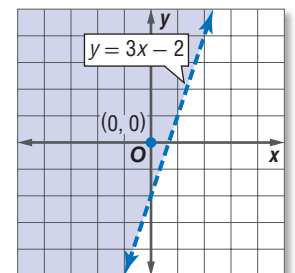
Graph $3x - y < 2$.

Step 1 First, solve for y in terms of x.
$$3x - y < 2$$
$$-y < -3x + 2$$
$$y > 3x - 2$$

Then, graph $y = 3x - 2$. Because the inequality involves $>$, graph the boundary with a dashed line.

Step 2 Select a test point in either half-plane. A simple choice is $(0, 0)$.

$3x - y < 2$	Original inequality
$3(0) - 0 < 2$	$x = 0$ and $y = 0$
$0 < 2$	true

Step 3 So, the half-plane containing the origin is the solution. Shade this half-plane.

✓ **Check Your Progress** Graph each inequality.

1A. $y > \frac{1}{2}x + 3$ **1B.** $x - 1 > y$

▷ **Personal Tutor** glencoe.com

EXAMPLE 2 **Graph an Inequality (≤ or ≥)**

Graph $x + 5y \le 10$.

Step 1 Solve for y in terms of x.

$x + 5y \le 10$ **Original inequality**

$5y \le -x + 10$ **Subtract x from each side and simplify.**

$y \le -\frac{1}{5}x + 2$ **Divide each side by 5.**

Graph $y = -\frac{1}{5}x + 2$. Because the inequality symbol is $\le$, graph the boundary with a solid line.

Step 2 Select a test point. Let's use (3, 3). Substitute the values into the original inequality.
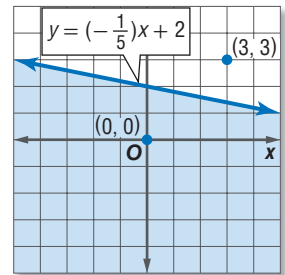
$x + 5y \le 10$ **Original inequality**

$3 + 5(3) \le 10$ **$x = 3$ and $y = 3$**

$18 \nleq 10$ **Simplify.**

Step 3 Since this statement is false, shade the other half-plane.

✅ **Check Your Progress** **Graph each inequality.**

2A. $x - y \le 3$ **2B.** $2x + 3y \ge 18$

▶ **Personal Tutor glencoe.com**

Solve Linear Inequalities We can use a coordinate plane to solve inequalities with one variable.

EXAMPLE 3 **Solve Inequalities From Graphs**

Use a graph to solve $3x + 5 < 14$.

Step 1 First graph the boundary, which is the related function. Replace the inequality sign with an equals sign, and get 0 on a side by itself.

$3x + 5 < 14$ **Original inequality**

$3x + 5 = 14$ **Change $<$ to $=$.**

$3x - 9 = 0$ **Subtract 14 from each side and simplify.**

Graph $y = 3x - 9$ with a dashed line.

Step 2 Choose (0, 0) as a test point. These values in the original inequality give us $5 < 14$.

Step 3 Since this statement is true, shade the half-plane containing the point (0, 0).

Notice that the x-intercept of the graph is at 3. Since the half-plane to the left of the x-intercept is shaded, the solution is $x < 3$.
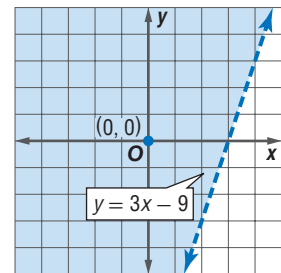
✅ **Check Your Progress** **Use a graph to solve each inequality.**

3A. $4x - 3 \ge 17$ **3B.** $-2x + 6 > 12$

▶ **Personal Tutor glencoe.com**

When using inequalities to solve real-world problems, the domain and the range are often restricted to nonnegative or whole numbers.

● Real-World **EXAMPLE 4** Write and Solve an Inequality

CLASS PICNIC A yearbook company promises to give the junior class a picnic if they spend at least \$28,000 on yearbooks and class rings. Each yearbook costs \$35, and each class ring costs \$140. How many yearbooks and class rings must the junior class buy to get their picnic?

Understand You know the cost of each item and the minimum amount the class needs to spend.

Plan Let x = the number of yearbooks and y = the number of class rings the class must buy. Write an inequality.

$35	times	the number of yearbooks	plus	$140	times	the number of rings	is at least	$28,000.
35	·	x	+	140	·	y	$\geq$	28,000

Solve Find the slope.

