CHAPTER 0

Preparing for Algebra

Chapter 0 contains lessons on topics from previous courses. You can use this chapter in various ways.

- Begin the school year by taking the Pretest. If you need additional review, complete the lessons in this chapter. To verify that you have successfully reviewed the topics, take the Posttest.
- As you work through the text, you may find that there are topics you need to review. When this happens, complete the individual lessons that you need.
- Use this chapter for reference. When you have questions about any of these topics, flip back to this chapter to review definitions or key concepts.

KYPS

Get Started on Chapter 0

You will review several concepts, skills, and vocabulary terms as you study Chapter 0. To get ready, identify important terms and organize your resources.

FOLDABLES® Study Organizer

Throughout this text, you will be invited to use Foldables to organize your notes.

Why should you use them?

- They help you organize, display, and arrange information.
- They make great study guides, specifically designed for you.
- You can use them as your math journal for recording main ideas, problem-solving strategies, examples, or questions you may have.
- They give you a chance to improve your math vocabulary.

How should you use them?

- Write general information – titles, vocabulary terms, concepts, questions, and main ideas – on the front tabs of your Foldable.
- Write specific information – ideas, your thoughts, answers to questions, steps, notes, and definitions – under the tabs.
- Use the tabs for:
  - math concepts in parts, like types of triangles,
  - steps to follow, or
  - parts of a problem, like *compare* and *contrast* (2 parts) or *what*, *where*, *when*, *why*, and *how* (5 parts).
- You may want to store your Foldables in a plastic zipper bag that you have three-hole punched to fit in your notebook.

When should you use them?

- Set up your Foldable as you begin a chapter, or when you start learning a new concept.
- Write in your Foldable every day.
- Use your Foldable to review for homework, quizzes, and tests.

New Vocabulary

English		Español
integer	• p. 7 •	entero
absolute value	• p. 11 •	valor absolute
opposites	• p. 11 •	opuestos
reciprocal	• p. 18 •	recíproco
perimeter	• p. 23 •	perímetro
circle	• p. 24 •	círculo
diameter	• p. 24 •	diámetro
center	• p. 24 •	centro
circumference	• p. 24 •	circunferencia
radius	• p. 24 •	radio
area	• p. 26 •	area
volume	• p. 29 •	volumen
surface area	• p. 31 •	area de superficie
probability	• p. 33 •	probabilidad
sample space	• p. 33 •	espacio muestral
complements	• p. 33 •	complementos
tree diagram	• p. 34 •	diagrama de árbol
odds	• p. 35 •	probabilidades
mean	• p. 37 •	media
median	• p. 37 •	mediana
mode	• p. 37 •	moda
range	• p. 38 •	rango
quartile	• p. 38 •	cuartil
lower quartile	• p. 38 •	cuartil inferior
upper quartile	• p. 38 •	cuartil superior
bar graph	• p. 40 •	gráfica de barras
histogram	• p. 40 •	histograma
line graph	• p. 41 •	gráfica lineal
circle graph	• p. 41 •	gráfica circular
outliers	• p. 42 •	valores atípicos

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- Study the chapter online
- Explore **Math in Motion**
- Get extra help from your own **Personal Tutor**
- Use **Extra Examples** for additional help
- Take a **Self-Check Quiz**
- **Review Vocabulary** in fun ways

CHAPTER 0

Pretest

For each problem, determine whether you need an estimate or an exact answer. Then solve.

1. **SHOPPING** Addison paid \$1.29 for gum and \$0.89 for a package of notebook paper. She gave the cashier a \$5 bill. If the tax was \$0.14, how much change should Addison receive?

2. **DISTANCE** Luis rode his bike 1.2 miles to his friend's house, then 0.7 mile to the video store, then 1.9 miles to the library. If he rode the same route back home, about how far did he travel in all?

Find each sum or difference.

3. $20 + (-7)$

4. $-15 + 6$

5. $-9 - 22$

6. $18.4 - (-3.2)$

7. $23.1 + (-9.81)$

8. $-5.6 + (-30.7)$

Find each product or quotient.

9. $11(-8)$

10. $-15(-2)$

11. $63 \div (-9)$

12. $-22 \div 11$

Replace each ● with <, >, or = to make a true sentence.

13. $\frac{7}{20}$ ● $\frac{2}{5}$

14. 0.15 ● $\frac{1}{8}$

15. Order 0.5, $-\frac{1}{7}$, -0.2, and $\frac{1}{3}$ from least to greatest.

Find each sum or difference. Write in simplest form.

16. $\frac{5}{6} + \frac{2}{3}$

17. $\frac{11}{12} - \frac{3}{4}$

18. $\frac{1}{2} + \frac{4}{9}$

19. $-\frac{3}{5} + \left(-\frac{1}{5}\right)$

Find each product or quotient.

20. $2.4(-0.7)$

21. $-40.5 \div (-8.1)$

Name the reciprocal of each number.

22. $\frac{4}{11}$

23. $-\frac{3}{7}$

Find each product or quotient. Write in simplest form.

24. $\frac{2}{21} \div \frac{1}{3}$

25. $\frac{1}{5} \cdot \frac{3}{20}$

26. $\frac{6}{25} \div \left(-\frac{3}{5}\right)$

27. $\frac{1}{9} \cdot \frac{3}{4}$

28. $-\frac{2}{21} \div \left(-\frac{2}{15}\right)$

29. $2\frac{1}{2} \cdot \frac{2}{15}$

Express each percent as a fraction in simplest form.

30. 20%

31. 7.5%

Use the percent proportion to find each number.

32. 18 is what percent of 72?

33. 35 is what percent of 200?

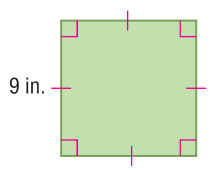
34. 4 is 60% of what number?

35. **TEST SCORES** James answered 14 items correctly on a 16-item quiz. What percent did he answer correctly?

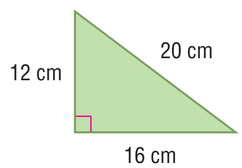
36. **BASKETBALL** Emily made 75% of the baskets that she attempted. If she made 9 baskets, how many attempts did she make?

Find the perimeter and area of each figure.

37.

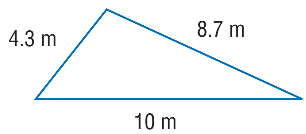


38.



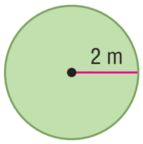
39. A parallelogram has side lengths of 7 inches and 11 inches. Find the perimeter.

40. **GARDENS** Find the perimeter of the garden.

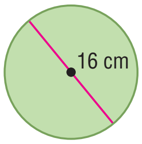


Find the circumference and area of each circle. Round to the nearest tenth.

41.



42.



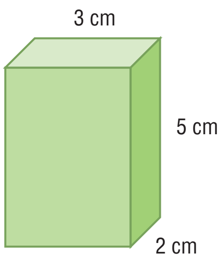
43. **BIRDS** The floor of a birdcage is a circle with a circumference of about 47.1 inches. What is the diameter of the birdcage floor? Round to the nearest inch.

Find the volume and surface area of each rectangular prism given the length, width, and height.

44. $\ell = 3$ cm, $w = 1$ cm, $h = 3$ cm

45. $\ell = 6$ ft, $w = 2$ ft, $h = 5$ ft

46. Find the volume and surface area of the rectangular prism.



One pencil is randomly selected from a case containing 3 red, 4 green, 2 black, and 6 blue pencils. Find each probability.

47. P(green)

48. P(red or blue)

49. Use a tree diagram to find the sample space for the event *a die is rolled, and a coin is tossed*. State the number of possible outcomes.

One coin is randomly selected from a jar containing 20 pennies, 15 nickels, 3 dimes, and 12 quarters. Find the odds of each outcome. Write in simplest form.

50. a penny

51. a penny or nickel

52. A coin is tossed 50 times. The results are shown in the table. Find the experimental probability of the coin landing heads up. Write as a fraction in simplest form.

Lands Face-Up	Number of Times
head	22
tails	28

Find the mean, median, and mode for each set of data.

53. {10, 11, 18, 24, 30}

54. {4, 8, 9, 9, 10, 14, 16}

55. Find the range, median, lower quartile, and upper quartile for {16, 19, 21, 24, 25, 31, 35}.

56. **SCHOOL** Devonte's scores on his first four Spanish tests are 92, 85, 90, and 92. What test score must Devonte earn on the fifth test so that the mean will be exactly 90?

57. **MUSIC** The table shows the results of a survey in which students were asked to choose which of the four instruments they would like to learn. Make a bar graph of the data.

Favourite Instrument	
Instrument	**Number of Students**
drums	8
guitar	12
piano	5
trumpet	234

58. Make a stem-and-leaf plot of the data: 42, 50, 38, 59, 50, 44, 46, 62, 47, 35, 55, and 56.

59. **EXPENSES** The table shows how Dylan spent his money at the fair. Make a circle graph of the data.

Money Spent at the Fair	
How Spent	**Amount ($)**
rides	6
food	10
games	4

0-1

Plan for Problem-Solving

Using the **four-step problem-solving plan** can help you solve any word problem.

Objective
Use the four-step problem-solving plan.

New Vocabulary
four-step problem-solving plan
defining a variable

Key Concept **Four-Step Problem-Solving Plan** For Your FOLDABLE

Step 1 Understand the problem.
Step 2 Plan the solution.
Step 3 Solve the problem.
Step 4 Check the solution.

Each step of the plan is important.

Step 1 Understand the Problem

To solve a verbal problem, first read the problem carefully and explore what the problem is about.

- Identify what information is given.
- Identify what you are asked to find.

Step 2 Plan the Solution

One strategy you can use to solve a problem is to write an equation. Choose a variable to represent one of the unspecific numbers in the problem. This is called **defining a variable**. Then use the variable to write expressions for the other unspecified numbers in the problem.

Step 3 Solve the Problem

Use the strategy you chose in Step 2 to solve the problem.

Step 4 Check the Solution

Check your answer in the context of the original problem.

- Does your answer make sense?
- Does it fit the information in the problem?

EXAMPLE 1

FLOORS Ling's hallway is 10 feet long and 4 feet wide. He paid $200.00 to tile his hallway floor. How much did Ling pay per square foot for the tile?

Understand We are given the measurements of the hallway and the total cost of the tile. We are asked to find the cost of each square foot of tile.

Plan Write an equation. Let f represent the cost of each square foot of tile. The area of the hallway is 10×4 or 40 ft^2.

40	times	the cost per square foot	equals	200
40	$\cdot$	f	$=$	200

Solve $40 \cdot f = 200$ Find f mentally by asking, "What number times 40 equals 200?"

$f = 5$

The tile cost $5 per square foot.

Check If the tile costs $5 per square foot, then 40 square feet of tile costs $5 \cdot 40$ or $200. The answer makes sense.

When an exact value is needed, you can use estimation to check your answer.

EXAMPLE 2

TRAVEL Emily's family drove 254.6 miles. Their car used 19 gallons of gasoline. Describe the car's gas mileage.

Understand We are given the total miles driven and how much gasoline was used. We are asked to find the gas mileage of the car.

Plan Write an equation. Let G represent the car's gas mileage.

gas mileage = number of miles ÷ number of gallons used

$G = 254.6 \div 19$

Solve $G = 254.6 \div 19$

$= 13.4$ mi/gal

The car's gas mileage is 13.4 miles per gallon.

Check Use estimation to check your solution.

260 mi ÷ 20 gal = 13 mi/gal

Since the solution 13.4 is close to the estimate, the answer is reasonable.

Real-World Link

In a recent year, an average of $2.9 billion was spent on grooming and boarding dogs in the United States.

Source: American Pet Products Manufacturers Association

Exercises

For each problem, determine whether you need an estimate or an exact answer. Then use the four step problem-solving plan to solve.

1. **DRIVING** While on vacation, the Jacobson family drove 312.8 miles the first day, 177.2 miles the second day, and 209 miles the third day. About how many miles did they travel in all?
2. **PETS** Ms. Hernandez boarded her dog at a kennel for 4 days. It cost $18.90 per day, and she had a coupon for $5 off. What was the final cost for boarding her dog?
3. **MEASUREMENT** William is using a 1.75-liter container to fill a larger container that holds 14 liters of water. About how many times will he need to fill the smaller container?
4. **SEWING** Fabric costs $5.15 per yard. The drama department needs 18 yards of the fabric for their new play. About how much should they expect to pay?
5. **MONEY** The table shows donations that students gave to help purchase a new tree for the school. How much money did they donate in all?

Number of Students	Amount of Each Donation
20	$2.50
15	$3.25

6. **SHOPPING** Is $12 enough to buy a half gallon of milk for $2.30, a bag of apples for $3.99, and four cups of yogurt that cost $0.79 each? Explain.

0-2 Real Numbers

Objective

Classify and use real numbers.