$35x + 140y - 35x \geq 28{,}000 - 35x$ **Subtract 35x from each side.**

$140y \geq -35x + 28{,}000$ **Divide each side by 140.**

$\dfrac{140y}{140} \geq \dfrac{-35x}{140} + \dfrac{28000}{140}$ **Simplify.**

$y \geq -0.25x + 200$ **Simplify.**

Because the yearbook company cannot sell a negative number of items, the domain and range must be positive numbers. Graph the boundary with a solid line. If we test (0, 0), the result is $0 \geq 28{,}000$, which is false. Shade the closed half-plane that does not include the origin. One solution is (500, 100), or 500 yearbooks and 100 class rings.
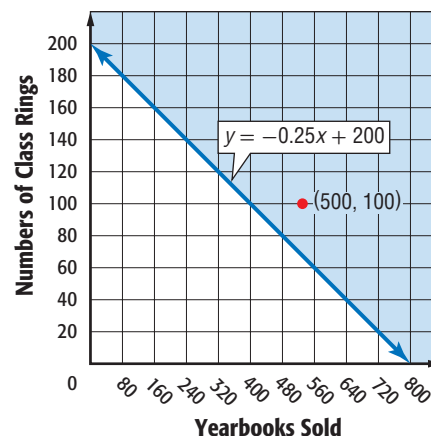
Check If we test (500, 100), the result is $100 \geq 75$, which is correct. Because the company cannot sell a fraction of an item, only points with whole-number coordinates can be solutions.

✔ Check Your Progress

4. MARATHONS Neil wants to run a marathon at a pace of at least 6 miles per hour. Write and graph an inequality for the miles y he will run in x hours.

▷ **Personal Tutor** glencoe.com

✅ Check Your Understanding

Graph each inequality.

1. $y > x + 3$ **2.** $y \geq -8$ **3.** $x + y > 1$

4. $y \leq x - 6$ **5.** $y < 2x - 4$ **6.** $x - y \leq 4$

Use a graph to solve each inequality.

7. $7x + 1 < 15$ **8.** $-3x - 2 \geq 11$

9. $3y - 5 \leq 34$ **10.** $4y - 21 > 1$

11. BUSINESS The Kowabunga Surf Shop sells skim boards for $115 and longboard surf boards for $685. The store has a weekly overhead of $2300.

   a. Write an inequality to describe this situation.

   b. How many skim boards and surf boards must the shop sell each week to make a profit?

Practice and Problem Solving

Graph each inequality.

12. $y < x - 3$ **13.** $y > x + 12$ **14.** $y \geq 3x - 1$

15. $y \leq -4x + 12$ **16.** $6x + 3y > 12$ **17.** $2x + 2y < 18$

18. $5x + y > 10$ **19.** $2x + y < -3$ **20.** $-2x + y \geq -4$

21. $8x + y \leq 6$ **22.** $10x + 2y \leq 14$ **23.** $-24x + 8y \geq -48$

Use a graph to solve each inequality.

24. $10x - 8 < 22$ **25.** $20x - 5 > 35$ **26.** $4y - 77 \geq 23$

27. $5y + 8 \leq 33$ **28.** $35x + 25 < 6$ **29.** $14x - 12 > -31$

30. DECORATING Sybrina is decorating her bedroom. She has $300 to spend on paint and bed linens. A gallon of paint costs $14, while a set of bed linens costs $60.

   a. Write an inequality for this situation.

   b. How many gallons of paint and bed linen sets can Sybrina buy and stay within her budget?

Use a graph to solve each inequality.

31. $3x + 2 < 0$ **32.** $4x - 1 > 3$ **33.** $-6x - 8 \geq -4$

34. $-5x + 1 < 3$ **35.** $-7x + 13 < 10$ **36.** $-4x - 4 \leq -6$

37 SOCCER The girls' soccer team has a booth at the local carnival to raise money and buy new goals for $2000. How many of each item must they sell to buy the goals?

   a. Write an inequality that represents this situation.

   b. Graph this inequality.

   c. Make a table of values that shows at least five possible solutions.

   d. Plot the possible solutions you found on your graph.

Support Girls Soccer

Hot Dogs $1.00
Sodas $1.25

Graph each inequality. Determine which of the ordered pairs are part of the solution set for each inequality.

38. $y \geq 6$; $\{(0, 4), (-2, 7), (4, 8), (-4, -8), (1, 6)\}$

39 $x < -4$; $\{(2, 1), (-3, 0), (0, -3), (-5, -5), (-4, 2)\}$

40. $2x - 3y \leq 1$; $\{(2, 3), (3, 1), (0, 0), (0, -1), (5, 3)\}$

41. $5x + 7y \geq 10$; $\{(-2, -2), (1, -1), (1, 1), (2, 5), (6, 0)\}$

42. $-3x + 5y < 10$; $\{(3, -1), (1, 1), (0, 8), (-2, 0), (0, 2)\}$

43. $2x - 2y \geq 4$; $\{(0, 0), (0, 7), (7, 5), (5, 3), (2, -5)\}$

44. RECYCLING A curbside recycling service will remove up to 50 pounds of plastic bottles and paper products each week. They charge $0.25 per pound of plastic and $0.75 per pound for paper products.

  a. Write an inequality that describes the pounds of each kind of product that can be included in the curbside service.

  b. Write an inequality that describes the charge for the curbside service.

  c. Graph each inequality.

  d. Compare the two graphs.

45. ⬢ **MULTIPLE REPRESENTATIONS** In this problem, you will investigate graphing compound inequalities on a coordinate plane. Use inequalities A and B.

  A. $7(y + 6) \leq 21x + 14$ **B.** $-3y \leq 3x + -12$
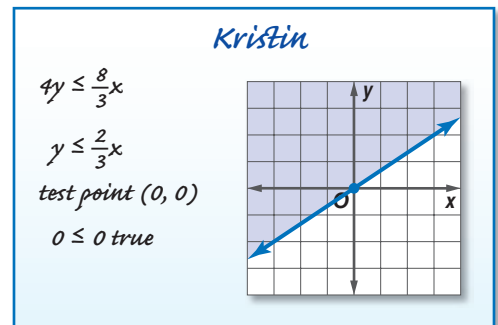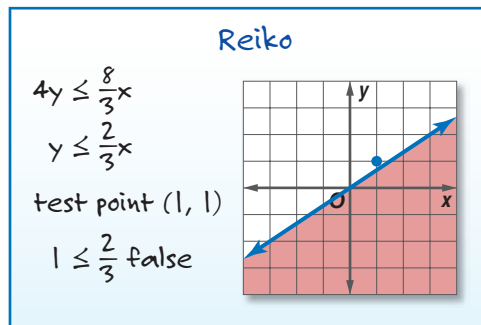
  a. NUMERICAL Solve each inequality for y.

  b. GRAPHICAL Graph both inequalities on one graph. Shade the half-plane that makes A true in red. Shade the half-plane that makes B true in blue.

  c. VERBAL What does the overlapping region represent?

H.O.T. Problems   Use **H**igher-**O**rder **T**hinking Skills

46. FIND THE ERROR Reiko and Kristin are solving $4y \leq \frac{8}{3}x$ by graphing. Is either of them correct? Explain your reasoning.

Reiko
$4y \leq \frac{8}{3}x$
$y \leq \frac{2}{3}x$
test point $(1, 1)$
$1 \leq \frac{2}{3}$ false

Kristin
$4y \leq \frac{8}{3}x$
$y \leq \frac{2}{3}x$
test point $(0, 0)$
$0 \leq 0$ true

**47. CHALLENGE** Graph $y > |x + 5|$.

48. REASONING Explain why a point on the boundary should not be used as a test point.

49. OPEN ENDED Write a two-variable inequality with a restricted domain and range to represent a real-world situation. Give the domain and range, and explain why they are restricted.

50. WRITING IN MATH Summarize the steps to graph an inequality in two variables.

51. What is the domain of this function?

A $0 \leq x \leq 3$

B $0 \leq x \leq 9$

C $0 \leq y \leq 9$

D $0 \leq y \leq 3$
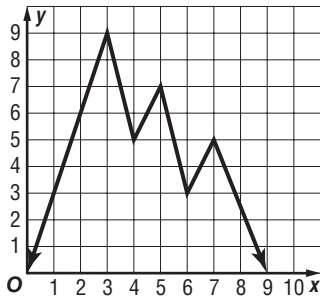
52. EXTENDED RESPONSE An arboretum will close for the winter when all of the trees have lost their leaves. The table shows the number of trees each day that still have leaves.

| Day	5	10	15	20
Trees with Leaves	325	260	195	130

a. Write an equation that represents the number of trees with leaves t after d days.

b. Find the y-intercept. What does it mean in the context of this problem?

c. After how many days will the arboretum close? Explain how you got your answer.

53. Which inequality best represents the statement below?

A jar contains 832 gumballs. Ebony's guess was within 46 pieces.

F $|g - 832| \leq 46$

G $|g + 832| \leq 46$

H $|g - 832| \geq 46$

J $|g + 832| \geq 46$

54. GEOMETRY If the rectangular prism has a volume of 10,080 cm³, what is the value of x?
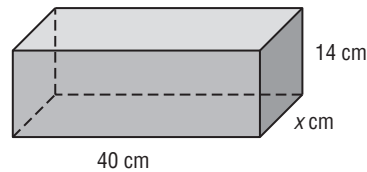
A 12 cm

B 14 cm

C 16 cm

D 18 cm

Spiral Review

Solve each open sentence. (Lesson 5-5)

55. $|y - 2| > 4$

56. $|t - 6| \leq 5$

57. $|3 + d| < -4$

Solve each compound inequality. (Lesson 5-4)

58. $4c - 4 < 8c - 16 < 6c - 6$

59. $5 < \frac{1}{2}p + 3 < 8$

60. $0.5n \geq -7$ or $2.5n + 2 \leq 9$

Write an equation of the line that passes through each pair of points. (Lesson 4-2)

61. $(1, -3)$ and $(2, 5)$

62. $(-2, -4)$ and $(-7, 3)$

63. $(-6, -8)$ and $(-8, -5)$

64. FITNESS The table shows the maximum heart rate to maintain during aerobic activities. Write an equation in function notation for the relation. Determine what would be the maximum heart rate to maintain in aerobic training for an 80-year-old. (Lesson 3-5)

Age (yr)	20	30	40	50	60	70
Pulse rate (beats/min)	175	166	157	148	139	130

Skills Review

65. WORK The formula $s = \dfrac{w - 10r}{m}$ is used to find keyboarding speeds. In the formula, s represents the speed in words per minute, w the number of words typed, r is the number of errors, and m is the number of minutes typed. Solve for r. (Lesson 2-8)

KY Program of Studies **HS-AT-S-EI13** Students will graph the solution set of a linear inequality and identify whether the solution set is an open or closed half-plane.