KY Program of Studies

HS-NPO-S-NS2 Students will locate the position of a real number on the number line, find its distance from the origin and find the distance between two numbers on the number line.
HS-NPO-S-NS5 Students will compare and contrast number systems, including complex numbers as solutions to quadratic equations that do not have real solutions. *Also addresses HS-NPO-S-NS3, HS-NPO-S-E7, and HS-NPO-S-E8.*

New Vocabulary

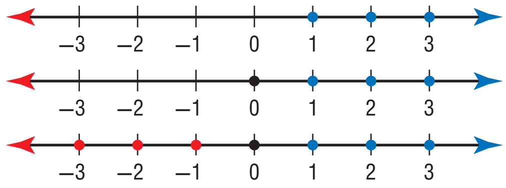
positive number
negative number
natural number
whole number
integer
rational number
square root
perfect square
irrational number
real number
graph
coordinate

A number line can be used to show the sets of natural numbers, whole numbers, integers, and rational numbers. Values greater than 0, or **positive numbers**, are listed to the right of 0, and values less than 0, or **negative numbers**, are listed to the left of 0.

natural numbers: 1, 2, 3, …

whole numbers: 0, 1, 2, 3, …

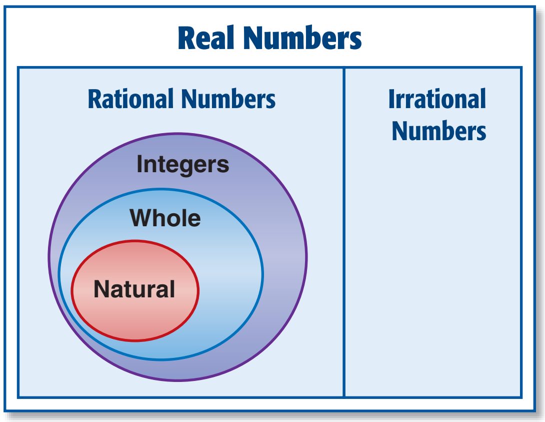
integers: …, −3, −2, −1, 0, 1, 2, 3, …



rational numbers: numbers that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$

A **square root** is one of two equal factors of a number. For example, one square root of 64, written as $\sqrt{64}$, is 8 since $8 \cdot 8$ or 8^2 is 64. Another square root of 64 is −8 since $(-8) \cdot (-8)$ or $(-8)^2$ is also 64. A number like 64, whose square root is a rational number, is called a **perfect square**. The square roots of a perfect square are rational numbers.

A number such as $\sqrt{3}$ is the square root of a number that is not a perfect square. It cannot be expressed as a terminating or repeating decimal; $\sqrt{3} \approx 1.73205\ldots$. Numbers that cannot be expressed as terminating or repeating decimals, or in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, are called **irrational numbers**. Irrational numbers and rational numbers together form the set of **real numbers**.



EXAMPLE 1 Classify Real Numbers

Name the set or sets of numbers to which each real number belongs.

a. $\frac{5}{22}$

Because 5 and 22 are integers and $5 \div 22 = 0.2272727\ldots$ or $0.2\overline{27}$, which is a repeating decimal, this number is a rational number.

b. $\sqrt{81}$

Because $\sqrt{81} = 9$, this number is a natural number, a whole number, an integer, and a rational number.

c. $\sqrt{56}$

Because $\sqrt{56} = 7.48331477\ldots$, which is not a repeating or terminating decimal, this number is irrational.

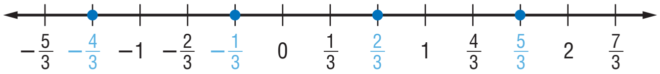
To graph a set of numbers means to draw, or plot, the points named by those numbers on a number line. The number that corresponds to a point on a number line is called the coordinate of that point. The rational numbers and the irrational numbers complete the number line.

> **StudyTip**
>
> **Graphs** The graph of $b \leq 4.5$ includes integers like 3 and -1, as well as rational numbers like $\frac{3}{8}$ and $-\frac{12}{13}$ and irrational numbers like $\sqrt{40}$ and π.

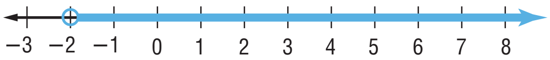
EXAMPLE 2 Graph Real Numbers

Graph each set of numbers on a number line.

a. $\left\{-\frac{4}{3}, -\frac{1}{3}, \frac{2}{3}, \frac{5}{3}\right\}$

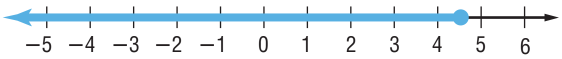


b. $x > -2$



The heavy arrow indicates that all numbers to the right of -2 are included in the graph. The circle at -2 indicates that -2 is not included in the graph.

c. $b \leq 4.5$



The heavy arrow indicates that all points to the left of 4.5 are included in the graph. The dot at 4.5 indicates that 4.5 is included in the graph.

d. $h \geq -3\frac{2}{5}$

The heavy arrow indicates that all points to the right of $-3\frac{2}{5}$ are included in the graph. The dot at $-3\frac{2}{5}$ indicates that $-3\frac{2}{5}$ is included in the graph.

Any repeating decimal can be written as a fraction.

EXAMPLE 3 Write Repeating Decimals as Fractions

Write $0.\overline{7}$ as a fraction in simplest form.

Step 1

$N = 0.777\ldots$	**Let N represent the repeating decimal.**
$10N = 10(0.777\ldots)$	**Since only one digit repeats, multiply each side by 10.**
$10N = 7.777\ldots$	**Simplify.**

Step 2 Subtract N from $10N$ to eliminate the part of the number that repeats.

$$10N = 7.777\ldots$$
$$\underline{-(N = 0.777\ldots)}$$

$9N = 7$	**Subtract.**
$\frac{9N}{9} = \frac{7}{9}$	**Divide each side by 9.**
$N = \frac{7}{9}$	**Simplify.**

Perfect squares can also be used to simplify square roots of rational numbers.

Key Concept — **Perfect Square** *For Your FOLDABLE*

Words Rational numbers with square roots that are rational numbers.

Examples 25 is a perfect square since $\sqrt{25} = 5$.

144 is a perfect square since $\sqrt{144} = 12$.

EXAMPLE 4 Square Roots

Simplify each square root.

a. $-\sqrt{\frac{49}{256}}$

$-\sqrt{\frac{49}{256}} = -\sqrt{\left(\frac{7}{16}\right)^2}$ — $7^2 = 49$ and $16^2 = 256$

$= -\frac{7}{16}$ — Simplify.

b. $\sqrt{\frac{41}{121}}$

$\sqrt{\frac{41}{121}} = \sqrt{\left(\frac{2}{11}\right)^2}$ — $2^2 = 4$ and $11^2 = 121$

$= \frac{2}{11}$ — Simplify.

StudyTip

Perfect Squares Keep a list of perfect squares in your notebook. Reference it when you need to simplify a square root.

You can estimate square roots of numbers that are not perfect squares.

EXAMPLE 5 Estimate Square Roots

Estimate each square root to the nearest whole number.

a. $\sqrt{15}$

Find the two perfect squares closest to 15. List some perfect squares.

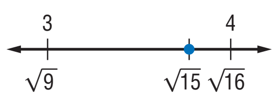
1, 4, 9, 16, 25, 36, …

15 is between 9 and 16.

$9 < 15 < 16$ — Write an inequality.

$\sqrt{9} < 15 < \sqrt{16}$ — Take the square root of each term.

$3 < \sqrt{15} < 4$ — Simplify.



Since 15 is closer to 16 than 9, the best whole-number estimate for $\sqrt{15}$ is 4.

b. $\sqrt{130}$

Find the two perfect squares closest to 121. List some perfect squares.

81, 100, 121, 144

130 is between 121 and 144.

$121 < 130 < 144$	**Write an inequality.**
$\sqrt{121} < \sqrt{130} < \sqrt{144}$	**Take the square root of each term.**
$11 < \sqrt{130} < 12$	**Simplify.**

Since 130 is closer to 121 than 144, the best whole number estimate for $\sqrt{130}$ is 11.

CHECK $\sqrt{130} \approx 11.4018\ldots$ **Use a calculator.**

Rounded to the nearest whole number, $\sqrt{130}$ is 11. So our estimate is valid.

StudyTip

Draw a Diagram Graphing points on a number line can help you analyze your estimate for accuracy.

Exercises

Name the set or sets of numbers to which each real number belongs.

1. $-\sqrt{64}$ **2.** $\frac{8}{3}$ **3.** $\sqrt{28}$ **4.** $\frac{56}{7}$

5. $-\sqrt{22}$ **6.** $\frac{36}{6}$ **7.** $-\frac{5}{12}$ **8.** $\frac{18}{3}$

9. $\sqrt{10.24}$ **10.** $\frac{-54}{19}$ **11.** $\sqrt{\frac{82}{20}}$ **12.** $-\frac{72}{8}$

Graph each set of numbers.

13. $\{-4, -2, 1, 5, 7\}$ **14.** $x < -3.5$ **15.** $x \geq -7$

16. $\{-4, -2, -1, 1, 3\}$ **17.** $\{\ldots -2, 0, 2, 4, 6\}$ **18.** $x > -12$

Write each repeating decimal as a fraction in simplest form.

19. $0.\overline{5}$ **20.** $0.\overline{4}$

21. $0.\overline{13}$ **22.** $0.\overline{21}$

Simplify each square root.

23. $-\sqrt{25}$ **24.** $\sqrt{1.44}$ **25.** $\pm\sqrt{\frac{16}{49}}$

26. $\sqrt{361}$ **27.** $\sqrt{49}$ **28.** $\pm\sqrt{0.64}$

29. $-\sqrt{6.25}$ **30.** $\sqrt{\frac{169}{196}}$ **31.** $\sqrt{\frac{25}{324}}$

Estimate each square root to the nearest whole number.

32. $\sqrt{31}$ **33.** $\sqrt{24}$ **34.** $\sqrt{112}$ **35.** $\sqrt{152}$

0-3

Operations with Integers

An integer is any number from the set {…, −3, −2, −1, 0, 1, 2, 3, …}. You can use a number line to add integers.

Objective

To add, subtract, multiply, and divide integers.

KY Program of Studies

HS-NPO-S-NS2 Students will locate the position of a real number on the number line, find its distance from the origin and find the distance between two numbers on the number line.

New Vocabulary

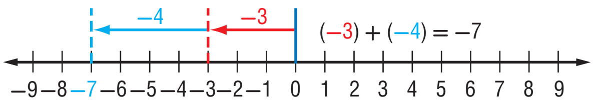
absolute value
opposites
additive inverses

EXAMPLE 1

Use a number line to find −3 + (−4).

Step 1 Draw an arrow from 0 to −3.

Step 2 Draw a second arrow 4 units to the left to represent adding −4.



The second arrow ends at −7. So, −3 + (−4) = −7.

You can also use absolute value to add integers. The **absolute value** of a number is its distance from 0 on the number line.

Same Signs (+ + or − −)		Different Signs (+ − or − +)	
3 + 5 = 8	3 and 5 are positive. Their sum is positive.	3 + (−5) = −2	−5 has the greater absolute value. Their sum is negative.
−3 + (−5) = −8	−3 and −5 are negative. Their sum is negative.	−3 + 5 = 2	5 has the greater absolute value. Their sum is positive.

EXAMPLE 2

Find −11 + (−7).

$-11 + (-7) = -(|-11| + |-7|)$ — **Add the absolute values. Both numbers are negative, so the sum is negative.**

$= -(11 + 7)$ — **Absolute values of nonzero numbers are always positive.**

$= -18$ — **Simplify.**

Every positive integer can be paired with a negative integer. These pairs are called **opposites**. A number and its opposite are **additive inverses** of each other. Additive inverses can be used when you subtract integers.

EXAMPLE 3

Find 18 − 23.

$18 - 23 = 18 + (-23)$ — **To subtract 23, add its inverse.**

$= -(|-23| - |18|)$ — **Subtract the absolute values. Because |−23| is greater than |18|, the result is negative.**

$= -(23 - 18)$ — **Absolute values of nonzero numbers are always positive.**

$= -5$ — **Simplify.**

StudyTip

Products and Quotients The product or quotient of two numbers having the *same sign* is positive. The product or quotient of two numbers having *different signs* is negative.

Same Signs (+ + or − −)		Different Signs (+ − or − +)	
3(5) = 15	3 and 5 are positive. Their product is positive.	3(−5) = −15	3 and −5 have different signs. Their product is negative.
−3(−5) = 15	−3 and −5 are negative. Their product is positive.	−3(5) = −15	−3 and 5 have different signs. Their product is negative.

EXAMPLE 4

Find each product or quotient.

a. 4(−5)
$4(-5) = -20$ — different signs ⟶ negative product

b. −51 ÷ (−3)
$-51 \div (-3) = 17$ — same sign ⟶ positive quotient

c. −12(−14)
$-12(-14) = 168$ — same sign ⟶ positive product

d. −63 ÷ 7
$-63 \div 7 = -9$ — different signs ⟶ negative quotient

Exercises

Find each sum or difference.

1. $-8 + 13$
2. $11 + (-19)$
3. $-19 - 8$
4. $-77 + (-46)$
5. $12 - 34$
6. $41 + (-56)$
7. $50 - 82$
8. $-47 - 13$
9. $-80 + 102$

Find each product or quotient.

10. $5(18)$
11. $60 \div 12$
12. $-12(15)$
13. $-64 \div (-8)$
14. $8(-22)$
15. $54 \div (-6)$
16. $30(14)$
17. $-23(5)$
18. $-200 \div 2$

19. WEATHER The outside temperature was −4°F in the morning and 13°F in the afternoon. By how much did the temperature increase?

20. DOLPHINS A dolphin swimming 24 feet below the ocean's surface dives 18 feet straight down. How many feet below the ocean's surface is the dolphin after it dives down?

21. MOVIES A movie theater gave out 50 coupons for $3 off each movie. What is the total amount of discounts provided by the theater?

22. WAGES Emilio earns $11 per hour. He works 14 hours a week. His employer withholds $32 from each paycheck for taxes. If he is paid weekly, what is the amount of his paycheck?

23. FINANCES Talia is working on a monthly budget. Her monthly income is $500. She has allocated $200 for savings, $100 for vehicle expenses, and $75 for clothing. How much is available to spend on entertainment?

0-4 Adding and Subtracting Rational Numbers

Objective

Add and subtract rational numbers.

KY Program of Studies

HS-NPO-S-NS1 Students will compare real numbers using order relations. HS-NPO-S-NO12 Students will develop fluency in operations with real numbers and matrices, using mental computation or paper-and-pencil calculations for simple cases and calculators and/or computers for more complicated cases.

You can use different methods to compare rational numbers. One way is to compare two fractions with common denominators. Another way is to compare decimals.

EXAMPLE 1

Replace ● with <, >, or = to make $\frac{2}{3}$ ● $\frac{5}{6}$ a true sentence.

Method 1 Write the fractions with the same denominator.

The least common denominator of $\frac{2}{3}$ and $\frac{5}{6}$ is 6.

$\frac{2}{3} = \frac{4}{6}$

$\frac{5}{6} = \frac{5}{6}$

Since $\frac{4}{6} < \frac{5}{6}$, $\frac{2}{3} < \frac{5}{6}$.

Method 2 Write as decimals.

Write $\frac{2}{3}$ and $\frac{5}{6}$ as decimals. You may want to use a calculator.

2 [÷] 3 [ENTER] .6666666667

so, $\frac{2}{3} = 0.\overline{6}$

5 [÷] 6 [ENTER] .8333333333

so, $\frac{5}{6} = 0.8\overline{3}$

Since $0.\overline{6} < 0.8\overline{3}$, $\frac{2}{3} < \frac{5}{6}$.

You can order rational numbers by writing all of the fractions as decimals.

EXAMPLE 2

Order $5\frac{2}{9}$, $5\frac{3}{8}$, 4.9, and $-5\frac{3}{5}$ from least to greatest.

$5\frac{2}{9} = 5.\overline{2}$ $\quad$ $5\frac{3}{8} = 5.375$

$4.9 = 4.9$ $\quad$ $-5\frac{3}{5} = -5.6$

$-5.6 < 4.9 < 5.\overline{2} < 5.375$. So, from least to greatest, the numbers are $-5\frac{3}{5}$, 4.9, $5\frac{2}{9}$, and $5\frac{3}{8}$.

To add or subtract fractions with the same denominator, add or subtract the numerators and write the sum or difference over the denominator.

EXAMPLE 3

Find each sum or difference. Write in simplest form.

a. $\frac{3}{5} + \frac{1}{5}$

$\frac{3}{5} + \frac{1}{5} = \frac{3+1}{5}$ **The denominators are the same. Add the numerators.**

$= \frac{4}{5}$ **Simplify.**

b. $\frac{7}{16} - \frac{1}{16}$

$\frac{7}{16} - \frac{1}{16} = \frac{7-1}{16}$ **The denominators are the same. Subtract the numerators.**

$= \frac{6}{16}$ **Simplify.**

$= \frac{3}{8}$ **Rename the fraction.**

c. $\frac{4}{9} - \frac{7}{9}$

$\frac{4}{9} - \frac{7}{9} = \frac{4-7}{9}$ **The denominators are the same. Subtract the numerators.**

$= -\frac{3}{9}$ **Simplify.**

$= -\frac{1}{3}$ **Rename the fraction.**

StudyTip

Mental Math If the denominators of the fractions are the same, you can use mental math to determine the sum or difference.

To add or subtract fractions with unlike denominators, first find the least common denominator (LCD). Rename each fraction with the LCD, and then add or subtract. Simplify if necessary.

EXAMPLE 4

Find each sum or difference. Write in simplest form.

a. $\frac{1}{2} + \frac{2}{3}$

$\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6}$ **The LCD for 2 and 3 is 6. Rename $\frac{1}{2}$ as $\frac{3}{6}$ and $\frac{2}{3}$ as $\frac{4}{6}$.**

$= \frac{3+4}{6}$ **Add the numerators.**

$= \frac{7}{6}$ or $\frac{1}{16}$ **Simplify.**

b. $\frac{3}{8} - \frac{1}{3}$

$\frac{3}{8} - \frac{1}{3} = \frac{9}{24} - \frac{8}{24}$ **The LCD for 8 and 3 is 24. Rename $\frac{3}{8}$ as $\frac{9}{24}$ and $\frac{1}{3}$ as $\frac{8}{24}$.**

$= \frac{9-8}{24}$ **Subtract the numerators.**

$= \frac{1}{24}$ **Simplify.**

c. $\frac{2}{5} - \frac{3}{4}$

$\frac{2}{5} - \frac{3}{4} = \frac{8}{20} - \frac{15}{20}$ **The LCD for 5 and 4 is 20. Rename $\frac{2}{5}$ as $\frac{8}{20}$ and $\frac{3}{4}$ as $\frac{15}{20}$.**

$= \frac{8-15}{20}$ **Subtract the numerators.**

$= -\frac{7}{20}$ **Simplify.**

StudyTip

Number Line Start at the first number. Then move 3.5 units left to find the solution.

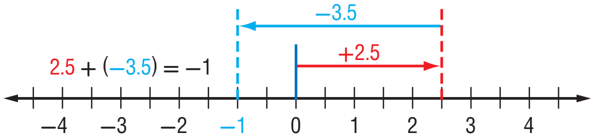
You can use a number line to add rational numbers.

EXAMPLE 5

Use a number line to find 2.5 + (−3.5).

Step 1 Draw an arrow from 0 to 2.5.

Step 2 Draw a second arrow 3.5 units to the left.



The second arrow ends at −1.

So, 2.5 + (−3.5) = −1.

You can also use absolute value to add rational numbers.

Same Signs (+ + or − −)		Different Signs (+ − or − +)	
3.1 + 2.5 = 5.6	3.1 and 2.5 are positive, so the sum is positive.	3.1 + (−2.5) = 0.6	3.1 has the greater absolute value, so the sum is positive.
−3.1 + (−2.5) = −5.6	−3.1 and −2.5 are negative, so the sum is negative.	−3.1 + 2.5 = −0.6	3.1 has the greater absolute value, so the sum is negative.

EXAMPLE 6

Find each sum.

a. $-13.12 + (-8.6)$

$-13.12 + (-8.6) = -(|-13.12| + |-8.6|)$ — **Both numbers are negative, so the sum is negative.**

$= -(13.12 + 8.6)$ — **Absolute values of nonzero numbers are always positive.**

$= -21.72$ — **Simplify.**

b. $\frac{7}{16} + \left(-\frac{3}{8}\right)$

$\frac{7}{16} + \left(-\frac{3}{8}\right) = \frac{7}{16} + \left(-\frac{6}{16}\right)$ — **The LCD is 16. Replace $-\frac{3}{8}$ with $-\frac{6}{16}$.**

$= \left(+\left|\frac{7}{16}\right| - \left|-\frac{6}{16}\right|\right)$ — **Subtract the absolute values. Because $\left|\frac{7}{16}\right|$ is greater than $\left|-\frac{6}{16}\right|$, the result is positive.**

$= +\left(\frac{7}{16} - \frac{6}{16}\right)$ — **Absolute values of nonzero numbers are always positive.**

$= \frac{1}{16}$ — **Simplify.**

To subtract a negative rational number, add its inverse.

EXAMPLE 7

Find $-32.25 - (-42.5)$.

$-32.25 - (-42.5) = -32.25 + 42.5$	To subtract -42.5, add its inverse.
$= \lvert 42.5 \rvert - \lvert -32.25 \rvert$	Subtract the absolute values. Because $\lvert 42.5 \rvert$ is greater than $\lvert -32.25 \rvert$, the result is positive.
$= 42.5 - 32.25$	Absolute values of nonzero numbers are always positive.
$= 10.25$	Simplify.

Exercises

Replace each ● with <, >, or = to make a true sentence.

1. $-\frac{5}{8}$ ● $\frac{3}{8}$
2. $\frac{4}{5}$ ● 710
3. $\frac{5}{6}$ ● 0.875
4. 1.2 ● $1\frac{2}{9}$
5. $\frac{8}{15}$ ● $0.5\overline{3}$
6. $-\frac{7}{11}$ ● $-\frac{2}{3}$

Order each set of rational numbers from least to greatest.

7. $3.8, 3.06, 3\frac{1}{6}, 3\frac{3}{4}$
8. $2\frac{1}{4}, 1\frac{7}{8}, 1.75, 2.4$
9. $0.11, -\frac{1}{9}, -0.5, \frac{1}{10}$
10. $-4\frac{3}{5}, -3\frac{2}{5}, -4.65, -4.09$

Find each sum or difference. Write in simplest form.

11. $\frac{2}{5} + \frac{1}{5}$
12. $\frac{3}{9} + \frac{4}{9}$
13. $\frac{5}{16} - \frac{4}{16}$
14. $\frac{6}{7} - \frac{3}{7}$
15. $\frac{2}{3} + \frac{1}{3}$
16. $\frac{5}{8} + \frac{7}{8}$
17. $\frac{4}{3} + \frac{4}{3}$
18. $\frac{7}{15} - \frac{2}{15}$
19. $\frac{1}{3} - \frac{2}{9}$
20. $\frac{1}{2} + \frac{1}{4}$
21. $\frac{1}{2} - \frac{1}{3}$
22. $\frac{3}{7} + \frac{5}{14}$
23. $\frac{7}{10} - \frac{2}{15}$
24. $\frac{3}{8} + \frac{1}{6}$
25. $\frac{13}{20} - \frac{2}{5}$

Real-World Link

About 97% of the water is saltwater from oceans. Of the freshwater, only 1% is on the surface from lakes, rivers, and swamps.

Source: U.S. Geological Survey

Find each sum or difference. Write in simplest form if necessary.

26. $-1.6 + (-3.8)$
27. $-32.4 + (-4.5)$
28. $-38.9 + 24.2$
29. $-9.16 - 10.17$
30. $26.37 + (-61.1)$
31. $72.5 - (-81.3)$
32. $43.2 + (-27.9)$
33. $79.3 - (-14)$
34. $1.34 - (-0.458)$
35. $-\frac{1}{6} - \frac{2}{3}$
36. $\frac{1}{2} - \frac{4}{5}$
37. $-\frac{2}{5} + \frac{17}{20}$
38. $-\frac{4}{5} + \left(-\frac{1}{3}\right)$
39. $-\frac{1}{12} - \left(-\frac{3}{4}\right)$
40. $-\frac{7}{8} - \left(-\frac{3}{16}\right)$

41. **GEOGRAPHY** About $\frac{7}{10}$ of the surface of Earth is covered by water. The rest of the surface is covered by land. How much of Earth's surface is covered by land?

0-5 Multiplying and Dividing Rational Numbers

Objective

Multiply and divide rational numbers.

KY Program of Studies

HS-NPO-S-E7 Students will estimate solutions to problems with real numbers in both realistic and mathematical situations.

New Vocabulary

multiplicative inverses
reciprocals

The product or quotient of two rational numbers having the *same sign* is positive. The product or quotient of two rational numbers having *different signs* is negative.

EXAMPLE 1

Find each product or quotient.

a. $7.2(-0.2)$

different signs ⟶ negative product

$7.2(-0.2) = -1.44$

b. $-23.94 \div (-10.5)$

same sign ⟶ positive quotient

$-23.94 \div (-10.5) = 2.28$

To multiply fractions, multiply the numerators and multiply the denominators. If the numerators and denominators have common factors, you can simplify before you multiply by canceling.

EXAMPLE 2

Find each product.

a. $\frac{2}{5} \cdot \frac{1}{3}$

$\frac{2}{5} \cdot \frac{1}{3} = \frac{2 \cdot 1}{5 \cdot 3}$ — Multiply the numerators. Multiply the denominators.

$= \frac{2}{15}$ — Simplify.

b. $\frac{3}{5} \cdot 1\frac{1}{2}$

$\frac{3}{5} \cdot 1\frac{1}{2} = \frac{3}{5} \cdot \frac{3}{2}$ — Write $1\frac{1}{2}$ as an improper fraction.

$= \frac{3 \cdot 3}{5 \cdot 2}$ — Multiply the numerators. Multiply the denominators.

$= \frac{9}{10}$ — Simplify.

c. $\frac{1}{4} \cdot \frac{2}{9}$

$\frac{1}{4} \cdot \frac{2}{9} = \frac{1}{\cancel{4}_2} \cdot \frac{\cancel{2}^1}{9}$ — Divide by the GCF, 2.

$= \frac{1 \cdot 1}{2 \cdot 9}$ or $\frac{1}{18}$ — Multiply the numerators. Multiply the denominators and simplify.

EXAMPLE 3

Find $-\left(\frac{3}{4}\right)\left(\frac{3}{8}\right)$.

$\left(-\frac{3}{4}\right)\left(\frac{3}{8}\right) = -\left(\frac{3}{4} \cdot \frac{3}{8}\right)$ — different signs ⟶ negative product

$= -\left(\frac{3 \cdot 3}{4 \cdot 8}\right)$ or $\frac{9}{32}$ — Multiply the numerators. Multiply the denominators and simplify

Two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals**.

EXAMPLE 4

Name the reciprocal of each number.

a. $\frac{3}{8}$

$\frac{3}{8} \cdot \frac{8}{3} = 1$ **The product is 1.**

The reciprocal of $\frac{3}{8}$ is $\frac{8}{3}$.

b. $2\frac{4}{5}$

$2\frac{4}{5} = \frac{14}{5}$ **Write $2\frac{4}{5}$ as $\frac{14}{5}$.**

$\frac{14}{5} \cdot \frac{5}{14} = 1$ **The product is 1.**

The reciprocal of $2\frac{4}{5}$ is $\frac{5}{14}$.

To divide one fraction by another fraction, multiply the dividend by the multiplicative inverse of the divisor.