You can use a graphing calculator to investigate the graphs of inequalities. Graphing calculators can only shade between two functions, so enter a lower boundary as well as an upper boundary for each inequality.
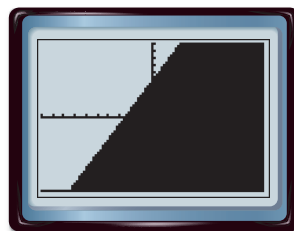

ACTIVITY 1   Less Than

Graph $y \leq 2x + 5$.

Clear all functions from the **Y=** list.

KEYSTROKES: Y= CLEAR

Graph $y \geq 2x + 5$ in the standard window.

KEYSTROKES: 2nd [DRAW] 7 (−) 10 , 2 X,T,θ,n + 5) ENTER

The lower boundary is **Ymin** or −10. The upper boundary is $y = 2x + 5$. All ordered pairs for which y is *less than or equal* to $2x + 5$ lie *below or on* the line and are solutions.

[−10, 10] scl: 1 by [−10, 10] scl: 1

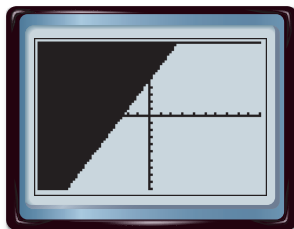**ACTIVITY 2   Greater Than**

Graph $y - 2x \geq 5$.

Clear the drawing that is currently displayed.

KEYSTROKES: 2nd [DRAW] 1

Rewrite $y - 2x \geq 5$ as $y \geq 2x + 5$ and graph it.

KEYSTROKES: 2nd [DRAW] 7 2 X,T,θ,n + 5 , 10) ENTER

This time, the lower boundary is $y = 2x + 5$. The upper boundary is **Ymax** or 10. All ordered pairs for which y is *greater than or equal to* $2x + 5$ lie *above or on* the line and are solutions.

[−10, 10] scl: 1 by [−10, 10] scl: 1

Exercises

1. Compare and contrast the two graphs shown above.

2. Graph $y \geq -3x + 1$ in the standard viewing window.
   a. What functions do you enter as the lower and upper boundaries?
   b. Using your graph, name four solutions of the inequality.

3. Suppose student water park tickets cost $16, and adult water park tickets cost $20. You would like to buy at least 10 tickets but spend no more than $200.
   a. Let x = number of student tickets and y = number of adult tickets. Write two inequalities, one representing the total number of tickets and the other representing the total cost of the tickets.
   b. Which inequalities would you use as the lower and upper bounds?
   c. Graph the inequalities. Use the viewing window [0, 20] scl: 1 by [0, 20] scl: 1.
   d. Name four possible combinations of student and adult tickets.

Chapter Summary

Key Concepts

Solving One-Step Inequalities (Lessons 5-1 and 5-2)

For all numbers a, b, and c, the following are true.

• If $a > b$ and c is positive, $ac > bc$.
• If $a > b$ and c is negative, $ac < bc$.

Multi-Step and Compound Inequalities (Lessons 5-3 and 5-4)

• Multi-step inequalities can be solved by undoing the operations in the same way you would solve a multi-step equation.
• A compound inequality containing *and* is only true if both inequalities are true.
• A compound inequality containing *or* is true if at least one of the inequalities is true.

Absolute Value Inequalities (Lesson 5-5)

• The absolute value of any number n is its distance from zero on a number line and is written as $|n|$.
• If $|x| = n$, then $x = -n$ or $x = n$.
• If $|x| < n$, then $x < n$ or $-x > n$.
• If $|x| > n$, then $x > n$ or $-x < n$.

Inequalities in Two Variables (Lesson 5-6)
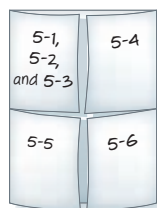
To graph an inequality:

Step 1 Graph the boundary. Use a solid line when the inequality contains $\leq$ or $\geq$. Use a dashed line when the inequality contains $<$ or $>$.

Step 2 Use a test point to determine which half-plane should be shaded.

Step 3 Shade the half-plane.

FOLDABLES® Study Organizer

Be sure the Key Concepts are noted in your Foldable.

Key Vocabulary

boundary (p. 315)

closed half-plane (p. 315)

compound inequality (p. 304)

half plane (p. 315)

intersection (p. 304)

open half-plane (p. 315)

set-builder notation (p. 284)

union (p. 305)

Vocabulary Check

State whether each sentence is *true* or *false*. If *false*, replace the underlined term to make a true sentence.

1. Set-builder notation is a <u>less</u> concise way of writing a solution set.

2. There are <u>two</u> types of compound inequalities.

3. If the graph of a compound inequality shows two functions overlapping, this is called the <u>union</u>.

4. A compound inequality containing *or* is true if one or more of the inequalities is true. Its graph is the <u>union</u> of the graphs of the two inequalities.

5. When the domain and range of an inequality are the set of real numbers, the graph of these points fill one of two regions on the coordinate plane called a <u>half-plane</u>.

6. A <u>point</u> defines the boundary between the two half-planes.

7. The <u>boundary</u> is the equation of the line that defines the edge of each half-plane.

8. The solution set to the inequality $y \geq x$ includes the <u>boundary</u>.

9. When solving an inequality, <u>multiplying</u> by a negative number reverses the inequality symbol.

10. The graph of a compound inequality that contains <u>*and*</u> is the intersection of the two inequalities.

Lesson-by-Lesson Review

HS-AT-S-VE01,
HS-AT-S-EI1

Solve each inequality. Then graph it on a number line.

11. $w - 4 > 9$

12. $x + 8 \leq 3$

13. $6 + h < 1$

14. $-5 < a + 2$

15. $13 - p \geq 15$

16. $y + 1 \leq 8$

17. FIELD TRIP Samantha's class is taking a field trip. The bus can hold 44 people. There are 35 students in Samantha's class. How many students can ride on the bus?

EXAMPLE 1

Solve $x - 9 < -4$. Then graph it on a number line.

$x - 9 < -4$ **Original inequality**

$x - 9 + 9 < -4 + 9$ **Add 9 to each side.**

$x < 5$ **Simplify.**

The solution set is $\{x \mid x < 5\}$.

HS-AT-S-EI3,
HS-AT-S-EI4

Solve each inequality. Check your solution.

18. $\frac{1}{3}x > 6$

19. $\frac{1}{5}g \geq -4$

20. $4p < 32$

21. $-55 \leq -5w$

22. $-2m > 100$

23. $\frac{2}{3}t < -48$

24. MOVIE RENTAL Jack has no more than $24 to spend on DVDs for a party. Each DVD rents for $4. Find the maximum number of DVDs Jack can rent for his party.

EXAMPLE 2

Solve $-14h < 56$. Check your solution.

$-14h < 56$ **Original inequality**

$\dfrac{-14h}{-14} > \dfrac{56}{-14}$ **Divide each side by −14.**

$h > -4$ **Simplify.**

CHECK To check, substitute three different values into the original inequality: −4, a number less than −4, and a number greater than −4.

HS-AT-S-VE01,
HS-AT-S-EI1

Solve each inequality. Check your solution.

25. $3h - 7 < 14$

26. $4 + 5b > 34$

27. $18 \leq -2x + 8$

28. $\frac{t}{3} - 6 > -4$

29. Four times a number decreased by 6 is less than −2. Define a variable, write an inequality, and solve for the number.

30. TICKET SALES The drama club collected $160 from ticket sales for the spring play. They need to collect at least $400 in order to pay for new lighting for the stage. If tickets sell for $3 each, how many more tickets need to be sold?

EXAMPLE 3

Solve $-6y - 13 > 29$. Check your solution.

$-6y - 13 > 29$ **Original inequality**

$-6y - 13 + 13 > 29 + 13$ **Add 13 to each side.**

$-6y > 42$ **Simplify.**

$\dfrac{-6y}{-6} < \dfrac{42}{-6}$ **Divide each side by −6 and change > to <.**

$y < -7$ **Simplify.**

The solution set is $\{y \mid y < -7\}$.

CHECK $-6y - 13 > 29$ **Original inequality**

$-6(-10) - 13 \overset{?}{>} 29$ **Substitute −10 for y.**

$47 > 29$ ✓   **Simplify.**

5-4 Solving Compound Inequalities (pp. 304–309)

HS-AT-S-VE01,
HS-AT-S-EI1

Solve each compound inequality. Then graph the solution set.

31. $m - 3 < 6$ and $m + 2 > 4$

32. $-4 < 2t - 6 < 8$

33. $3x + 2 \leq 11$ or $5x - 8 > 22$

34. KITES A large dragon kite can be flown in wind speeds no less than 7 miles per hour and no more than 16 miles per hour. Write an inequality that represents the wind speeds for which the kite can be flown.

EXAMPLE 4

Solve $-3w + 4 > -8$ and $2w - 11 > -19$. Then graph the solution set.

$$-3w + 4 > -8 \qquad \text{and} \qquad 2w - 11 > -19$$
$$w < 4 \qquad\qquad\qquad w > -4$$

To graph the solution set, graph $w < 4$ and graph $w > -4$. Then find the intersection.

```
←+─⊕─+─+─+─+─+─+─⊕─+→
 −5−4−3−2−1  0  1  2  3  4  5
```

5-5 Inequalities Involving Absolute Value (pp. 310–314)

HS-NPO-S-N04,
HS-AT-S-EI4

Solve each inequality. Then graph the solution set.