EXAMPLE 5

Find each quotient.

a. $\frac{1}{3} \div \frac{1}{2}$

$\frac{1}{3} \div \frac{1}{2} = \frac{1}{3} \cdot \frac{2}{1}$ **Multiply $\frac{1}{3}$ by $\frac{2}{1}$, the reciprocal of $\frac{1}{2}$.**

$= \frac{2}{3}$ **Simplify.**

b. $\frac{3}{8} \div \frac{2}{3}$

$\frac{3}{8} \div \frac{2}{3} = \frac{3}{8} \cdot \frac{3}{2}$ **Multiply $\frac{3}{8}$ by $\frac{3}{2}$, the reciprocal of $\frac{2}{3}$.**

$= \frac{9}{16}$ **Simplify.**

c. $\frac{3}{4} \div 2\frac{1}{2}$

$\frac{3}{4} \div 2\frac{1}{2} = \frac{3}{4} \div \frac{5}{2}$ **Write $2\frac{1}{2}$ as a mixed number.**

$= \frac{3}{4} \cdot \frac{2}{5}$ **Multiply $\frac{3}{4}$ by $\frac{2}{5}$, the reciprocal of $2\frac{1}{2}$.**

$= \frac{6}{20}$ or $\frac{3}{10}$ **Simplify.**

d. $-\frac{1}{5} \div \left(-\frac{3}{10}\right)$

$-\frac{1}{5} \div \left(-\frac{3}{10}\right) = -\frac{1}{5} \cdot \left(-\frac{10}{3}\right)$ **Multiply $-\frac{1}{5}$ by $-\frac{10}{3}$, the reciprocal of $-\frac{3}{10}$.**

$= \frac{10}{15}$ or $\frac{2}{3}$ **Same sign ⟶ positive quotient; simplify.**

StudyTip

Use Estimation You can justify your answer by using estimation. $\frac{3}{8}$ is close to $\frac{1}{2}$ and $\frac{2}{3}$ is close to 1. So, $\frac{1}{2}$ divided by 1 is $\frac{1}{2}$.

StudyTip

Negative Fractions A negative fraction can be written as $-\frac{1}{2}$ or $\frac{-1}{2}$.

Exercises

Find each product or quotient. Round to the nearest hundredth if necessary.

1. $6.5(0.13)$

2. $-5.8(2.3)$

3. $42.3 \div (-6)$

4. $-14.1(-2.9)$

5. $-78 \div (-1.3)$

6. $108 \div (-0.9)$

7. $0.75(-6.4)$

8. $-23.94 \div 10.5$

9. $-32.4 \div 21.3$

Find each product. Simplify before multiplying if possible.

10. $\frac{3}{4} \cdot \frac{1}{5}$

11. $\frac{2}{5} \cdot \frac{3}{7}$

12. $-\frac{1}{3} \cdot \frac{2}{5}$

13. $-\frac{2}{3} \cdot \left(-\frac{1}{11}\right)$

14. $2\frac{1}{2} \cdot \left(-\frac{1}{4}\right)$

15. $3\frac{1}{2} \cdot 1\frac{1}{2}$

16. $\frac{2}{9} \cdot \frac{1}{2}$

17. $\frac{3}{2} \cdot \left(-\frac{1}{3}\right)$

18. $\frac{1}{3} \cdot \frac{6}{5}$

19. $-\frac{9}{4} \cdot \frac{1}{18}$

20. $\frac{11}{3} \cdot \frac{9}{44}$

21. $\left(-\frac{30}{11}\right) \cdot \left(-\frac{1}{3}\right)$

22. $-\frac{3}{5} \cdot \frac{5}{6}$

23. $\left(-\frac{1}{3}\right)\left(-7\frac{1}{2}\right)$

24. $\frac{2}{7} \cdot 4\frac{2}{3}$

Name the reciprocal of each number.

25. $\frac{6}{7}$

26. $\frac{1}{22}$

27. $-\frac{14}{23}$

28. $2\frac{3}{4}$

29. $-5\frac{1}{3}$

30. $3\frac{3}{4}$

Find each quotient.

31. $\frac{2}{3} \div \frac{1}{3}$

32. $\frac{16}{9} \div \frac{4}{9}$

33. $\frac{3}{2} \div \frac{1}{2}$

34. $\frac{3}{7} \div \left(-\frac{1}{5}\right)$

35. $-\frac{9}{10} \div 3$

36. $\frac{1}{2} \div \frac{3}{5}$

37. $2\frac{1}{4} \div \frac{1}{2}$

38. $-1\frac{1}{3} \div \frac{2}{3}$

39. $\frac{11}{12} \div 1\frac{2}{3}$

40. $4 \div \left(-\frac{2}{7}\right)$

41. $-\frac{1}{3} \div \left(-1\frac{1}{5}\right)$

42. $\frac{3}{25} \div \frac{2}{15}$

43. **PIZZA** A large pizza at Pizza Shack has 12 slices. If Bobby ate $\frac{1}{4}$ of the pizza, how many slices of pizza did he eat?

44. **MUSIC** Samantha practices the flute for $4\frac{1}{2}$ hours each week. How many hours does she practice in a month?

45. **BAND** How many band uniforms can be made with $131\frac{3}{4}$ yards of fabric if each uniform requires $3\frac{7}{8}$ yards?

46. **CARPENTRY** How many boards, each 2 feet 8 inches long, can be cut from a board 16 feet long if there is no waste?

47. **SEWING** How many 9-inch ribbons can be cut from $1\frac{1}{2}$ yards of ribbon?

0-6 The Percent Proportion

Objective
Use and apply the percent proportion.

Vocabulary
percent
percent proportion

A **percent** is a ratio that compares a number to 100. To write a percent as a fraction, express the ratio as a fraction with a denominator of 100. Fractions should be expressed in simplest form.

EXAMPLE 1

Express each percent as a fraction.

a. 79%

$79\% = \frac{79}{100}$ — **Definition of percent**

b. 107%

$107\% = \frac{107}{100}$ — **Definition of percent**

$= 1\frac{7}{100}$ — **Simplify.**

c. 0.5%

$0.5\% = \frac{0.5}{100}$ — **Definition of percent**

$= \frac{5}{1000}$ — **Multiply the numerator and denominator by 10 to eliminate the decimal.**

$= \frac{1}{200}$ — **Simplify.**

In the **percent proportion**, the ratio of a part of something to the whole (base) is equal to the percent written as a fraction.

$$\frac{a}{b} = \frac{p}{100}$$

part → a; whole → b; percent → p

Example: 25% of 40 is 10. (percent: 25%; whole: 40; part: 10)

You can use the percent proportion to find the part.

EXAMPLE 2

40% of 30 is what number?

$\frac{a}{b} = \frac{p}{100}$ — **The percent is 40, and the base is 30. Let *a* represent the part.**

$\frac{a}{30} = \frac{40}{100}$ — **Replace *b* with 30 and *p* with 40.**

$100a = 30(40)$ — **Find the cross products.**

$100a = 1200$ — **Simplify.**

$\frac{100a}{100} = \frac{1200}{100}$ — **Divide each side by 100.**

$a = 12$ — **Simplify.**

The part is 12. So, 40% of 30 is 12.

You can also use the percent proportion to find the percent of the base.

EXAMPLE 3

SURVEYS Kelsey took a survey of some of the students in her lunch period. 42 out of the 70 students Kelsey surveyed said their family had a pet. What percent of the students had pets?

$\frac{a}{b} = \frac{p}{100}$ The part is 42, and the base is 70. Let p represent the percent.

$\frac{42}{70} = \frac{p}{100}$ Replace a with 42 and b with 70.

$4200 = 70p$ Find the cross products.

$\frac{4200}{70} = \frac{70p}{70}$ Divide each side by 70.

$60 = p$ Simplify.

The percent is 60, so $\frac{60}{100}$ or 60% of the students had pets.

StudyTip

Percent Proportion In percent problems, the whole, or base usually follows the word *of*.

EXAMPLE 4

67.5 is 75% of what number?

$\frac{a}{b} = \frac{p}{100}$ The percent is 75, and the part is 67.5. Let b represent the base.

$\frac{67.5}{b} = \frac{75}{100}$ Replace a with 67.5 and p with 75.

$6750 = 75b$ Find the cross products.

$\frac{6750}{75} = \frac{75b}{75}$ Divide each side by 75.

$90 = b$ Simplify.

The base is 90, so 67.5 is 75% of 90.

Exercises

Express each percent as a fraction in simplest form.

1. 5%
2. 60%
3. 11%
4. 120%
5. 78%
6. 2.5%
7. 0.6%
8. 0.4%
9. 1400%

Use the percent proportion to find each number.

10. 25 is what percent of 125?
11. 16 is what percent of 40?
12. 14 is 20% of what number?
13. 50% of what number is 80?
14. What number is 25% of 18?
15. Find 10% of 95.

Use the percent proportion to find each number.

16. What percent of 48 is 30?
17. What number is 150% of 32?
18. 5% of what number is 3.5?
19. 1 is what percent of 400?
20. Find 0.5% of 250.
21. 49 is 200% of what number?
22. 15 is what percent of 12?
23. 36 is what percent of 24?

24. **BASKETBALL** Madeline usually makes 85% of her shots in basketball. If she attempts 20, how many will she likely make?

25. **TEST SCORES** Brian answered 36 items correctly on a 40-item test. What percent did he answer correctly?

26. **CARD GAMES** Juanita told her dad that she won 80% of the card games she played yesterday. If she won 4 games, how many games did she play?

StudyTip

Word Problems
When a problem starts with the result and asks for something that happened earlier, work backward.

27. **SOLUTIONS** A glucose solution is prepared by dissolving 6 milliliters of glucose in 120 milliliters of pure solution. What is the percent of glucose in the pure solution?

28. **DRIVER'S ED** Kara needs to get a 75% on her driving education test in order to get her license. If there are 35 questions on the test how many does she need to answer correctly?

29. **HEALTH** The U.S. Food and Drug Administration require food manufacturers to label their products with a nutritional label. The nutritional label below shows a portion of the information from a package of macaroni and cheese.

Nutrition Facts

Serving Size 1 cup (228g)		
Servings per container 2		
Amount per serving		
Calories 250	Calories from Fat 110	
		%Daily value*
Total Fat	12g	18%
Saturated Fat	3g	15%
Cholesterol	30mg	10%
Sodium	470mg	20%
Total Carbohydrate	31g	10%
Dietary Fiber	0g	0%
Sugars	5g	
Protein	5g	
Vitamin A 4%	•	Vitamin C 2%
Calcium 20%	•	Iron 4%

a. The label states that a serving contains 3 grams of saturated fat, which is 15% of the daily value recommended for a 2000-Calorie diet. How many grams of saturated fat are recommended for a 2000-Calorie diet?

b. The 470 milligrams of sodium (salt) in the macaroni and cheese is 20% of the recommended daily value. What is the recommended daily value of sodium?

c. For a healthy diet, the National Research Council recommends that no more than 30 percent of the total Calories come from fat. What percent of the Calories in a serving of this macaroni and cheese come from fat?

30. **TEST SCORES** The table shows the number of points each student in Will's study group earned on the most recent math test. There were 88 points possible on the test.

Name	Will	Penny	Cheng	Minowa	Rob
Score	72	68	81	87	75

a. Find Will's percent correct on the test.

b. Find Cheng's percent correct on the test.

c. Find Rob's percent correct on the test.

d. What was the highest percentage? The lowest?

31. **PET STORE** In a pet store, 15% of the animals are hamsters. If the store has 40 animals, how many of them are hamsters?

0-7 Perimeter

Objective

Find the perimeter of two-dimensional figures.

KY Program of Studies

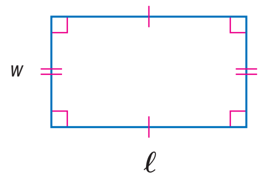
HS-M-S-SM2 Students will apply to both real world and mathematical situations US Customary and metric systems of measurement.

New Vocabulary

perimeter
circle
diameter
circumference
center
radius

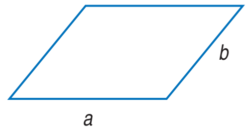
Perimeter is the distance around a geometric figure. Perimeter is measured in linear units.

Rectangle



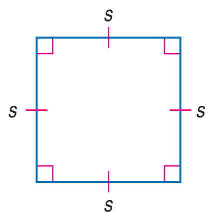
$P = 2(\ell + w)$ or
$P = 2\ell + 2w$

Parallelogram



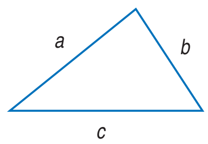
$P = 2(a + b)$ or
$P = 2a + 2b$

Square



$P = 4s$

Triangle

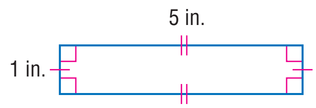


$P = a + b + c$

EXAMPLE 1

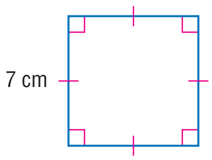
Find the perimeter of each figure.

a. a rectangle with a length of 1 inch and a width of 5 inches



$P = 2(\ell + w)$	**Perimeter**
$= 2(5 + 1)$	**$\ell = 5$, $w = 1$**
$= 2(6)$	**Add.**
$= 12$	The perimeter is 12 inches.

b. a square with a side length of 7 centimeters

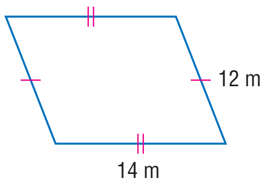


$P = 4s$	**Perimeter formula**
$= 4(7)$	**Replace s with 7.**
$= 28$	The perimeter is 28 centimeters.