35. $|x - 4| < 9$

36. $|p + 2| > 7$

37. $|2c + 3| \leq 11$

38. $|f - 9| \geq 2$

39. $|3d - 1| \leq 8$

40. $\left|\dfrac{4b - 2}{3}\right| < 12$

41. $\left|\dfrac{2t + 6}{2}\right| > 10$

42. $|-4y - 3| < 13$

43. $|m + 19| \leq 1$

44. $|-k - 7| \geq 4$

EXAMPLE 5

Solve $|x - 6| < 9$. Then graph the solution set.

Case 1 $x - 6$ is positive.

$$x - 6 < 9$$
$$x < 15$$

Case 2 $x - 6$ is negative.

$$-(x - 6) < 9$$
$$x > -3$$

The solution set is $\{x \mid -3 < x < 15\}$.

```
←+─⊖─+─+─+─+─+─+─+─+─⊖─+─+→
 −4 −2  0  2  4  6  8 10 12 14 16 18
```

5-6 Graphing Inequalities in Two Variables (pp. 315–320)

HS-AT-S-EI13

Graph each inequality.

45. $y > x - 3$

46. $y < 2x + 1$

47. $3x - y \leq 4$

48. $y \geq -2x + 6$

49. $5x - 2y < 10$

50. $3x + 4y > 12$

Graph each inequality. Determine which of the ordered pairs are part of the solution set for each inequality.

51. $y \leq 4$; $\{(3, 6), (1, 2), (-4, 8), (3, -2), (1, 7)\}$

52. $-2x + 3y \geq 12$; $\{(-2, 2), (-1, 1), (0, 4), (2, 2)\}$

53. BAKERY Ben has $24 to spend on cookies and cupcakes at the bakery. Each large cookie is $2, and each cupcake is $3. Write and graph an inequality that represents what Ben can buy.

EXAMPLE 6

Graph $2x - y > 3$.

Solve for y in terms of x.

$$2x - y > 3 \qquad \text{Original inequality}$$
$$y < 2x - 3 \qquad \text{Multiply each side by } -1.$$

Graph the boundary using a dashed line. Choose $(0, 0)$ as a test point.

$$2(0) - 0 \overset{?}{>} 3$$
$$0 \not> 3$$

Since 0 is not greater than 3, shade the plane that does not contain $(0, 0)$.

Solve each inequality. Then graph it on a number line.

1. $x - 9 < -4$
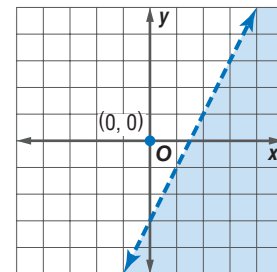
2. $6p \geq 5p - 3$

3. **MULTIPLE CHOICE** Drew currently has 31 comic books in his collection. His friend Connor has 58 comic books. How many more comic books does Drew need to add to his collection in order to have a larger collection than Connor?

   A no more than 21

   B 27

   C at least 28

   D more than 30

Solve each inequality. Check your solution.

4. $\frac{1}{5}h > 3$

5. $7w \leq -42$

6. $-\frac{2}{3}t \geq 24$

7. $-9m < -36$

8. $3c - 7 < 11$

9. $\frac{g}{4} + 3 \leq -9$

10. $-2(x - 4) > 5x - 13$

11. **ZOO** The 8th grade science class is going to visit the local zoo. The class can spend up to $300 on admission.

| Zoo Admission	
Visitor	**Cost**
Student	$8
Adult	$10

   a. Write an inequality for this situation.

   b. If there are 32 students in the class and 1 adult will attend for every 8 students, how much will admission be?

Solve each compound inequality. Then graph the solution set.

12. $y - 8 < -3$ or $y + 5 > 19$

13. $-11 \leq 2h - 5 \leq 13$

14. $3z - 2 > -5$ and $7z + 4 < -17$

Define a variable, write an inequality, and solve the problem. Check your solution.

15. The difference of a number and 4 is no more than 8.

16. Nine times a number decreased by four is at least twenty-three.

17. **MULTIPLE CHOICE** Write a compound inequality for the graph shown below.
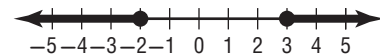
   F $-2 \leq x < 3$ H $x < -2$ or $x \geq 3$

   G $x \leq -2$ or $x \geq 3$ J $-2 < x \leq 3$

Solve each inequality. Then graph the solution set.

18. $|p - 5| < 3$

19. $|2f + 7| \geq 21$

20. $|-4m + 3| \leq 15$

21. $\left|\frac{x - 3}{4}\right| > 5$

22. **RETAIL** A sporting goods store is offering a $15 coupon on any pair of shoes.

   a. The most expensive pair of shoes is $149.95 and the least expensive pair of shoes is $24.95. What is the range of prices for customers who have the coupons?

   b. You have a choice of buying a pair of shoes with a regular price of $109.95 using the coupon or having a 15% discount on the price. Which option is best?