EXAMPLE 2

Find the perimeter of each figure.

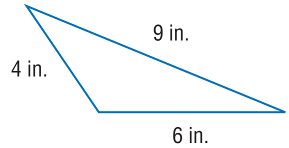
a.



$P = 2(a + b)$	**Perimeter formula**
$= 2(14 + 12)$	***a* = 14, *b* = 12**
$= 2(26)$	**Add.**
$= 52$	**Multiply.**

The perimeter of the parallelogram is 52 meters.

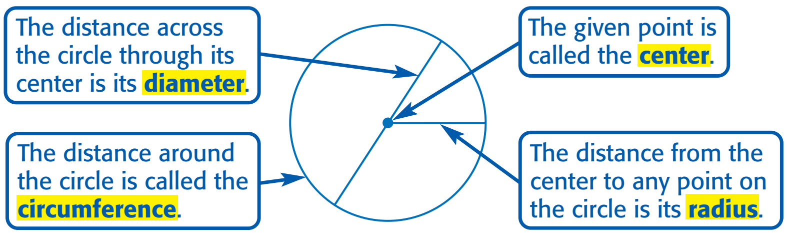
b.



$P = a + b + c$	**Perimeter formula**
$= 4 + 6 + 9$	***a* = 4, *b* = 6, *c* = 9**
$= 19$	**Add.**

The perimeter of the triangle is 19 inches.

A **circle** is the set of all points in a plane that are the same distance from a given point.



The formula for the circumference of a circle is $C = \pi d$ or $C = 2\pi r$.

EXAMPLE 3

Find the circumference of each circle to the nearest tenth.

a. The radius is 4 feet.

$C = 2\pi r$	**Circumference formula**
$= 2\pi(4)$	**Replace *r* with 4.**
$= 8\pi$	**Simplify.**

The exact circumference is 8π feet.

8 [π] [ENTER] 25.13274123

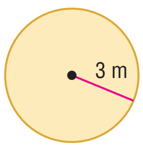
The circumference is about 25.1 feet.

b. The diameter is 15 centimeters.

$C = \pi d$	**Circumference formula**
$= \pi(15)$	**Replace d with 15.**
$= 15\pi$	**Simplify.**
≈ 47.1	**Use a calculator to evaluate 15π.**

The circumference is about 47.1 centimeters.

c.



$C = 2\pi r$	**Circumference formula**
$= 2\pi(3)$	**Replace *r* with 3.**
$= 6\pi$	**Simplify.**
≈ 18.8	**Use a calculator to evaluate 6π.**

The circumference is about 18.8 meters.

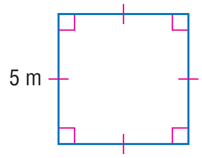
Exercises

StudyTip

Congruent Marks The hash marks on the figures indicate sides that have the same length.

Find the perimeter of each figure.

1.



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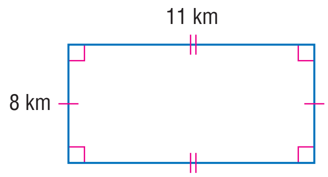

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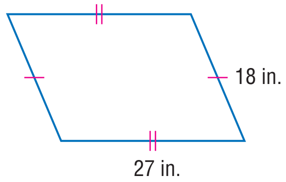

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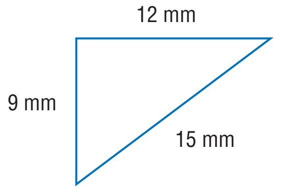


5. a square with side length 8 inches

6. a rectangle with length 9 centimeters and width 3 centimeters

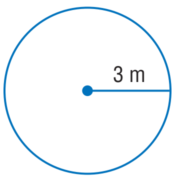
7. a triangle with sides 4 feet, 13 feet, and 12 feet

8. a parallelogram with side lengths $6\frac{1}{4}$ inches and 5 inches

9. a quarter-circle with a radius of 7 inches

Find the circumference of each circle. Round to the nearest tenth.

10.



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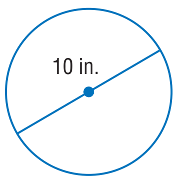

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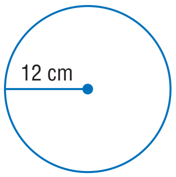


13. **GARDENS** A square garden has a side length of 5.8 meters. What is the perimeter of the garden?

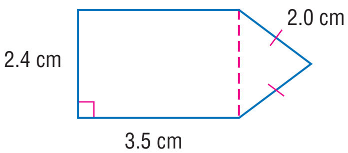
14. **ROOMS** A rectangular room is $12\frac{1}{2}$ feet wide and 14 feet long. What is the perimeter of the room?

15. **CYCLING** The tire for a 10-speed bicycle has a diameter of 27 inches. Find the distance the bicycle will travel in 10 rotations of the tire. Round to the nearest tenth.

16. **GEOGRAPHY** Earth's circumference is approximately 25,000 miles. If you could dig a tunnel to the center of the Earth, how long would the tunnel be? Round to the nearest tenth mile.

Find the perimeter of each figure. Round to the nearest tenth if necessary.

17.



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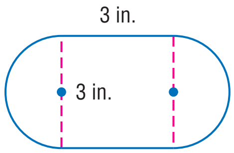

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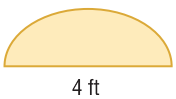

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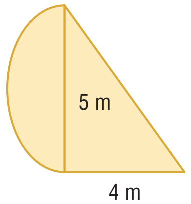


0-8 Area

Objective

Find the area of two-dimensional figures.

KY Program of Studies

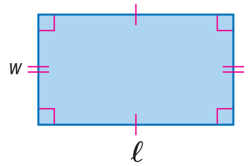
HS-M-S-SM2 Students will apply to both real world and mathematical situations US Customary and metric systems of measurement.
HS-NPO-S-NO12
Students will develop fluency in operations with real numbers and matrices, using mental computation or paper-and-pencil calculations for simple cases and calculators and/or computers for more complicated cases.

New Vocabulary

area

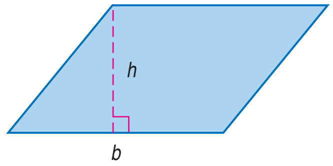
Area is the number of square units needed to cover a surface. Area is measured in square units.

Rectangle



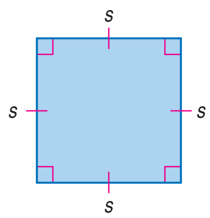
$A = \ell w$

Parallelogram



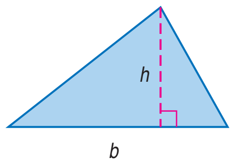
$A = bh$

Square



$A = s^2$

Triangle

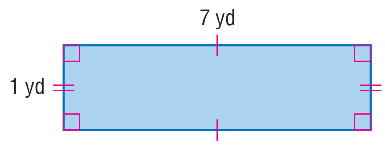


$A = \frac{1}{2}bh$

EXAMPLE 1

Find the area of each figure.

a. a rectangle that has a length of 7 yards and a width of 1 yard

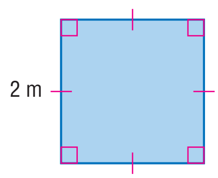


$A = \ell w$ **Area formula**

$= 7(1)$ **$\ell = 7, w = 1$**

$= 7$ The area of the rectangle is 7 square yards.

b. a square that has a side length of 2 meters



$A = s^2$ **Area formula**

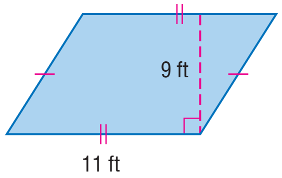
$= 2^2$ **$s = 2$**

$= 4$ The area is 4 square meters.

EXAMPLE 2

Find the area of each figure.

a. a parallelogram that has a base of 11 feet and a height of 9 feet



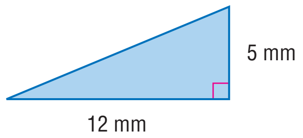
$A = bh$ **Area formula**

$= 11(9)$ **$b = 11$, $h = 9$**

$= 99$ **Multiply.**

The area is 99 square feet.

b. a triangle that has a base of 12 millimeters and a height of 5 millimeters



$A = \frac{1}{2}bh$ **Area formula**

$= \frac{1}{2}(12)(5)$ **$b = 12$, $h = 5$**

$= 30$ **Multiply.**

The area is 30 square millimeters.

The formula for the area of a circle is $A = \pi r^2$.

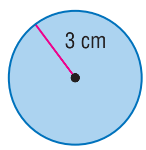
EXAMPLE 3

Find the area of each circle to the nearest tenth.

StudyTip

Mental Math You can use mental math to check your solutions. Square the radius and then multiply by 3.

a. The radius is 3 centimeters.



$A = \pi r^2$ **Area formula**

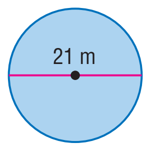
$= \pi(3)^2$ **Replace r with 3.**

$= 9\pi$ **Simplify.**

≈ 28.3 **Use a calculator to evaluate 9π.**

The area is about 28.3 square centimeters.

b. The diameter is 21 meters.



$A = \pi r^2$ **Area formula**

$= \pi(10.5)^2$ **Replace r with 10.5.**

$= 110.25\pi$ **Simplify.**

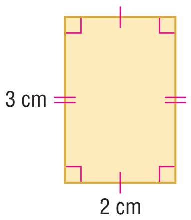
$= 346.4$ **Use a calculator to evaluate 110.25π.**

The area is about 346.4 square meters.

Exercises

Find the area of each figure.

1.



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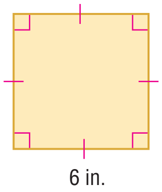

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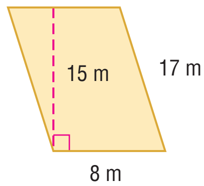


Find the area of each figure.

4. a triangle with a base 12 millimeters and height 11 millimeters

5. a square with side length 9 feet

6. a rectangle with length 8 centimeters and width 2 centimeters

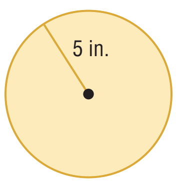
7. a triangle with a base 6 feet and height 3 feet

8. a quarter-circle with a diameter of 4 meters

9. a semi-circle with a radius of 3 inches

Find the area of each circle. Round to the nearest tenth.

10.



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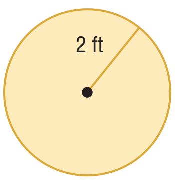

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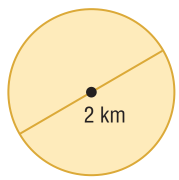


13. The radius is 4 centimeters.

14. The radius is 7.2 millimeters.

15. The diameter is 16 inches.

16. The diameter is 25 feet.

17. **RECREATION** The Granville Parks and Recreation Department uses an empty city lot for a community vegetable garden. Each participant is allotted a space of 18 feet by 90 feet for a garden. What is the area of each plot?

18. **CAMPING** The square floor of a tent has an area of 49 square feet. What is the side length of the tent?

19. **PUBLIC SAFETY** The sound emitted from the siren of a tornado warning system can be heard for a 2.5-mile radius. Find the area of the region that hears the siren. Round to the nearest tenth square mile.

20. **HISTORY** Stonehenge is an ancient monument in Wiltshire, England. The giant stones of Stonehenge are arranged in a circle 30 meters in diameter. Find the area of the circle. Round to the nearest tenth square meter.

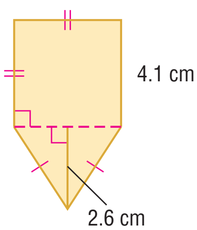
Real-World Link

It is estimated that Stonehenge was built around 2300 B.C. The construction of the monument took place in three phases.

Source: Window on Britain

Find the area of each figure. Round to the nearest tenth.

21.



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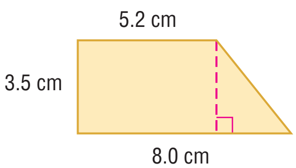

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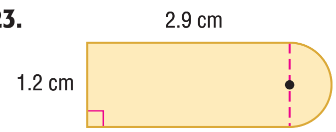


0-9 Volume

Objective

Find the volume of rectangular prisms.

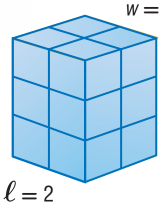
KY Program of Studies

HS-M-S-MPA3 Students will determine the surface area and volume of right rectangular prisms, pyramids, cylinders, cones and spheres in realistic problems.
HS-G-S-CG7 Students will investigate conjectures and solve problems involving two-dimensional figures and three-dimensional objects represented graphically.

New Vocabulary

volume

Volume is the measure of space occupied by a solid. Volume is measured in cubic units.



To find the volume of a rectangular prism, multiply the length times the width times the height. The formula for the volume of a rectangular prism is shown below.

$$V = \ell \cdot w \cdot h$$

The prism at the right has a volume of $2 \cdot 2 \cdot 3$ or 12 cubic units.

EXAMPLE 1

Find the volume of each rectangular prism.

a. The length is 8 centimeters, the width is 1 centimeter, and the height is 5 centimeters.

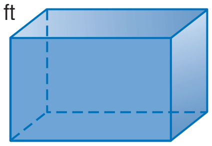
$V = \ell \cdot w \cdot h$ — **Volume formula**

$= 8 \cdot 1 \cdot 5$ — **Replace ℓ with 8, w with 1, and h with 5.**

$= 40$ — **Simplify.**

The volume is 40 cubic centimeters.

b.



The prism has a length of 4 feet, width of 2 feet, and height of 3 feet.