Graph each inequality.

23. $y < 4x - 1$ 24. $2x + 3y \geq 12$

25. Graph $y > -2x + 5$. Then determine which of the ordered pairs in $\{(-2, 0), (-1, 5), (2, 3), (7, 3)\}$ are in the solution set for the inequality.

26. **PRESCHOOL** Mrs. Jones is buying new books and puzzles for her preschool classroom. Each book costs $6, and each puzzle costs $4. She has a budget of $96. Write and graph an inequality to determine how many books and puzzles she can buy.

Write and Solve an Inequality

Many multiple-choice items will require writing and solving inequalities.
Follow the steps below to help you successfully solve these types of problems.

Strategies for Writing and Solving Inequalities

Step 1

Read the problem statement carefully.

Ask yourself:

- What am I being asked to solve?
- What information is given in the problem?
- What are the unknowns for which I need to solve?

Step 2

Translate the problem statement into an inequality.

- Assign variables to the unknown(s).
- Write the word sentence as a mathematical number sentence looking for words such as *greater than*, *less than*, *no more than*, *up to*, or *at least to* indicate the type of inequality as well as where to place the inequality sign.

Step 3

Solve the inequality.

- Solve for the unknowns in the inequality.
- Remember that multiplying or dividing each side by a negative number reverses the sign of the inequality.
- Check your answer to make sure it makes sense.

EXAMPLE

Read the problem. Identify what you need to know. Then use the information in the problem to solve. Show your work.

Pedro has earned scores of 89, 74, 79, 85, and 88 on his tests this semester. He needs a test average of at least 85 in order to earn an A for the semester. There will be one more test given this semester.

A Write an inequality to model the situation.

B What score must he have on his final test to earn an A for the semester?

⋮ Read the problem carefully. You are given Pedro's first 5 test scores and told that he needs an average of *at least* 85 after his next test to earn an A for the semester.

a. Write the inequality.

> **Words**   Pedro needs a test average of *at least* 85.
>
> ▼
>
> **Variable**   Let *t* represent Pedro's score on the final test.
>
> ▼
>
> **Equation** $\dfrac{89 + 74 + 79 + 85 + 88 + t}{6} \geq 85$
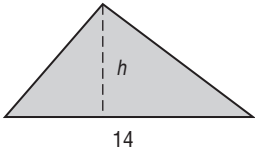
b. Solve the inequality for *t*.

$$\frac{89 + 74 + 79 + 85 + 88 + t}{6} \geq 85$$
$$89 + 74 + 79 + 85 + 88 + t \geq 85(6)$$
$$415 + t \geq 510$$
$$t \geq 95$$

So, Pedro's final test score must be greater than or equal to 95 in order to earn an A for the semester.

Exercises

Read each problem. Identify what you need to know. Then use the information in the problem to solve.

1. Craig has $20 to order a pizza. The pizza costs $12.50 plus $0.95 per topping. If there is also a $3 deliver fee, how many toppings can Craig order?

2. To join an archery club, Nina had to pay an initiation fee of $75, plus $40 per year in membership dues.

   a. Write an equation to model the total cost, *y*, of belonging to the club for *x* years.

   b. How many years will it take her to spend more than $400 to belong to the club?

3. The area of the triangle below is no more than 84 square millimeters. What is the height of the triangle?

4. Rosa earns $200 a month delivering newspapers, plus an average of $11 per hour babysitting. If her goal is to earn at least $295 this month, how many hours will she have to babysit?

5. To earn money for a new bike, Ethan is selling some of his baseball cards. He has saved $245. If the bike costs $1400, and he can sell 154 cards, for how much money will he need to sell each card to reach his goal?

6. In a certain lacrosse league, there can be no more than 22 players on each team, and no more than 10 teams per age group. There are 6 age groups.

   a. Write an inequality to represent this situation.

   b. What is the greatest number of players that can play lacrosse in this league?

7. Sarah has $120 to shop for herself and to buy some gifts for 6 of her friends. She has purchased a shirt for herself for $32. What is the maximum that she can spend on each friend?

Multiple Choice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.
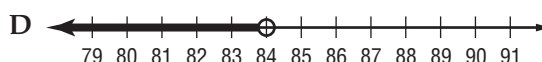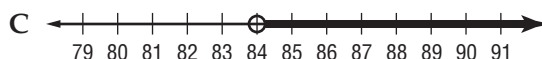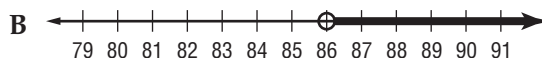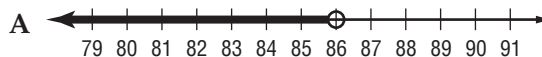
1. Miguel received a $100 gift certificate for a graduation gift. He wants to buy a CD player that costs $38 and CDs that cost $12 each. Which of the following inequalities represents how many CDs Miguel can buy?