$V = \ell \cdot w \cdot h$ — **Volume formula**

$= 4 \cdot 2 \cdot 3$ — **Replace ℓ with 4, w with 2, and h with 3.**

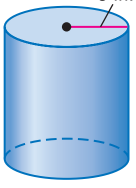
$= 24$ — **Simplify.**

The volume is 24 cubic feet.

The volume of a solid is the product of the area of the base and the height of the solid. For a cylinder, the area of the base is πr^2. So the volume is $V = \pi r^2 h$.

EXAMPLE 2

Find the volume of the cylinder.



$V = \pi r^2 h$ — **Volume of a cylinder**

$= \pi(3^2)6$ — **$r = 3$, $h = 6$**

$= 54\pi$ — **Simplify.**

≈ 169.6 — **Use a calculator.**

The volume is 169.6 cubic inches.

Exercises

Find the volume of each rectangular prism given the length, width, and height.

1. $\ell = 5$ cm, $w = 3$ cm, $h = 2$ cm

2. $\ell = 10$ m, $w = 10$ m, $h = 1$ m

3. $\ell = 6$ yd, $w = 2$ yd, $h = 4$ yd

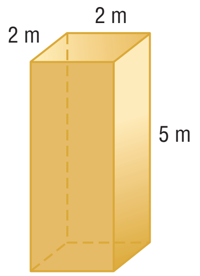
4. $\ell = 2$ in., $w = 5$ in., $h = 12$ in.

5. $\ell = 13$ ft, $w = 9$ ft, $h = 12$ ft

6. $\ell = 7.8$ mm, $w = 0.6$ mm, $h = 8$ mm

Find the volume of each rectangular prism.

7.



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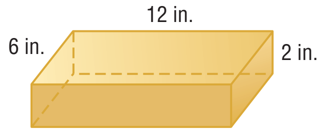


9. **GEOMETRY** A cube measures 3 meters on a side. What is its volume?

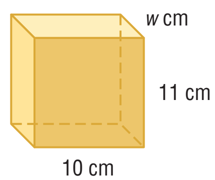
10. **AQUARIUMS** An aquarium is 8 feet long, 5 feet wide, and 5.5 feet deep. What is the volume of the tank?

StudyTip

Draw a Diagram Draw a diagram to organize the information given in the problem.

11. **COOKING** What is the volume of a microwave oven that is 18 inches wide by 10 inches long with a depth of $11\frac{1}{2}$ inches?

12. **BOXES** A cardboard box is 32 inches long, 22 inches wide, and 16 inches tall. What is the volume of the box?



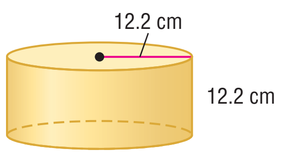
13. **SWIMMING POOLS** A children's rectangular pool holds 480 cubic feet of water. What is the depth of the pool if its length is 30 feet and its height is 16 feet?

14. **BAKING** A rectangular cake pan has a volume of 234 cubic inches. If the length of the pan is 9 inches and the width is 13 inches, what is the height of the pan?

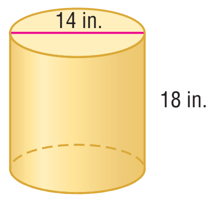
15. **GEOMETRY** The volume of the rectangular prism at the right is 440 cubic centimeters. What is the width?

Find the volume of each cylinder.

16.



17.



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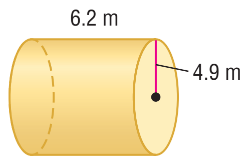


19. **FIREWOOD** Firewood is usually sold by a measure known as a *cord*. A full cord may be a stack $8 \times 4 \times 4$ feet or a stack $8 \times 8 \times 2$ feet.

a. What is the volume of a full cord of firewood?

b. A "short cord" of wood is $8 \times 4 \times$ the length of the logs. What is the volume of a short cord of $2\frac{1}{2}$-foot logs?

c. If you have an area that is 12 feet long and 2 feet wide in which to store your firewood, how high will the stack be if it is a full cord of wood?

0-10

Surface Area

Surface area is the sum of the areas of all the surfaces, or faces, of a solid. Surface area is measured in square units.

Objective

Find the surface area of rectangular prisms.

KY Program of Studies

HS-M-S-MPA3 Students will determine the surface area and volume of right rectangular prisms, pyramids, cylinders, cones and spheres in realistic problems.
HS-G-S-CG7 Students will investigate conjectures and solve problems involving two-dimensional figures and three-dimensional objects represented graphically.

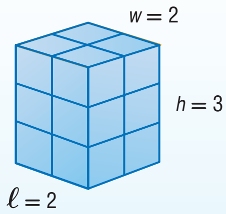
New Vocabulary

surface area

Key Concept — Surface Area

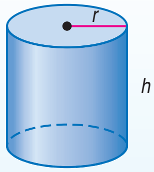
For Your FOLDABLE

Prism



$S = 2\ell w + 2\ell h + 2wh$

Cylinder

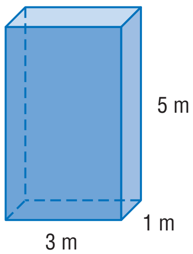


$S = 2\pi rh + 2\pi r^2$

EXAMPLE

Find the surface area of each solid. Round to the nearest tenth if necessary.

a.

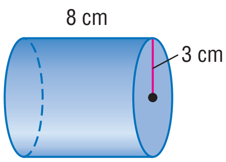


The prism has a length of 3 meters, width of 1 meter, and height of 5 meters.

$S = 2\ell w + 2\ell h + 2wh$	Surface area formula
$= 2(3)(1) + 2(3)(5) + 2(1)(5)$	$\ell = 3, w = 1, h = 5$
$= 6 + 30 + 10$	Multiply.
$= 46$	Add.

The surface area is 46 square meters.

b.



The height of the cylinder is 8 centimeters and the radius of the base is 3 centimeters. The surface area is the sum of the area of each base, $2\pi r^2$, and the area of the side, given by the circumference of the base times the height or $2\pi rh$.

$S = 2\pi rh + 2\pi r^2$	Formula for surface area of a cylinder.
$= 2\pi(3)(8) + 2\pi(3^2)$	$r = 3, h = 8$
$= 48\pi + 18\pi$	Simplify.
$\approx 207.3 \text{ cm}^2$	Use a calculator.

Exercises

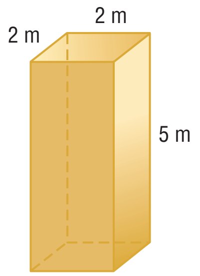
Find the surface area of each rectangular prism given the length, width, and height.

1. $\ell = 6$ in., $w = 1$ in., $h = 4$ in
2. $\ell = 8$ m, $w = 2$ m, $h = 2$ m
3. $\ell = 10$ mm, $w = 4$ mm, $h = 5$ mm
4. $\ell = 6.2$ cm, $w = 1$ cm, $h = 3$ cm
5. $\ell = 7$ ft, $w = 2$ ft, $h = \frac{1}{2}$ ft
6. $\ell = 7.8$ m, $w = 3.4$ m, $h = 9$ m

StudyTip

Alternate Method Another way to find the surface area of a solid is to draw the net of the solid on grid paper.

Find the surface area of each solid.

7.


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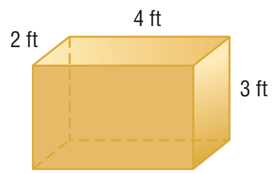

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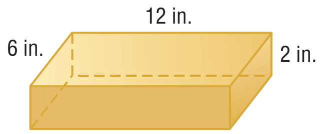

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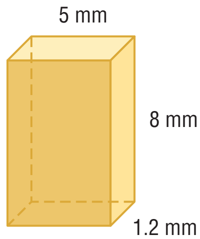

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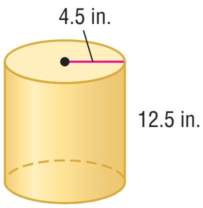

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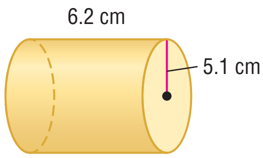


13. **GEOMETRY** What is the surface area of a cube with a side length of 2 meters?
14. **GIFTS** A gift box is in the shape of a rectangular prism 14 inches long, 5 inches wide, and 4 inches high. If the box is to be covered in fabric, how much fabric is needed if there is no overlap?
15. **BOXES** A new refrigerator is shipped in a box 34 inches deep, 66 inches high, and $33\frac{1}{4}$ inches wide. What is the surface area of the box in square feet? Round to the nearest square foot. (*Hint:* $1\text{ ft}^2 = 144\text{ in}^2$)
16. **PAINTING** A cabinet is 6 feet high, 3 feet wide, and 2 feet long. The entire outside surface of the cabinet is being painted except for the bottom. What is the surface area of the cabinet that is being painted?
17. **SOUP** A soup can is 4 inches tall and has a diameter of $3\frac{1}{4}$ inches. How much paper is needed for the label on the can? Round your answer to the nearest tenth.
18. **CRAFTS** For a craft project, Sarah is covering all the sides of a box with stickers. The length of the box is 8 inches, the width is 6 inches, and the height is 4 inches. If each sticker has a length of 2 inches and a width of 4 inches, how many stickers does she need to cover the box?

0-11 Simple Probability and Odds

Objective

Find the probability and odds of simple events.

KY Program of Studies

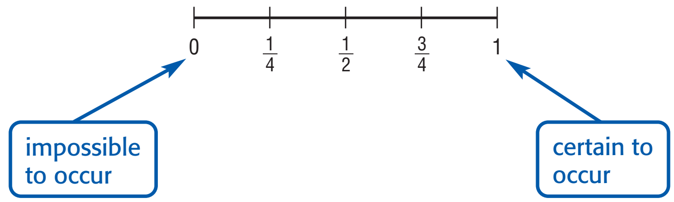
HS-DAP-S-P7 Students will explain how probability quantifies the likelihood that an event occurs in terms of numbers.

New Vocabulary

probability
sample space
equally likely
tree diagram
odds
complements
theoretical probability
experimental probability

The **probability** of an event is the ratio of the number of favorable outcomes for the event to the total number of possible outcomes. When you roll a die, there are six possible outcomes: 1, 2, 3, 4, 5, or 6. This list of all possible outcomes is called the **sample space**.

When there are n outcomes and the probability of each one is $\frac{1}{n}$, we say that the outcomes are **equally likely**. For example, when you roll a die, the 6 possible outcomes are equally likely because each outcome has a probability of $\frac{1}{6}$. The probabilty of an event is always between 0 and 1, inclusive. The closer a probability is to 1, the more likely it is to occur.



EXAMPLE 1

a. A die is rolled. Find the probability of rolling a 1 or 5.

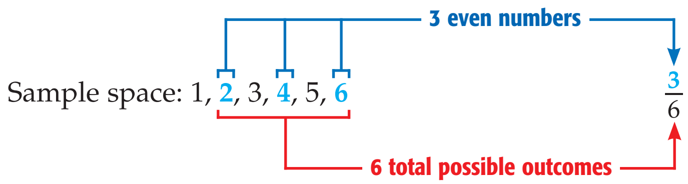
There are six possible outcomes. There are two favorable outcomes, 1 and 5.

$$\text{probability} = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}} = \frac{2}{6}$$

So, $P(1 \text{ or } 5) = \frac{2}{6}$ or $\frac{1}{3}$.

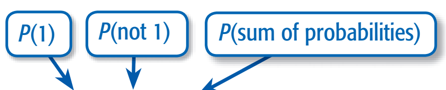
b. rolling an even number

Three of the six outcomes are even numbers. So, there are three favorable outcomes.



So, $P(\text{even number}) = \frac{3}{6}$ or $\frac{1}{2}$.

The events for rolling a 1 and for *not* rolling a 1 are called **complements**.



$$\frac{1}{6} + \frac{5}{6} = \frac{6}{6} \text{ or } 1$$

The sum of the probabilities for any two complementary events is always 1.

EXAMPLE 2

A bowl contains 5 red chips, 7 blue chips, 6 yellow chips, and 10 green chips. One chip is randomly drawn. Find each probability.

a. blue

There are 7 blue chips and 28 total chips.

$$P(\text{blue chip}) = \frac{7}{28} \quad \begin{array}{l} \leftarrow \text{number of favorable outcomes} \\ \leftarrow \text{number of possible outcomes} \end{array}$$

$$= \frac{1}{4}$$

The probability can be stated as $\frac{1}{4}$, 0.25, or 25%.

b. red or yellow

There are 5 + 6 or 11 chips that are red or yellow.

$$P(\text{red or yellow}) = \frac{11}{28} \quad \begin{array}{l} \leftarrow \text{number of favorable outcomes} \\ \leftarrow \text{number of possible outcomes} \end{array}$$

$$\approx 0.39$$

The probability can be stated as $\frac{11}{28}$, 0.39, or 39%.

c. not green

There are 5 + 7 + 6 or 18 chips that are not green.

$$P(\text{not green}) = \frac{18}{28} \quad \begin{array}{l} \leftarrow \text{number of favorable outcomes} \\ \leftarrow \text{number of possible outcomes} \end{array}$$

$$= \frac{9}{14} \text{ or about } 0.64$$

The probability can be stated as $\frac{9}{14}$, about 0.64, or about 64%.

StudyTip

Alternate Method A chip drawn will either be green or not green. So, another method for finding *P*(not green) is to find *P*(green) and subtract that probability from 1.

One method used for counting the number of possible outcomes is to draw a **tree diagram**. The last column of a tree diagram shows all of the possible outcomes.

EXAMPLE 3

School baseball caps come in blue, yellow, or white. The caps have either the school mascot or the school's initials. Use a tree diagram to determine the number of different caps possible.

Color	Design	Outcomes
blue	mascot	blue, mascot
	initials	blue, initials
yellow	mascot	yellow, mascot
	initials	yellow, initials
white	mascot	white, mascot
	initials	white, initials

The tree diagram shows that there are 6 different caps possible.

This example is an illustration of the **Fundamental Counting Principle**, which relates the number of outcomes to the number of choices.

Key Concept — Fundamental Counting Principle

For Your FOLDABLE

Words If event *M* can occur in *m* ways and is followed by event *N* that can occur in *n* ways, then the event *M* followed by *N* can occur in $m \cdot n$ ways.

Example If there are 4 possible sizes for fish tanks and 3 possible shapes, then there are $4 \cdot 3$ or 12 possible fish tanks.

EXAMPLE 4

a. An ice cream shop offers one, two, or three scoops of ice cream from among 12 different flavors. The ice cream can be served in a wafer cone, a sugar cone, or in a cup. Use the Fundamental Counting Principle to determine the number of choices possible.

There are 3 ways the ice cream is served, 3 different servings, and there are 12 different flavors of ice cream.

We can use the Fundamental Counting Principle to find the number of possible choices.

number of scoops		number of flavors		number of serving options		number of choices of ordering ice cream
3	·	12	·	3	=	108

So, there are 108 different ways to order ice cream.

b. Jimmy needs to make a password for his log-on name on a website. The password can be any digit 0-9 but the digits may not repeat. How many possible passwords are there?

If the first digit is a 4, then the next digit cannot be a 4.

We can use the Fundamental Counting Principle to find the number of possible passwords.

1st digit		2nd digit		3rd digit		number of passwords
10	·	9	·	8	=	720

So, there are 720 possible passwords.

The **odds** of an event occurring is the ratio that compares the number of ways an event can occur (successes) to the number of ways it cannot occur (failures).

StudyTip

Odds The sum of the number of successes and the number of failures equals the sample space, or the number of possible outcomes.

EXAMPLE 5

Find the odds of rolling a number less than 3.

There are six possible outcomes; 2 are successes and 4 are failures.

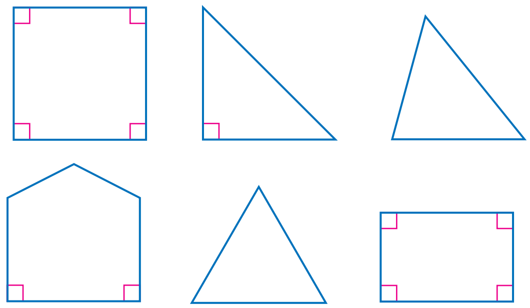
So, the odds of rolling a number less than 3 are $\frac{1}{2}$ or 1:2.

Exercises

One coin is randomly selected from a jar containing 70 nickels, 100 dimes, 80 quarters, and 50 one-dollar coins. Find each probability.

1. P(quarter)
2. P(dime)
3. P(quarter or nickel)
4. P(value greater than \$0.10)
5. P(value less than \$1)
6. P(value at most \$1)

One of the polygons below is chosen at random. Find each probability.

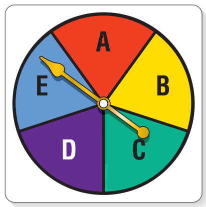


7. P(triangle)
8. P(pentagon)
9. P(not a quadrilateral)
10. P(more than 2 right angles)

StudyTip

Counting Outcomes When counting possible outcomes, make a column in your tree diagram for each part of the event.

Use a tree diagram to find the sample space for each event. State the number of possible outcomes.



11. The spinner at the right is spun and two coins are tossed.
12. At a restaurant, you have several choices of sides to have with breakfast. You can choose white or whole wheat toast. You can choose sausage links, sausage patties, or bacon.
13. How many different 3-character codes are there using A, B, or C for the first character, 8 or 9 for the second character, and 0 or 1 for the third character?

A bag is full of different colored marbles. The probability of randomly selecting a red marble from the bag is $\frac{1}{8}$. The probability of selecting a blue marble is $\frac{13}{24}$. Find each probability.

14. P(not red)
15. P(not blue)

Find the odds of each outcome if a computer randomly picks a letter in the name THE UNITED STATES OF AMERICA.

16. the letter A
17. the letter T
18. a vowel
19. a consonant

Margaret wants to order a sub at the local deli.

20. Find the number of possible orders of a sub with one topping and one dressing option.
21. Find the number of possible ham subs with mayonnaise, any combination of toppings or no toppings at all.
22. Find the number of possible orders of a sub with any combination of dressing and/or toppings.

Subs	
ham, salami, roast beef, turkey, bologna, pepperoni	
Dressing	**Toppings**
mayonnaise, mustard, vinegar, oil	lettuce, onions, peppers, olives

0-12 Mean, Median, Mode, Range, and Quartiles

Objective
Calculate the measures of central tendency of a set of data.

Vocabulary
- measures of central tendency
- mean
- median
- mode
- measures of variation
- range
- quartiles
- lower quartile
- upper quartile

Measures of central tendency are numbers used to represent a set of data. Three types of measures of central tendency are mean, median, and mode. The **mean** is the sum of the numbers in a set of data divided by the number of items.

EXAMPLE 1

Katherine is running a lemonade stand. She made $3.50 on Tuesday, $4.00 on Wednesday, $5.00 on Thursday, and $4.50 on Friday. What was her mean daily profit?

$$\text{mean} = \frac{\text{sum of daily profits}}{\text{number of days}}$$

$$= \frac{\$3.50 + \$4.00 + \$5.00 + \$4.50}{4}$$

$$= \frac{\$17.00}{4} \text{ or } \$4.25$$

Katherine's mean daily profit was $4.25.

The **median** is the middle number in a set of data when the data are arranged in numerical order. If there is an even number of data, the median is the mean of the two middle numbers.

EXAMPLE 2

The table shows the number of hits Marcus made for his team. Find the median of the data.

Team Played	Number of Hits by Marcus
Badgers	3
Hornets	6
Bulldogs	5
Vikings	2
Rangers	3
Panthers	7

To find the median, order the numbers from least to greatest. The median is in the middle.

2, 3, 3, 5, 6, 7

$$\frac{3 + 5}{2} = 4$$

There is an even number of items. Find the mean of the middle two.

The median number of hits is 4.

The **mode** is the number or numbers that appear most often in a set of data. If no item appears most often, the set has no mode.

EXAMPLE 3

The table shows the heights in inches of the members of a college men's basketball team. What is the mode of the heights?

Men's Basketball Team				
74	78	79	80	78
72	81	83	76	78
76	75	77	79	72

78 occurs three times. 72, 76, and 79 each occur twice. All the other heights occur once.

Since 78 occurs most frequently, the mode height is 78.

You can use measures of central tendency to solve problems.

EXAMPLE 4

SCHOOL **On her first five history tests, Yoko received the following scores: 82, 96, 92, 83, and 91. What test score must Yoko earn on the sixth test so that her average (mean) for all six tests is 90?**

$$\text{mean} = \frac{\text{sum of the first five scores} + \text{sixth score}}{\text{total number of tests}}$$ **Write an equation.**

$$90 = \frac{82 + 96 + 92 + 83 + 91 + x}{6}$$ **Use *x* to represent the sixth score.**

$$90 = \frac{444 + x}{6}$$ **Simplify.**

$$540 = 444 + x$$ **Multiply each side by 6.**

$$96 = x$$ **Subtract 444 from each side.**

To have an average score of 90, Yoko must earn a 96 on the sixth test.

Measures of variation are used to describe the distribution of the data. One measure, the difference between the greatest and the least data values, is called the **range** of the temperatures.

StudyTip

Describing Data The measures of variation including range describe how the data in a set vary. This is another way to describe data.

EXAMPLE 5

The times in minutes it took Olivia to walk to school each day this week are 18, 15, 15, 12, and 14. Find the range of the times.

$\text{range} = \text{greatest value} - \text{least value}$ **Write an equation.**

$= 18 - 12 \text{ or } 6$ **The greatest value is 18, and the least value is 12.**

The range of the times is 6 minutes.

In a set of data, the **quartiles** are values that separate the data into four equal subsets, each containing one fourth of the data. Q_1, Q_2, and Q_3 are used to represent the three quartiles. Q_1 is the **lower quartile**. It divides the lower half of the data into two equal parts. Q_2 is the median since it separates the data into two equal parts. Q_3 is the **upper quartile**. It divides the upper half of the data into two equal parts.

EXAMPLE 6

Find the median, lower quartile, and upper quartile of the data shown below.

22, 16, 35, 26, 14, 17, 28, 29, 21, 17, 20

Order the data from least to greatest. Then use the list to determine the quartiles.

14, 16, 17, 17, 20, 21, 22, 26, 28, 29, 35

Q_1 = 17, Q_2 = 21, Q_3 = 28

The median (Q_2) is 21, the lower quartile (Q_1) is 17, and the upper quartile (Q_3) is 28.

Exercises

> **StudyTip**
>
> **Calculating Measures of Central Tendency** Remember to order the data from least to greatest before finding the median of the data.

Find the mean, median, and mode for each set of data.

1. {1, 2, 3, 5, 5, 6, 13}
2. {3, 5, 8, 1, 4, 11, 3}
3. {52, 53, 53, 53, 55, 55, 57}
4. {8, 7, 5, 19}
5. {3, 11, 26, 4, 1}
6. {201, 201, 200, 199, 199}
7. {4, 5, 6, 7, 8}
8. {3, 7, 21, 23, 63, 27, 29, 95, 23}

Find the range, median, lower quartile, and upper quartile for each set of data.

9. {4, 7, 11, 19, 26, 26, 32}
10. {62, 65, 67, 68, 73, 80, 81, 83, 99}
11. {17, 9, 10, 17, 18, 5, 2}
12. {33, 38, 29, 25, 41, 40}
13. {10, 9, 8, 7, 6, 5, 4}
14. {111, 109, 112, 114, 119, 112}
15. **SCHOOL** The table shows the cost of some school supplies. Find the mean, median, and mode costs.

Cost of School Supplies	
Supply	**Cost**
pencils	\$0.50
pens	\$2.00
paper	\$2.00
pocket Folder	\$1.25
calculator	\$5.25
notebook	\$3.00
eraser	\$2.50
markers	\$3.50

16. **NUTRITION** The table shows the number of servings of fruits and vegetables that Cole eats one week. Find the range, median, lower quartile, and upper quartile of the data.

Fruit and Vegetable Servings	
Day	**Number of Servings**
Monday	5
Tuesday	7
Wednesday	5
Thursday	4
Friday	3
Saturday	3
Sunday	8

17. **SCHOOL** Bill's scores on his first four science tests are 86, 90, 84, and 91. What test score must Bill earn on the fifth test so that his average (mean) will be exactly 88?
18. **BOWLING** Sue's average for 9 games of bowling is 108. What is the lowest score she can receive for the tenth game to have an average of 110?
19. **SCHOOL** Olivia has an average score of 92 on five French tests. If she earns a score of 96 on the sixth test, what will her new average score be?
20. **JOBS** The number of hours Maria and her friends each work at their part-time jobs is 20, 10, 8, 5, 25, 12 and 10 hours. Find the average amount of time Maria and her friends work at their jobs. Round to the nearest hour.
21. **MOVIES** At a movie theater, ten movies are playing and their lengths are 105, 95, 115, 120, 150, 130, 100, 125, 110, and 135 minutes. Find the average length of a movie playing at this theater. Round to the nearest tenth.
22. **BASKETBALL** The heights of players of a girls' basketball team are shown. Find the average height of the team. Round to the nearest tenth.

Height of Players (in.)			
72	71	69	66
62	70	64	69
67	65	65	70

0-13 Representing Data

Objective

To represent sets of data using different visual displays.

KY Program of Studies

HS-DAP-S-DR2 Students will apply histograms, parallel box plots and scatterplots **to display data.**

New Vocabulary

frequency table
bar graph
histogram
line graph
stem-and-leaf plot
circle graph
box-and-whisker plot
interquartile range
outliers

Data can be displayed and organized by different methods. In a **frequency table**, you use tally marks to record and display the frequency of events. A **bar graph** compares different categories of data by showing each as a bar with a length that is related to the frequency.

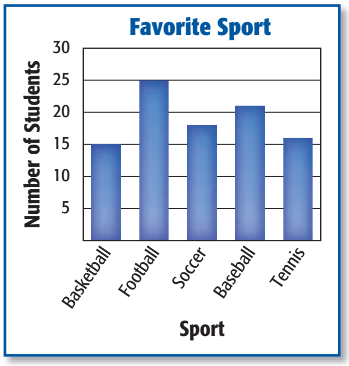
EXAMPLE 1

The frequency table shows the results of a survey of students' favorite sports. Make a bar graph to display the data.

Sport	Tally	Frequency
basketball	||||/ ||||/ ||||/	15
football	||||/ ||||/ ||||/ ||||/ ||||/	25
soccer	||||/ ||||/ ||||/ |||	18
baseball	||||/ ||||/ ||||/ ||||/		21
tennis	||||/ ||||/ ||||/		16

Step 1 Draw a horizontal axis and a vertical axis. Label the axes as shown. Add a title.

Step 2 Draw a bar to represent each sport. The vertical scale is the number of students who chose each sport. The horizontal scale identifies the sport chosen.



A **histogram** is a type of bar graph used to display numerical data that have been organized into equal intervals.