A $n \le 6$

B $n \ge 5$

C $n < 5$

D $n \le 5$

2. Craig is paid time-and-a-half for any additional hours over 40 that he works.

Time	Pay Rate
Up to 40 hours	$12.80/hr
Additional hours worked over 40	$19.20/hr

If Craig's goal is to earn at least $600 next week, what is the minimum number of hours he needs to work?

F   43 hours   H 44 hours

G 45 hours   J   46 hours

3. Which equation has a slope of $-\frac{2}{3}$ and a y-intercept of 6?

A $y = 6x + \frac{2}{3}$ C $y = -\frac{2}{3}x + 6$

B $y = -\frac{2}{3}x - 6$ D $y = 6x - \frac{2}{3}$

4. The highest score that is on record on a video game is 10,219 points. The lowest score on record is 257 points. Which of the following inequalities best shows the range of scores recorded on the game?

F $x \le 10{,}219$

G $x \ge 257$

H $257 < x < 10{,}219$

J $257 \le x \le 10{,}219$

5. Kyle scored 14 points in his last basketball game, bringing his total points for the season to over 100. Which number line represents the number of points Kyle had scored prior to the last game?

A
79 80 81 82 83 84 85 86 87 88 89 90 91

B
79 80 81 82 83 84 85 86 87 88 89 90 91

C
79 80 81 82 83 84 85 86 87 88 89 90 91

D
79 80 81 82 83 84 85 86 87 88 89 90 91

6. The girls' volleyball team is selling T-shirts and pennants to raise money for new uniforms. The team hopes to raise at least $250.

Item	Price
T-shirt	$10
Pennant	$4

Which of the following combinations of items sold would meet this goal?

F   16 T-shirts and 20 pennants

G 20 T-shirts and 12 pennants

H 18 T-shirts and 18 pennants

J   15 T-shirts and 25 pennants

7. What type of line does not have a defined slope?

A horizontal   C perpendicular

B parallel   D vertical

8. Which expression below illustrates the Associative Property?

F $abc = bac$

G $2(x - 3) = 2x - 6$

H $(p + 3) - t = p(3 - t)$

J $5 + (-5) = 0$

Test-TakingTip

Question 2 You can check your answer by finding Craig's earnings for the hours worked.

Short Response/Gridded Response

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. Solve $-4 < 3x + 8 \leq 23$.

10. GRIDDED RESPONSE Tien is saving money for a new television. She needs to save at least $720 to pay for her expenses. Each week Tien saves $50 toward her new television. How many weeks will it take so she can pay for the television?

11. Write an inequality that best represents the graph.
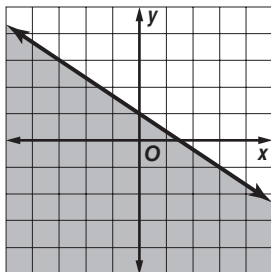
12. Solve $|x - 4| < 2$.

13. GRIDDED RESPONSE Daniel wants to ship a set of golf clubs and several boxes of golf balls in a box that can hold up to 20 pounds. If the set of clubs weighs 9 pounds and each box of golf balls weighs 12 ounces, how many boxes of golf balls can Daniel ship?

14. Graph the solution set for the inequality $3x - 6 \leq 4x - 4 \leq 3x + 1$.

15. Write an equation that represents the data in the table.

x	y
3	12.5
4	16
5	19.5
6	23
7	26.5

16. A sporting goods company near the beach rents bicycles for $10 plus $5 per hour. Write an equation in slope-intercept form that shows the total cost, y, of renting a bicycle for x hours. How much would it cost Emily to rent a bicycle for 6 hours?

Extended Response

Record your answers on a sheet of paper. Show your work.

17. Theresa is saving money for a vacation. She needs to save at least $640 to pay for her expenses. Each week, she puts $35 towards her vacation savings.

a. Let w represent the number of weeks Theresa saves money. Write an inequality to model the situation.

b. Solve the inequality from part a. What is the minimum number of weeks Theresa must save money in order to reach her goal?

c. If Theresa were to save $45 each week instead, by how many weeks would the minimum savings time be decreased?

Need Extra Help?																	
If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Go to Lesson or Page...	5-3	5-2	4-1	5-4	5-1	5-6	3-3	1-3	5-4	5-2	5-6	5-5	5-3	5-4	2-1	4-2	5-2
KY Program of Studies	MA-HS-5.3.1	MA-HS-5.3.1	MA-HS-5.1.5	MA-HS-5.3.1	MA-HS-5.3.1	MA-HS-5.3.3	MA-HS-1.4.1	MA-HS-1.5.1	MA-HS-5.3.1	MA-HS-5.3.1	MA-HS-5.3.3	MA-HS-5.3.1	MA-HS-5.3.1	MA-HS-5.3.1	MA-HS-5.3.1	MA-HS-5.3.3	MA-HS-5.3.1