EXAMPLE 2

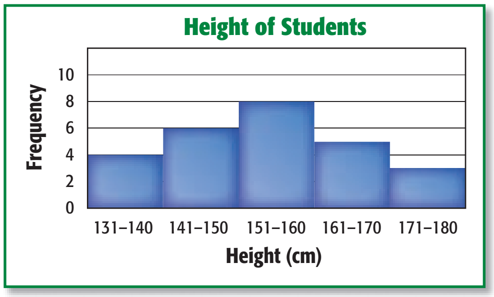
The frequency table shows the heights of students in a class. Make a histogram of the data.

Heights of Students		
Height (cm)	**Tally**	**Frequency**
131–140	||||	4
141–150	||||/		6
151–160	||||/ |||	8
161–170	||||/	5
171–180	|||	3

Step 1 Draw and label a horizontal and vertical axis. Include a title.

Step 2 Show the intervals from the frequency table on the horizontal axis.

Step 3 For each height interval, draw a bar whose height is given by the frequencies. There is no space between the bars.



Another way to represent data is by using a line graph. A **line graph** usually shows how data change over a period of time.

EXAMPLE 3

Sales at the Marshall High School Store are shown in the table. Make a line graph of the data.

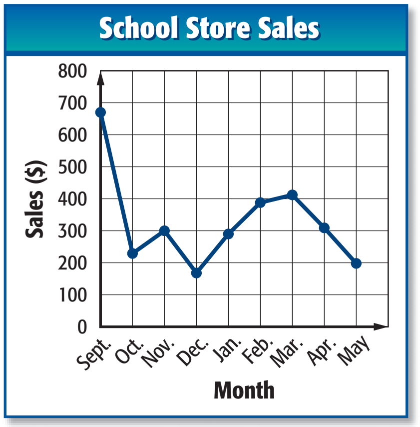
School Store Sales Amounts

September	$670	**December**	$168	**March**	$412
October	$229	**January**	$290	**April**	$309
November	$300	**February**	$388	**May**	$198

Step 1 Draw a horizontal axis and a vertical axis and label them as shown. Include a title.

Step 2 Plot the points to represent the data.

Step 3 Draw a line connecting each pair of consecutive points.



Data can also be organized and displayed by using a stem-and-leaf plot. In a **stem-and-leaf plot**, the greatest common place value is used for the *stems*. The numbers in the next greatest place value are used to form the *leaves*.

EXAMPLE 4

The speeds (mph) of 20 of the fastest land animals are listed at the right. Use the data to make a stem-and-leaf plot.

42	40	40	35	50
32	50	36	50	40
45	70	43	45	32
40	35	61	48	35

Source: *The World Almanac*

The greatest place value is tens. So, 32 miles per hour would have a stem of 3 and a leaf of 2.

Stem	Leaf
3	2 2 5 5 5 6
4	0 0 0 0 2 3 5 5 8
5	0 0 0
6	1
7	0

Key: 3/2 = 32

Real-World Link

The fastest animal on land is the cheetah. Cheetahs can run at speeds up to 60 miles per hour.

Source: Infoplease

A **circle graph** is a graph that shows the relationship between parts of the data and the whole. The circle represents all of the data.

EXAMPLE 5

The table shows how Lily spent 8 hours of one day at summer camp.

Summer Camp	
Activity	**Hours**
canoeing	3
crafts	1
eating	2
hiking	2

First, find the ratio that compares the number of hours for each activity to 8. Then multiply each ratio by 360 to find the number of degrees for each section of the graph.

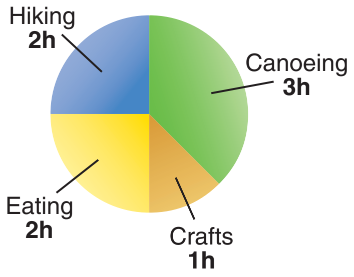
Canoeing: $\frac{3}{8} \cdot 360° = 135°$

Crafts: $\frac{1}{8} \cdot 360° = 45°$

Eating: $\frac{2}{8} \cdot 360° = 90°$

Hiking: $\frac{2}{8} \cdot 360° = 90°$

Summer Camp



StudyTip

Interquartile Range When the interquartile range is a small value, the data in the set are close together. A large interquartile range means that the data are spread out.

Data can be organized and displayed by dividing a set of data into four parts using the median and quartiles. This is a **box-and-whisker plot**. The box in a box-and-whisker plot represents the interquartile range. The **interquartile range** is the difference between the upper and lower quartiles. Data that are more than 1.5 times the value of the interquartile range beyond the quartiles are called **outliers**.

EXAMPLE 6

Draw a box-and-whisker plot for these data.

14, 30, 16, 20, 18, 16, 20, 18, 22, 13, 8

Step 1 Order the data from least to greatest. Then determine the quartiles.

8, 13, **14**, 16, 16, **18**, 18, 20, **20**, 22, 30

Q_1 (14), Q_2 (18), Q_3 (20)

Determine the interquartile range.

$$\text{IQR} = Q_3 - Q_1$$
$$= 20 - 14 \text{ or } 6$$

Check to see if there are any outliers.

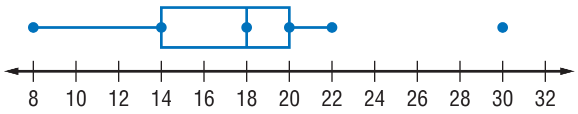
$14 - 1.5(6) = 5$ $\quad$ $20 + 1.5(6) = 29$

Numbers less than 5 or greater than 29 are outliers.

The only outlier is 30.

Step 2 Draw a number line that includes the least and greatest numbers in the data. Place bullets above the number line to represent the three quartile points, any outliers, the least number that is not an outlier, and the greatest number that is not an outlier.

Step 3 Draw the box and the whiskers. The vertical rules go through the quartiles.



Exercises

StudyTip

Best Representation A set of data can be represented by several different displays. There is usually one type of graph that is best for displaying the data.

1. SURVEYS Alana surveyed several students to find the number of hours of sleep they typically get each night. The results are shown in the table. Make a bar graph of the data.

Hours of Sleep					
Alana	8	Kwam	7.5	Tomas	7.75
Nick	8.25	Kate	7.25	Sharla	8.5

2. PLAYS The frequency table at the right shows the ages of people attending a high school play. Make a histogram to display the data.

Age	Tally	Frequency
10–19	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	47
20–39	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |||	43
40–59	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	31
60–79	𝍸 |||	8

3. LAWN CARE Marcus started a lawn care service. The chart shows how much money he made over the first 8 weeks of summer break. Make a line graph of the data.

Lawn Care Profits ($)								
Week	1	2	3	4	5	6	7	8
Profit	25	40	45	50	75	85	95	95

Use each set of data to make a stem-and-leaf plot and a box-and-whisker plot.

4. {65, 63, 69, 71, 73, 59, 60, 70, 72, 66, 71, 58}

5. {31, 30, 28, 26, 22, 34, 26, 31, 47, 32, 18, 33, 26, 23, 18}

6. MONEY The table shows how Ping spent his allowance of $40. Make a circle graph of the data.

Allowance	
How Spent	**Amount ($)**
savings	15
downloaded music	8
snacks	5
T-shirt	12

7. JOGGING The table shows the number of miles Hannah jogged each day for 10 days. Make a line graph of the data.

Day	1	2	3	4	5	6	7	8	9	10
Miles Jogged	2	2	3	3.5	4	4.5	2.5	3	4	5

CHAPTER 0 Posttest

For each problem, determine whether you need an estimate or an exact answer. Then use the four-step problem-solving plan to solve.

1. **DISTANCE** Luis rode his bike 1.2 miles to his friend's house, then 0.7 mile to the video store, then 1.9 miles to the library. If he rode the same route back home, about how far did he travel in all?

2. **SHOPPING** The regular price of a T-shirt is $9.99. It is on sale for 15% off. Sales tax is 6%. If you give the cashier a $10 bill, how much change will you receive?

Find each sum or difference.

3. $-31 + (-4)$
4. $48 - 55$
5. $-71 - (-10)$
6. $31 - 42.9$
7. $-11.5 + 8.1$
8. $-0.38 - (-1.06)$

Find each product or quotient.

9. $-21(-5)$
10. $-81 \div (-3)$
11. $-120 \div 8$
12. $-39 \div -3$

Replace each ● with <, >, or = to make a true sentence.

13. -0.62 ● $-\frac{6}{7}$
14. $\frac{12}{44}$ ● $\frac{8}{11}$
15. Order $4\frac{4}{5}$, 4.85, $2\frac{5}{8}$, and 2.6 from least to greatest.

Find each sum or difference. Write in simplest form.

16. $\frac{1}{7} + \frac{5}{7}$
17. $\frac{7}{8} - \frac{1}{8}$
18. $\frac{1}{6} + \left(-\frac{1}{2}\right)$
19. $-\frac{1}{12} - \left(-\frac{3}{4}\right)$

Find each product or quotient.

20. $-1.2(9.3)$
21. $-20.93 \div (-2.3)$
22. $10.5 \div (-1.2)$
23. $(-3.4)(-2.8)$

Name the reciprocal of each number.

24. 6
25. $1\frac{2}{5}$
26. $-2\frac{3}{7}$
27. $-\frac{1}{2}$
28. $\frac{4}{3}$
29. $5\frac{1}{3}$

Find each product or quotient. Write in simplest form.

30. $\frac{2}{5} \cdot \frac{5}{9}$
31. $\frac{4}{5} \div \frac{1}{5}$
32. $-\frac{7}{8} \cdot 2$
33. $\frac{1}{3} \div 2\frac{1}{4}$
34. $-6 \cdot \left(-\frac{3}{4}\right)$
35. $\frac{7}{18} \div \left(-\frac{14}{15}\right)$

36. **PICNIC** Joseph is mixing $5\frac{1}{2}$ gallons of orange drink for his class picnic. Every $\frac{1}{2}$ gallon requires 1 packet of orange drink mix. How many packets of orange drink mix does Joseph need?

Express each percent as a fraction in simplest form.

37. 6%
38. 140%

Use the percent proportion to find each number.

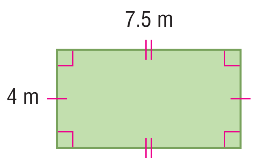
39. 50% of what number is 31?
40. What number is 110% of 51?
41. Find 8% of 95.

42. **SOLUTIONS** A solution is prepared by dissolving 24 milliliters of saline in 150 milliliters of pure solution. What is the percent of saline in the pure solution?

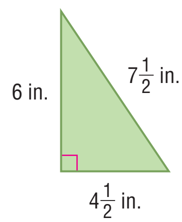
43. **SHOPPING** Marta got 60% off a pair of shoes. If the shoes cost $9.75 (before sales tax), what was the original price of the shoes?

Find the perimeter and area of each figure.

44.

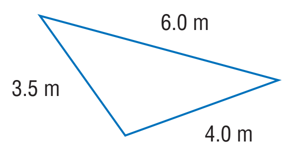


45.

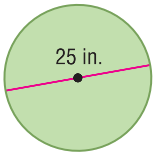


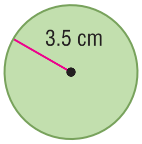
46. A parallelogram has a base of 20 millimeters and a height of 6 millimeters. Find the area.

47. **GARDENS** Find the perimeter of the garden.



Find the circumference and area of each circle. Round to the nearest tenth.

48. 

49. 

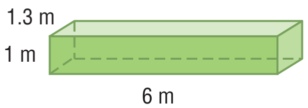
50. **PARKS** A park has a circular area for a fountain that has a circumference of about 16 feet. What is the radius of the circular area? Round to the nearest tenth.

Find the volume and surface area of each rectangular prism given the length, width, and height.

51. $\ell = 1.5$ m, $w = 3$ m, $h = 2$ m

52. $\ell = 4$ in., $w = 1$ in., $h = \frac{1}{2}$ in.

53. Find the volume and surface area of the rectangular prism.



One marble is randomly selected from a jar containing 3 red, 4 green, 2 black, and 6 blue marbles. Find each probability.

54. P(red or blue)

55. P(green or red)

56. P(not black)

57. P(not blue)

58. **Use a tree diagram to find the sample space for the event. State the number of possible outcomes.**

A movie theater is offring snack specials. You can choose a small, medium, large, or jumbo popcorn with or without butter, and soda or bottled water.

One coin is randomly selected from a jar containing 20 pennies, 15 nickels, 3 dimes, and 12 quarters. Find the odds of each outcome. Write in simplest form.

59. a dime

60. a value less than \$0.25

61. a value greater than \$0.10

62. a value less than \$0.05

63. **SCHOOL** In a science class, each student must choose a lab project from a list of 15, write a paper on one of 6 topics, and give a presentation about one of 8 subjects. How many ways can students choose to do their assignments?

64. **GAMES** Marcos has been dealt seven different cards. How many different ways can he play his cards if he is required to play one card at a time?

Find the mean, median, and mode for each set of data.

65. {99, 88, 88, 92, 100}

66. {30, 22, 38, 41, 33, 41, 30, 24}

67. Find the range, median, lower quartile, and upper quartile for {77, 75, 72, 70, 79, 77, 70, 76}.

68. **TESTS** Kevin's scores on the first four science tests are 88, 92, 82, and 94. What score must he earn on the fifth test so that the mean will be 90?

69. **FOOD** The table shows the results of a survey in which students were asked to choose their favorite food. Make a bar graph of the data.

Favorite Foods

Food	Number of Students
pizza	15
chicken nuggets	10
cheesy potatoes	8
ice cream	5

70. Make a box-and-whisker plot of the following data: 26, 18, 26, 29, 18, 20, 35, 32, 31, 24, 26, and 22.

71. **BUDGET** The table shows how Kat spends her allowance. Make a circle graph of the data.

Category	Amount (\$)
Savings	25
Clothes	10
Entertainment	15